

An energy-based approach for estimates of the stress-strain fields near crack-like notches

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Abstract

Estimates of the elastic-plastic stress-strain distribution near cracks and crack-like notches under maximum load and in plane stress were made using a modified form of the Equivalent Strain Energy Density Hypothesis (here denoted as the GM approach). According to this hypothesis both, the elastic W_S and the elastic-plastic W_σ strain energy densities, have approximately the same value in the region ahead of a crack/crack-like notch. In the present paper, the W_S was estimated integrating the stress-strain relations in the linear-elastic regime. Using the classical linear elastic fracture mechanics, the principal stresses in the solution for W_S were left in terms of the well known stress intensity factor. This parameter is believed to be the characteristic variable for crack-like notches. On the other hand, the W_σ was estimated by integration of the bi-axial constitutive relation for a strain-hardening material using the deformation plasticity theory. Equating the expressions for both energies, an implicit relation for the maximum stress (σ_{1max}) as a function of the distance from the crack tip (r) and the ratio between the two non-zero principal stresses (λ), was obtained. A computer code was developed to numerically solve this implicit relation for σ_{1max} at discrete values of r and for $0.2 \leq \lambda \leq 1$. One fixed K value and three structural materials were considered for the simulation. The resultant matrix of estimated values for maximum stress field within the plastic zone is presented in various non-dimensional plots. The λ parameter did not exhibit a strong influence in the crack/notch stress-strain tip fields. For comparison, the classical HRR solution for elastic-plastic stress fields in cracks was also plotted in the same graph and good agreement was found with the GM approach. The results can be easily extended to cyclic loading for using in fatigue design.

Keywords: strain energy density, linear elastic fracture mechanics, crack-like notches.

1 Introduction

Fracture and fatigue behavior of structures with cracks and notches primarily depends on the stress-strain fields in the vicinity of these discontinuities. For cracks and in small scale yielding conditions, the crack tip stress-strain fields according to the classical Linear Elastic Fracture Mechanics (LEFM) do control the fatigue and fracture process, as has been amply demonstrated by the practice of engineering

in the last decades. This happens in spite of the existence of a small volume of material in front of the crack (plastic zone) where the stresses are well above the yield point.

The analysis of stress in notches, needed almost exclusively for fatigue design purposes, is performed with the help of the fatigue stress concentration factor k_f . Again the material behavior is assumed to be linear elastic and the little plasticity, if present, is believed to be confined at a pretty small region that it can be neglected. By increasing the nominal stress by a factor of k_f (or by decreasing the strength) it is possible to obtain the characteristic variable (maximum local stress) at the tip of the notch.

Despite the success of the above engineering solutions, many attempts were and are made to find and use in design the stress-strain fields within the plastic zones. A closed form solution for cracks was developed by Hutchinson [1] and Rice & Rosengren [2] and named *HRR* solution since then. For notches, approximate methods were also developed, the two most used being due to the works of Neuber [3] and Glinka [4]. Both of the approaches, however, return only scalar values for stress and strain at the tip of the notch and not the stress-strain fields ahead of the notch. In many cases the stress-strain field is an important parameter. For example, the fatigue limit in sharp notches, according to the line method [5], is believed to be the average stress range over certain critical distance, also called the process zone. The present paper deals with the analogies between crack and notches so as to develop a new method to estimate the monotonic stress-strain fields near crack-like notches in plane stress and on the crack plane.

2 Approximate methods to calculate inelastic stress and strain at the tip of notched members (for elastic nominal stress only)

Notched engineering members are often subjected to nominal stresses that causes localized yielding. For elastic behavior the values of local strain and stress (ϵ , σ) can be estimated on the basis of the theoretical stress (the same as strain) concentration factor k_t . At the notch tip a uniaxial state of stress exists. For this reason, after yielding, i.e., when the product $k_t \cdot \sigma_n > S_y$, where σ_n is the nominal stress and S_y the yield strength, the localized stresses and strains at the notch tip continue to vary according to the monotonic stress-strain response of the material, which means that the stress diminishes and the strain grows. In these cases it is necessary to define separate stress k_σ and strain k_ϵ concentration factors as follows:

$$k_\sigma = \frac{\sigma}{\sigma_n}, \quad k_\epsilon = \frac{\epsilon}{\epsilon_n} \quad (1)$$

As in the case of stress, ϵ_n is the nominal strain. Some approximate methods have been developed for estimating these notch stresses and strains. Among them, Neuber [3] and Glinka [4] rules are the most widely used and are well described in textbooks like Dowling [6]. Neuber's rule can be mathematically represented by the following equation:

$$k_t = \sqrt{k_\sigma k_\epsilon} \quad (2)$$

For localized yielding and uniaxial state of stress $\epsilon_n = \sigma_n/E$, where E is the elasticity modulus, applies. One equation relating the product of local stress and strain and the nominal stress times the stress concentration factor can be obtained substituting this relation in Eq. (1) and the result in Eq. (2):

$$k_t = \sqrt{k_\sigma k_\epsilon} = \sqrt{\frac{\sigma}{\sigma_n} \frac{\epsilon}{\epsilon_n}} = \sqrt{\frac{\sigma}{\sigma_n} \frac{E \epsilon}{\sigma_n}} \quad (3)$$

$$\sigma \epsilon = \frac{(k_t \sigma_n)^2}{E}$$

Equation (3) means that, knowing few properties of materials (E , H , n), (being H and n the strength coefficient and the strain hardening exponent, respectively), the true stress-strain curve, the applied load and the theoretical stress concentration factor, it is possible to obtain a solution for the local stress and strain at the tip of the notch (σ_{tip} and ϵ_{tip}). In $\epsilon\sigma$ coordinates, Eq. (3) is a hyperbola ($(k_t \cdot \sigma_n)^2/E$ is a constant) whose intersection with equations that model the monotonic or cyclic stress-strain behavior of material, Ramberg-Osgood $\epsilon = \sigma/E + (\sigma/H)^{(1/n)}$ e.g., returns the desired values stress and strain at the notch tip.

On the other hand, Glinka's approach [4] is based on the equivalence between the plastic W_σ and the elastic strain energy density W_S at the notch tip and can be mathematically enunciated as in Eq. (4):

$$\frac{(k_t \sigma_n)^2}{2E} = \frac{\sigma^2}{2E} + \frac{\sigma}{n+1} \left(\frac{\sigma}{H}\right)^{\frac{1}{n}} \quad (4)$$

Using some type of iteration or numerical method it is possible to solve Eq. (4) for local stress at the notch tip σ_{tip} . Local strains ϵ_{tip} are then obtained by substituting the local stress in the stress-strain curve of the material. This method generally gives estimates of local strains and stresses lower than those obtained by Neuber's rule. It is worthy to mention that both, Neuber's and Glinka's methods, in the form of Eq. (3) and Eq. (4) are only valid while the nominal stress σ_n is under the yield strength S_y of material. It should be emphasized that the results corresponds only to the tip of notches.

3 Modifications proposed to Glinka's method to calculate inelastic stress-strain fields at notched members

Moving into the interior of the notch the state of stress becomes biaxial or triaxial. Suppose that we are only interested in the crack plane and in the surface of the solid, thereby dealing with plane stress. Some modifications are necessary to Glinka's approach in order to deal with this bi-axial stress state. The elastic strain energy density W_S in terms of principal stresses and in plane stress is [7]:

$$W_S = \frac{1}{2E} (\sigma_{1e}^2 + \sigma_{2e}^2 - 2\nu \sigma_{1e} \sigma_{2e}) \quad (5)$$

The subscript e in Eq. (5) denotes an elastic stress. For simplicity's sake we have considered that the non-zero components of principal stresses are σ_1 and σ_2 . To express W_S as a function of the

distance r from the notch tip we recall that, for crack-like notches, close relationship exists between stress concentration analysis and crack analysis, the latter realized by means of fracture mechanics [8]. In fact, on the crack/notch plane ($\theta = 0$) and in accordance with the classical Irwin's solution [9] for the stress tensor at the crack tip, $\tau_{xy} = 0$ and both σ_x and σ_y are the same and also main stresses ($\sigma_x = \sigma_y = \sigma_{1e} = \sigma_{2e}$). For maximum load Eq. 5 then becomes:

$$W_S = \frac{(1 - \nu)}{E} \sigma_{1e}^2 \quad (6)$$

The plastic zone size in $\theta = 0$ for materials with a power-hardening stress-strain curve was derived (from *HRR* solution) by Rice [10] and Schwalbe [11]:

$$r_{y_sh} = \frac{1}{(1 + n) \pi} \left(\frac{K}{S_y} \right)^2 \quad (7)$$

In Eq. (7) K is the well known stress intensity factor of fracture mechanics. As r_{y_sh} by Eq. (7) is $2/(1 + n)$ times the plastic zone size r_y deducted from Irwin's solution $r_y = 1/(2\pi) \cdot (K/S_y)^2$, the elastic stress field near a crack tip and consequently the elastic strain energy density should be increased by a factor of $\sqrt{2/(1 + n)}$:

$$W_S = \frac{2}{(1 + n)} \frac{(1 - \nu)}{E} \sigma_{1e}^2 (K, r) \quad (8)$$

For now let the W_S as a function of σ_{1e} which in turn is a function of K and r . According to the strain energy density hypothesis *SEDH*, in small scale yielding, the strain energy density inside the notch/crack plastic zone W_σ can be calculated on the basis of a linear elastic analysis, it means $W_S = W_\sigma$. The term W_σ is the area under the strain-stress curve, provided this curve represents the true behavior of material under the actual state of stress. For plane stress, proportional loading (which means that principal stresses maintain constant ratios) and using the deformation plasticity theory, a useful relationship between the elastic-plastic principal strain ϵ_1 and the elastic-plastic principal stress σ_1 can be obtained, for a Ramberg-Osgood type stress-strain curve of material, as described in Dowling [6]:

$$\epsilon_1 = (1 - \lambda \nu) \frac{\sigma_1}{E} + (1 - 0.5 \lambda) \left(1 - \lambda + \lambda^2 \right)^{\frac{1-n}{2n}} \left(\frac{\sigma_1}{H} \right)^{\frac{1}{n}} \quad (9)$$

The parameter λ in Eq. (9) is defined as the ratio between principal (in plane) stresses $\lambda = \sigma_2/\sigma_1$. Figure 1 shows an example of the estimates effects that different λ ratios have on the stress-strain curve of a steel with $E = 207 \text{ GPa}$, $H = 1655 \text{ MPa}$ and $n = 0.131$. In this case, only for maximum clarity purposes in the graph, the range of λ has been defined between $-1 \leq \lambda \leq 1$.

Each point in the plastic zone (at different distances r from the notch/crack tip) will behave according to Eq. (9) which is shown in Figure 2 as function of r . Also shown in the figure is the area corresponding to the elastic-plastic strain energy density W_σ . The available expression for the stress-strain curve is in the form $\epsilon_1 = f(\sigma_1)$, so we need to calculate first the complementary strain energy density W_σ^* by simple integration:

$$W_{\sigma^*} = \int_0^{\sigma_{1\max}(r)} \varepsilon_1(r) d\sigma_1(r) \tag{10}$$

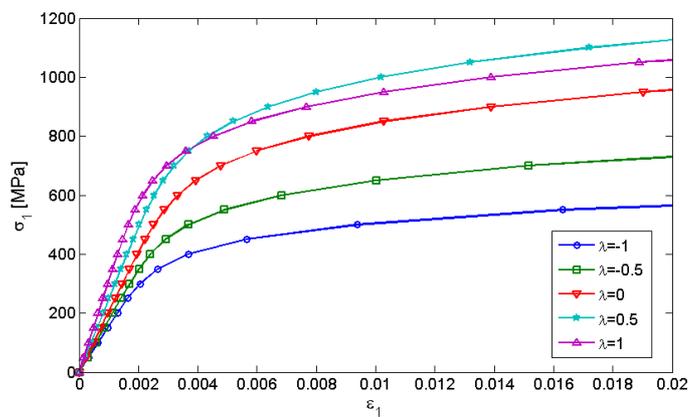


Figure 1: Effect of the ratio between principal stresses $\lambda = \sigma_2/\sigma_1$ on a Ramberg-Osgood stress-strain curve according to Eq. (9) for a steel ($\nu \cong 1/3$) with $E = 207 \text{ GPa}$, $H = 1655 \text{ MPa}$ and $n = 0.131$.

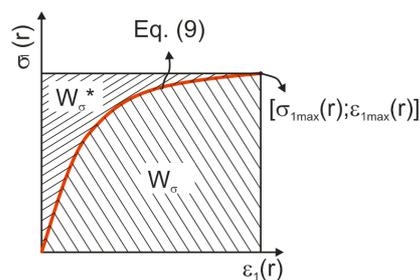


Figure 2: Elastic-plastic strain energy density W_{σ} and the complementary energy W_{σ^*} at each point (r fixed) within the plastic zone of a crack/notch. The elastic-plastic behavior of material is in accordance with the deformation plasticity theory.

Since proportional loading has been assumed $\lambda = \text{constant}$, the stress σ_2 grows in the same proportion as does σ_1 between zero and σ_{1max} . By substituting Eq. (9) into Eq. (10) and performing the integration comes up to:

$$W_{\sigma^*} = (1 - \lambda\nu) \frac{\sigma_{1max}^2(r)}{2E} + \frac{n(1 - 0.5\lambda)(1 - \lambda + \lambda^2)^{\frac{1-n}{2n}}}{(1+n)H^{1/n}} \sigma_{1max}^{\frac{1}{n}+1}(r) \quad (11)$$

From Figure 2 the total area for maximum load is:

$$\sigma_{1max}(r) \cdot \varepsilon_{1max}(r) = W_{\sigma} + W_{\sigma^*} \quad (12)$$

In Eq. (12) $\varepsilon_{1max}(r) = \varepsilon_{1max}(\sigma_{1max}(r))$ from Eq. (9). Performing the required substitutions and manipulating gives the desired elastic-plastic strain energy density for plane stress:

$$W_{\sigma} = \frac{(1 - \lambda\nu)}{2E} \sigma_{1max}^2(r) + \frac{(1 - 0.5\lambda)(1 - \lambda + \lambda^2)^{\frac{1-n}{2n}} \sigma_{1max}(r)}{(1+n)} \left(\frac{\sigma_{1max}(r)}{H} \right)^{\frac{1}{n}} \quad (13)$$

Equation (13) reduces, as expected, to the right hand side of Eq. (4) for the uniaxial state of stress ($\lambda = 0$) existent at the crack/notch tip ($r = 0$). Equating the Eq. (8) with Eq. (13) it is possible to obtain, for maximum load, an expression for the principal elastic-plastic stress-strain field $\sigma_{1max}(r)$ within the plastic zone:

$$\begin{aligned} \frac{2}{(1+n)} \frac{(1-\nu)}{E} \sigma_{1e}^2(K, r) &= \frac{(1-\lambda\nu)}{2E} \sigma_{1max}^2(r) + \dots \\ \dots + \frac{(1-0.5\lambda)(1-\lambda+\lambda^2)^{\frac{1-n}{2n}} \sigma_{1max}(r)}{(1+n)} &\left(\frac{\sigma_{1max}(r)}{H} \right)^{\frac{1}{n}} \end{aligned} \quad (14)$$

A non-singular expression for $\sigma_{1e}(K, r)$ which, in turn, depends on the applied nominal stress, is needed. The following section is dedicated to this problem.

4 Non-singular elastic stress fields on the crack plane

The elastic singularity of Irwin's solution was completely resolved by Creager & Paris's solution [12]:

$$\begin{aligned} \sigma_y(r)|_{\theta=0} &= \frac{K}{\sqrt{2\pi r}} \left[1 + \frac{\rho}{2r} \right] \\ \sigma_{tip} = k_t \cdot \sigma_n &= \sigma_y|_{r=\rho/2} = \frac{2K}{\sqrt{\pi\rho}} \end{aligned} \quad (15)$$

This equation, however, depends on the notch/crack tip radius ρ . In order to eliminate this parameter we use a procedure already discussed in Durán et al. [13]. First, an expression for crack tip opening displacement *CTOD* is needed. This can be obtained by doubling the displacement u_y of a physical crack of effective size ($a + r_{y_sh}$) [14]:

$$u_y = \frac{4}{E'} K \sqrt{\frac{r_{y_sh}}{2\pi}} \quad (16)$$

where $E' = E$ for plane stress and $E' = E/(1-\nu^2)$ for plane strain. Substitution of Eq. (7) into Eq. (16) and doubling the result gives the CTOD for a strain hardening material:

$$CTOD = 2u_y = \frac{8}{E'} K \sqrt{\frac{K^2}{2\pi(1+n)\pi S_y^2}} = \frac{8K^2}{\pi E' S_y} \sqrt{\frac{1}{2(1+n)}} \quad (17)$$

Now using the relation $\rho = CTOD/2$, the elastic stress distribution in the plastic zone for $\theta = 0$, or equivalently, the elastic point stresses as function of the distance from crack-tip $\sigma_y(r) = k_t \cdot \sigma_n$, as predicted by Creager & Paris [12] can be obtained for a given K :

$$\begin{aligned} \frac{\rho}{2} &= \frac{CTOD}{4} = \frac{2K^2}{\pi E' S_y} \sqrt{\frac{1}{2(1+n)}} \\ \sigma_y(r) &= \frac{K}{\sqrt{2\pi r}} \left(1 + \frac{\rho}{2r}\right) = \frac{K}{\sqrt{2\pi r}} \left(1 + \frac{2}{r} \frac{K^2}{\pi E' S_y} \sqrt{\frac{1}{2(1+n)}}\right) \end{aligned} \quad (18)$$

Obviously the $\sigma_y(r)$ is the $\sigma_{1e}(K,r)$ we are interested in for using in Eq. (14). After substitution of Eq. (18) into Eq. (14) we have an analytical implicit expression for the principal elastic-plastic stress field $\sigma_{1max}(r)$ within the plastic zone and for a given λ ratio:

$$\begin{aligned} \frac{(1-\nu)}{(1+n)} \frac{K^2}{E} \frac{1}{\pi r} \left(1 + \frac{2}{r} \frac{K^2}{\pi E S_y} \sqrt{\frac{1}{2(1+n)}}\right)^2 &= \frac{(1-\lambda\nu)}{2E} \sigma_{1max}^2(r) + \dots \\ \dots + \frac{(1-0.5\lambda)(1-\lambda+\lambda^2)^{\frac{1-n}{2n}} \sigma_{1max}(r)}{(1+n)} \left(\frac{\sigma_{1max}(r)}{H}\right)^{\frac{1}{n}} \end{aligned} \quad (19)$$

Note that for plane stress we use $E' = E$. Theoretically Eq. (18) would be valid only for distances $r > \rho/2$ from the crack tip. Like the *HRR* solution, however, inelastic stress-strain from Eq. (19) in the region of crack-tip blunting ($r < 2CTOD$) would not be valid because of the very intense deformation that there exists, which invalidates any analysis based on the consideration of small-scale yielding.

5 Results

A computer code was developed for numerically solving the Eq. 19 at discrete values of r ($2CTOD \leq r \leq r_{y_sh}$) and for $0.2 \leq \lambda \leq 1$. Each element of the matrix of σ_{1max} values with r columns and λ rows are then substituted into the Eq. (9) to obtain, if desired, the corresponding $\epsilon_{1max}(r,\lambda)$ values. Only stress fields are presented in this work.

Non-dimensional plots of stress fields against the distance from the crack/notch tip were used in order to evaluate the approach developed in this paper. The well established *HRR* solution

$\sigma_y/S_y=(r_{y_sh}/r)^{(n/n+1)}$ was also plotted for comparison. In this paper we are dealing only with crack-like notches. Consequently, there is an associated stress intensity factor K which include parameters like the nominal stress σ_n , the size of the notch a and the geometry factor F . By fixing a K value, for example, $K = 20 \text{ MPa.m}^{1/2}$, the plots become independent of these parameters. Figures 3 - 5 show the stress fields as predicted by the Glinka modified (GM) approach (this paper) for extreme values of λ and its comparison with the well known HRR solution. Note that for plane stress with $\sigma_3 = 0$, the parameter λ should be always greater than zero.

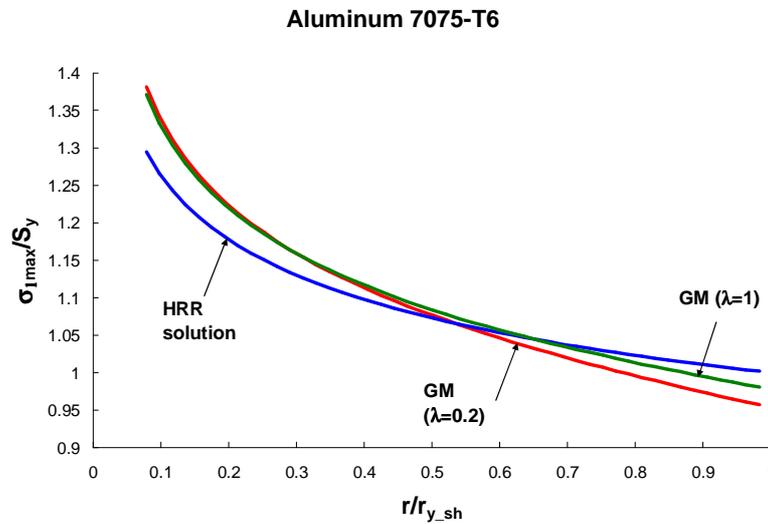


Figure 3: Stress field within the plastic zone for a crack/notch with $K = 20 \text{ MPa.m}^{1/2}$ in a Aluminum 7075 T-6 part as predicted by Glinka modified (GM) approach (this paper) for extreme values of $\lambda = \sigma_2/\sigma_1$ and comparison with the well known HRR solution.

6 Discussion

As already mentioned, Glinka's approach [4] has its main field of application in finding approximate values of the stress-strain at the notch tip after yielding. Both, Neuber and Glinka's approaches are also adapted for alternate loading and then used in low cycle fatigue design. In high cycle fatigue, however, the well known notch sensitivity phenomenon causes the fatigue limit not to be determined only by the maximum stress value at the notch tip, mainly for crack-like notches. Knowing the stress fields in these cases are of major importance. The basic idea under the extension of the Glinka approach deep inside of the plastic zone, as was done in this paper, and therefore dealing with stress fields in place of scalar values at the tip of the notches, is for its use in high cycle fatigue design. The preliminary

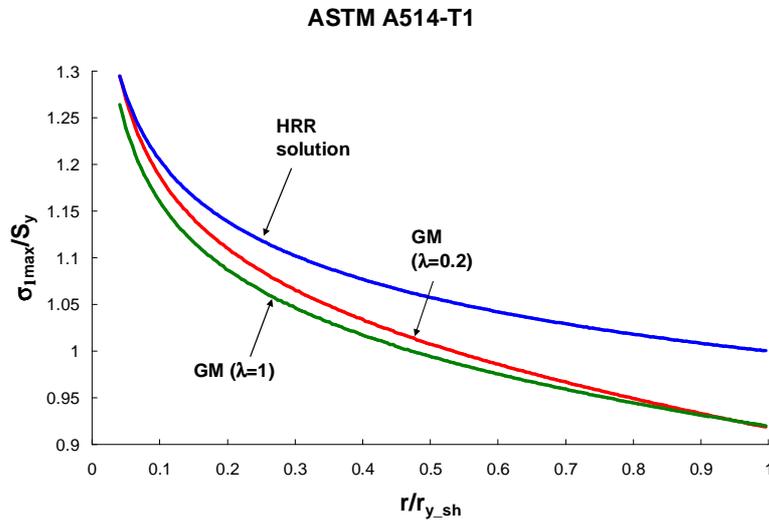


Figure 4: Stress field within the plastic zone for a crack/notch with $K = 20 \text{ MPa}\cdot\text{m}^{1/2}$ in a ASTM A514-T1 steel part as predicted by Glinka modified (*GM*) approach (this paper) for extreme values of $\lambda = \sigma_2/\sigma_1$ and comparison with the well known *HRR* solution.

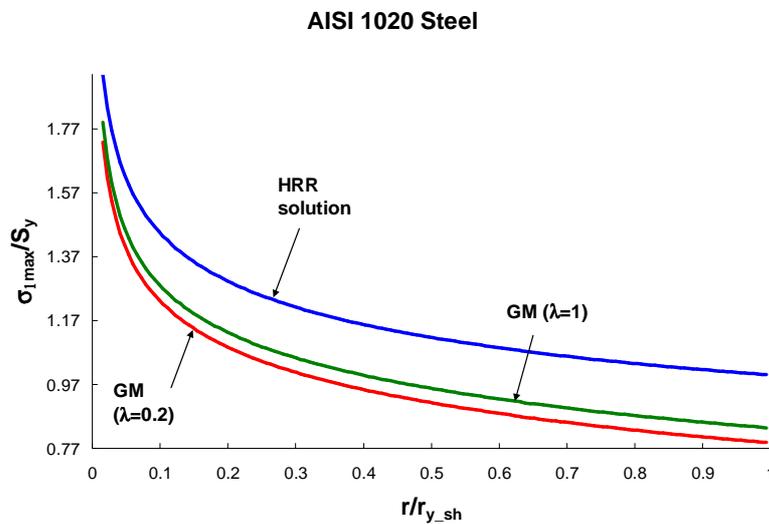


Figure 5: Stress field within the plastic zone for a crack/notch with $K = 20 \text{ MPa}\cdot\text{m}^{1/2}$ in a AISI 1020 Steel part as predicted by Glinka modified (*GM*) approach (this paper) for extreme values of $\lambda = \sigma_2/\sigma_1$ and comparison with the well known *HRR* solution.

results showed in this work for monotonic loading and the possibility of extending them for fatigue loading constitute a good and encouraging sign.

The *GM* approach is strongly dependent on some properties of materials, as can be seen in Eq. (19). Of course, non-dimensional plots avoid, in part, this dependence. The strain hardening exponent n is a non-dimensional quantity so that normalized plots are still functions of the materials involved. Three materials were used in the simulation and its properties were taken mainly from Dowling [6]. These properties are summarized in table 1.

Table 1: Properties of the three materials used for simulation in the present paper [6].

Material	True Fracture		Strength Coefficient H [MPa]	Strain Hardening Exponent n
	Strength [MPa]	Strain		
Aluminum 7075-T6	744	0.41	827	0.113
ASTM A514-T1	1213	1.08	1103	0.088
AISI 1020 steel	713	0.96	737	0.19

Besides the reasonably good agreement between the *HRR* solution and the *GM* method showed in Figs. 3-5, the most interesting fact to note after analyzing these figures is that the *GM* approach has a normalized stress gradient similar to the *HRR* solution in all the graphs. According to the *HRR* solution this gradient is $n/(n+1)$ while the *GM* approach (Eq. (19)) does not have an explicit form of this parameter. This is an excellent indication about the quality on the predictions of the stress fields by the proposed method.

It should be noted that there was almost no influence of λ parameter in the results. Glinka [15] has already used the *SEDH* so as to get the stress field inside the plastic zone. Although he used a somewhat different approach, the expression for the stress field he got was independent of the λ parameter. It is worthy to remember that nowadays the widespread use of modern and efficient computers facilitates the solution of equations like Eq. 19 and the understanding of effects such as the influence of λ parameter on the estimates of stress fields in crack-like notches.

7 Conclusion

After some modifications to the equivalent strain energy density hypothesis, a new method to estimate the stress fields in crack-like notches has been developed. Good agreement was found when comparing the results with a well established analytical solution. The most encouraging result obtained in the present paper is related to the fact that the normalized stress field gradient by the *GM* method is very similar to that of the *HRR* solution, with differences as low as 15 % in some cases.

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