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# ANALYSIS OF CONTROL LAWS FOR UNMANNED UNDERWATER VEHICLES

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**Abstract.** *Control law design for unmanned underwater vehicles presents extra difficulties, usually associated with differences between the nominal dynamic model and physical plant. These differences are mainly due to non modeled dynamics and parametric uncertainties. Therefore, it is essential that the control law designed to be implemented in practice has robustness characteristics which enables it to maintain performance, even in the presence of parametric variations between nominal model and physical plant, for example. Accordingly, this article shows results of tests with three different control laws designed to control a underwater ROV-type vehicle (Remotely Operated Vehicle). Were implemented and tested in simulation the following control laws: sliding mode with simple surface; sliding mode with integral surface; PID control used with feedback linearization. These tests with the control laws showed no significant differences when the nominal model was considered identical to physical plant. Several simulations were performed considering parametric variations between nominal model and physical plant. Other tests were performed considering non-modeled dynamics, such as sea currents. The worst results were obtained with sliding mode with simple surface control. The sliding mode control with integral surface showed similar results to those obtained with the PID. However, the PID (proportional, integral and derivative) implemented with feedback linearization technique presented the best results.*
***Keywords**: Modeling; Control; Robustness; Simulation; ROV .*

## 1. INTRODUCTION

Several scientific studies have been conducted focusing on the use of robots in underwater environment. These robots are used mainly because of physical limitations for the conduct of human tasks on the marine environment. In this context, the non-manned underwater vehicles are presented as a viable solution, with a large amount of applications. The use of these vehicles will tend to grow to the extent that the technical project are improved and costs reduced (Barros and Soares, 2002, Tavares 2003).

In Brazil there are few studies in the field of underwater robotics. Dominguez (1989) did a study on the modeling and developed a program for dynamic simulation of underwater vehicles. Cunha (1992) proposed an adaptive control system for tracking of trajectory. Hsu et al. (2000) presented a procedure for identifying the dynamic model of the thrusters. Barros and Soares (2002) proposed the low-cost vehicle that can operate as a ROV (Remotly Operated Vehicle) and AUV (Autonomus Underwater Vehicle). Souza and Maruyama (2002) investigated different techniques to control for positioning of vehicles.

Underwater vehicles have complex control projects, due to nonlinearity in their dynamics, uncertainties in models and, due to the presence of disturbances that are difficult to measure or estimate (Yoerger, 1985).

The strategy of using variable structure control (sliding mode) was developed in the Soviet Union in the 60s (see, for example, Emelyanov, 1967), and was employed in various non-linear systems. In Sens *et. al.* 2006 sliding control was developed and tested by simulations. In the present work sliding control was tested in two ways, ie with simple surface and with integral surface. These two control laws were compared with a PID (Proportional, Integral and Derivative) control used with feedback linearization. This PID control was implemented considering a window (floating window) of only *n* points to add the sum of the integral component. Several simulations were performed, considering parametric variations and non modeled dynamics. The main objective of the research reported in this article was to determine if the sliding control (in both ways) could provide superior performance to that obtained using PID with feedback linearization and integral floting window.

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## 2. MATHEMATIC MODELS

The mathematical modeling of underwater vehicles is divided into kinematic and dynamic models. It is necessary to work with two systems of reference: one fixed to the vehicle (body frame) and another at a fixed point on Earth (inertial frame). Underwater vehicles dynamics is modeled in the body frame. During the simulations, in each step of integration of differential equations is made a transformation from body to inertial frame, using the kinematic model. In this article it was used Euler angles formalism for determining the kinematic model (Fossen, 1994). The following general equations are used to model the kinematic and dynamic of the vehicle (Tavares, 2003):

$$\mathbf{M}\dot{\mathbf{v}}_{\mathrm{r}} + \mathbf{C}(\mathbf{v}_{\mathrm{r}})\mathbf{v}_{\mathrm{r}} + \mathbf{D}(\mathbf{v}_{\mathrm{r}})\mathbf{v}_{\mathrm{r}} + \mathbf{g}(\boldsymbol{\eta}) = \boldsymbol{\tau} \tag{1}$$

$$\dot{\boldsymbol{\eta}} = \mathbf{J}(\boldsymbol{\eta})\mathbf{v}_{r} \tag{2}$$

$$\mathbf{P}\dot{\mathbf{n}} = \boldsymbol{\tau}_{\mathrm{m}} - \boldsymbol{\tau}_{\mathrm{r}} \tag{3}$$

The equation (1) describes the dynamics of interaction between the fluid and the underwater vehicle, expressed in the body frame. In this equation, $\mathbf{v}_{\mathrm{r}} = \mathbf{v} - \mathbf{v}_{\mathrm{c}}$ is the relative velocity (difference between the velocities of the vehicle and the current marine); **M** is the matrix of inertia associated with the rigid body and the additional mass; $\mathbf{C}(\mathbf{v}_{\mathrm{r}})$ is the matrix of Coriolis and centripetal terms, also associated with rigid body and the additional mass; $\mathbf{D}(\mathbf{v}_{\mathrm{r}})$ is the matrix with drag and lift terms; $\boldsymbol{\tau}$ is the vector with forces and moments of thrusters; $\mathbf{g}(\boldsymbol{\eta})$ is the vector of forces and moments produced by weight and thrust; $\boldsymbol{\eta}=\left[\mathrm{x,y,z},\phi,\theta,\psi\right]^{\mathrm{T}}$ is the vector with the position and orientation of the vehicle, expressed in the inertial frame, where x, y, and z are the coordinates of the origin of the body reference and $\phi$, θ and ψ are the angles of orientation.

Equation (2) describes the kinematic model, to make the transformation between the body and inertial reference systems. It was use the following notation: $\dot{\boldsymbol{\eta}}$ is the velocity vector of the vehicle in inertial frame; $\mathbf{J}(\boldsymbol{\eta})$ is the matrix of transformation from body to the inertial frame, using Euler angles. In equations (1) and (2), all matrices have dimension 6x6 and all vectors have dimension 6x1. The elements of these matrices are presented in detail in Fossen (1994) and Tavares (2003). Equation (3) describes the dynamics of the thrusters. **P** is the matrix of inertia, $\dot{\mathbf{n}}$ the angular acceleration vector, $\boldsymbol{\tau}_{\mathrm{m}}$ the vector of motor torques and $\boldsymbol{\tau}_{\mathrm{r}}$ the vector of resistant torques. Considering that the vehicle has *p* thrusters, the inertia matrix has dimension *p*x*p*, while the vectors $\boldsymbol{\tau}_{\mathrm{m}}$ and $\boldsymbol{\tau}_{\mathrm{r}}$ have size *p*x1.

## 3. SLIDING MODE CONTROL

This section introduces the theoretical development of the sliding control (Slotine and Li, 1991). Considering the system in the state space form, the central idea of the method is to design the control so that all system trajectories converge to the surface of control, and that all trajectories that start inside the surface will remain there indefinitely. On this defined surface, the trajectories slide assynthotically to the desired values, and this phase is known as sliding mode.

Considering a single input, the nonlinear system can be put in the following state form:

$$x^{n} = f(\mathbf{x}) + b(\mathbf{x})u \tag{4}$$

$\mathbf{x} = [x\ \dot{x}\ \ldots\ x^{n-1}]^{T}$ is the state vector, $x$ is the output of interest, $u$ is the single input with the control signal and the generic functions $f(\mathbf{x}),\ b(\mathbf{x})$ are known, but with a range of uncertainties. Considering $\mathbf{x_d}$ the state vector of reference, the tracking error is given by $\tilde{\mathbf{x}}=\mathbf{x}-\mathbf{x_d}$. The control problem consists in the tracking of $\mathbf{x_d}$ with minimum error.

The sliding surface S(t) in $\mathbf{R}^{\mathrm{n}}$ space is defined by the equation s($\mathbf{x}$,t)=0, where:

$$s(\mathbf{x},t) = \left(\frac{d}{dt} + \lambda\right)^{n-1} \tilde{x} \tag{5}$$

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$\lambda$ is a positive constant. Equation (5) is derived and appears $x^n$, which is replacing in (4) determining the control signal **u**. For example, if the dynamic system is defined by:

$$\ddot{x} = f(x, \dot{x}, t) + u(t) \tag{6}$$

sliding surface is defined by s (t) = 0, and s(t) given in (05) with $n$ = 2. Considering the time derivative in equation (5) and using (6) has been:

$$\dot{s} = \ddot{x} - \ddot{x}_d + \lambda\dot{\tilde{x}} = f + u - \ddot{x}_d + \lambda\dot{\tilde{x}} \tag{7}$$

In the absence of modeling errors and disturbances, the control with this simple surface has the form:

$$\hat{u} = -\hat{f} + \ddot{x}_d - \lambda\dot{\tilde{x}} \tag{8}$$

where $\hat{f}$ is the estimative of the function *f*. A similar result can be obtained considering a integral surface:

$$s = \left(\frac{d}{dt} + \lambda\right)^2 \left(\int_0^t \tilde{x}dr\right) = \dot{\tilde{x}} + 2\lambda\tilde{x} + \lambda^2 \int_0^t \tilde{x}dr \tag{9}$$

In the absence of modeling errors and disturbances, the control with this integral surface would be:

$$\hat{u} = -\hat{f} + \ddot{x}_d - 2\lambda\dot{\tilde{x}} - \lambda^2\tilde{x} \tag{10}$$

To deal with the uncertainties of the model, there is also a discontinuous term function of the signal of *s*:

$$u = \hat{u} - k(x, \dot{x}, t) sign(s) \tag{11}$$

Function *sign* was implemented in the form:

$$sign(s) = \begin{cases} 1, \; if \; s > 0 \\ \quad 0, \qquad if \; s = 0 \\ -1, \; if \; s < 0 \end{cases} \tag{12}$$

To avoid the phenomenon of chattering, it modifies the function (12), setting up a "boundary layer" of width $\phi$ within which the transition of the signal happens. It is used:

$$u = \hat{u} - k(x, \dot{x}, t)\mathbf{sat}(s/\phi) \tag{13}$$

with

$$sat(s/\phi) = \begin{cases} sign(s), \; if \; |s/\phi| > 1 \\ s/\phi, \; if \; |s/\phi| <= 1 \end{cases} \tag{14}$$

For the multivariable control is needed to adapt the theory above and this is the case of underwater vehicles, whose output is the vector is:

$$x = \boldsymbol{\eta} = [\mathrm{x}, \mathrm{y}, \mathrm{z}, \phi, \theta, \psi]^{\mathrm{T}} \tag{15}$$

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## 4. PID CONTROL

PID control (Proportional, Integral and Derivative) is widely used in applications in the real world of engineering. Mathematic model (equations (1) and (2)) can be rewritten as:

$$\begin{aligned} \mathbf{M}\dot{\mathbf{v}} + \mathbf{n}(\mathbf{v}, \boldsymbol{\eta}) &= \boldsymbol{\tau} \\ \dot{\boldsymbol{\eta}} &= \mathbf{J}(\boldsymbol{\eta})\mathbf{v} \end{aligned} \tag{16}$$

In the present work PID control was implemented with the linearization by feedback technique. In this case, control torque can be expressed as:

$$\boldsymbol{\tau} = \mathbf{M}\mathbf{a}_{v} + \mathbf{n}(\mathbf{v}, \boldsymbol{\eta}) \tag{17}$$

Using PID, $\mathbf{a}_v$ can be expressed as:

$$\mathbf{a}_v = \mathbf{a}_\mathbf{r} + K_p\mathbf{e} + K_d\dot{\mathbf{e}} + K_i\int\mathbf{e} \tag{18}$$

$\mathbf{a}_v$ is the desired acceleration of the vehicle is the state error, both expressed in the body frame. It is usual to define the acceleration, velocity and position of the vehicle in the inertial frame and then get the same reference in the body frame using the kinematic relations.

The PID control tested in this article was implemented using a fixed amount of points (error values) for the integral component, equivalent to twenty points. This strategy is called here, floating window with *n* values of errors. At time *j*, when the *j* error is inserted is removed from the sum the *j-n* error. This strategy allows larger stability for the implementation of the discrete PID.

## 5. SIMULATION RESULTS

The reference trajectories were generated imposing the following second order differential equation:

$$\ddot{\boldsymbol{\eta}}_d + 2\xi\omega_n\dot{\boldsymbol{\eta}}_d + \omega_n^2\boldsymbol{\eta}_d = \omega_n^2\mathbf{r}_\eta \tag{19}$$

where $\boldsymbol{\eta}_d$ is the vector with the position and orientation of reference in each discrete time, $\mathbf{r}_\eta$ is the vector with the desired final position and orientation, $\xi$ is the damping ratio and $\omega_n$ the non-damped natural frequency. Note that $\xi$ and $\omega_n$ determine the desired trajectory characteristics. Analyzing the equation (19), $\boldsymbol{\eta}_d(\infty) = \mathbf{r}_\eta$, ie in steady state, the positions and orientations of reference are equal to the desired positions and orientations.

The simulations were performed considering the dynamic model of the vehicle NEROV (Norwegian Experimental Remotely Operated Vehicle) and the parameters were extracted from Tavares, 2003. In this vehicle there are six actuators (DC thrusters) and this architecture allows controlling all six degrees of freedom.

The goal of all simulations was to take the vehicle's initial position (10*m*, 10*m*, 10*m*), with initial angles of roll, pitch and yaw equal to 45° to the final position (0 *m*, 0 *m*, 0 *m*), with final angles of roll, pitch and yaw null.

Figures 1 and 2 show results of a simulation considering the nominal model equal to physical plant, ie, without parametric variations, in the case of sliding control with simple surface and with integral surface. It can be seen that in this case, the sliding control with simple surface shows behavior very close to the sliding with integral surface and both present good performance.

The simulations whose results are seen in Figures 3 and thereafter were based on a parametric change between model and plant (the thrust of the model is considered 3% greater than the thrust of the physical plant).

Figures 3 and 4 show results obtained with the sliding control with simple surface. There is a poor performance, visible mainly in the vertical position, indicating that this control does not support parametric variations.

This same simulation was repeated using the sliding control with integral surface and the results can be seen in Figures 5, 6 and 7. It can be observed that there was a loss of performance with respect to the case without parametric variation. A stead state error of approximately 0.2m persists in the vertical position of the vehicle.

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The PID control was tested in the same situation seen before, ie, considering parametric variation, and the results can be seen in Figures 8, 9 and 10. Stead state error at the vertical position is close to zero and this control presents the best behavior.

## 6. CONCLUSIONS

In this work was implemented and tested in simulation the following control laws: sliding mode with simple surface; sliding mode with integral surface; PID control used with feedback linearization. These tests with the control laws showed no significant differences when the nominal model was considered identical to physical plant. Others simulations were also performed based on parametric variations between nominal model and physical plant. It was found that the most sensitive parameters of the model are related to the mass and volume of the vehicle and the density of the fluid. These parameters directly influence the forces of weight and thrust, responsible for restoring torques. Other tests were also performed considering non-modeled dynamics, such as sea currents (omitted in this article for saving space). The worst results were obtained with sliding mode with simple surface control. The sliding mode control with integral surface showed similar results to those obtained with the PID. However, the PID (proportional, integral and derivative) implemented with feedback linearization and flouting window at integral component presented the best results. To test and compare different control techniques is always a difficult task because the results are very dependent on control projects. However, these control laws are designed to provide approximately the same performance when the nominal model is considered identical to the physical plant. In summary, this article found the control law that showed the greatest ability to maintain stability and performance, even in the presence of parametric variations and non-modeled dynamics.

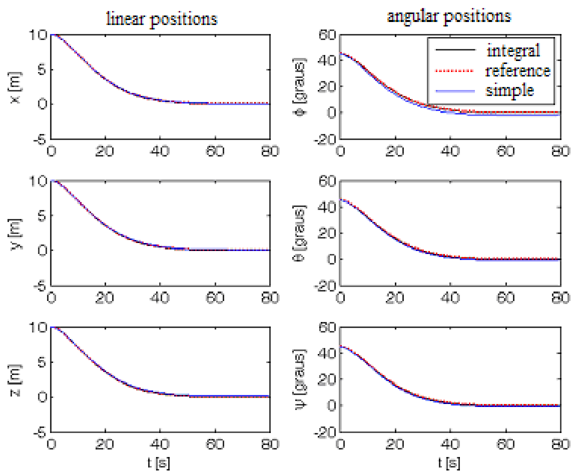


Figure 1 – Linear and angular positions in the inertial frame.

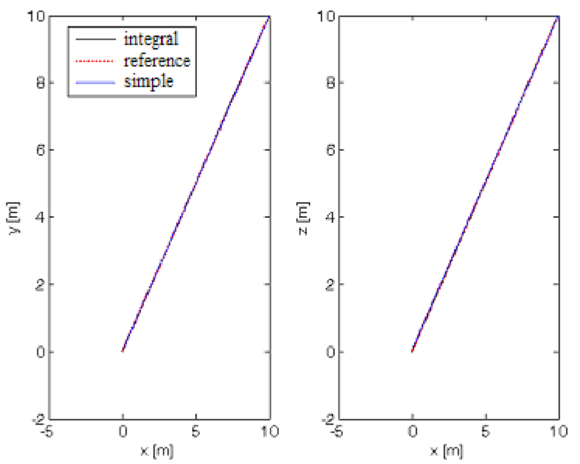


Figure 2 – Trajectories in the planes XY e XZ (inertial frame).

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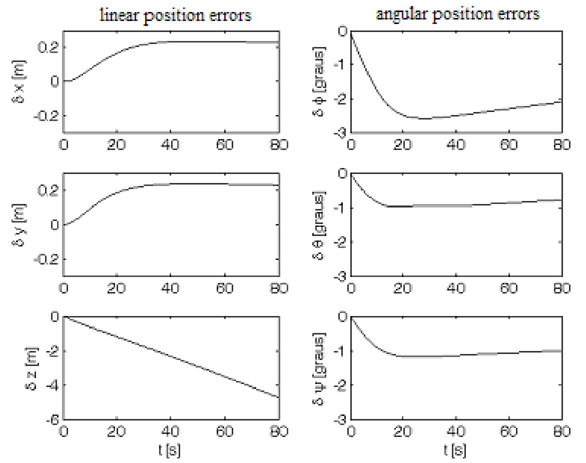


Figure 3 – Linear and angular position errors (sliding control with simple surface, considering parametric variation).

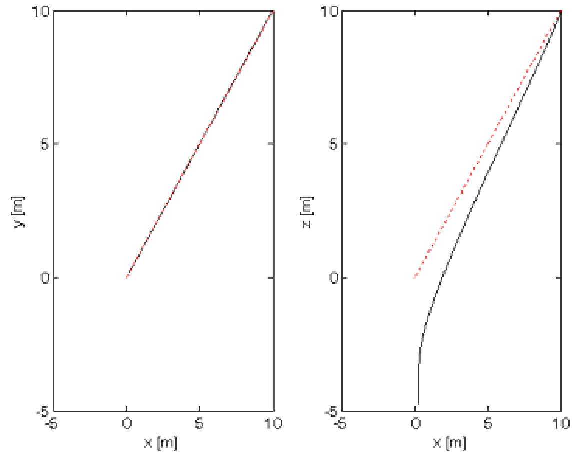


Figure 4 – Trajectories in the planes XY e XZ (sliding control with simple surface, considering parametric variation).

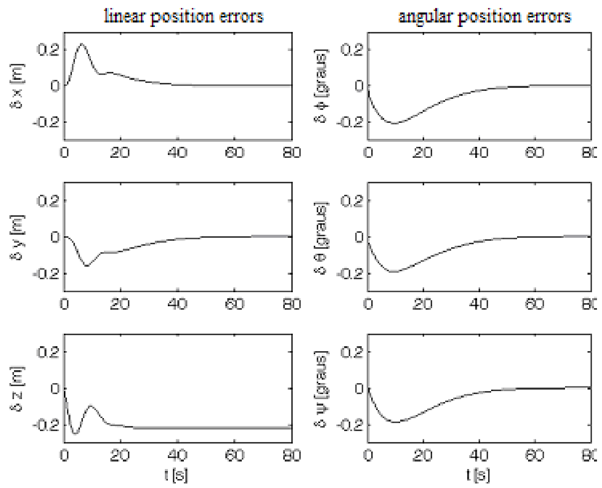


Figure 5 – Linear and angular position errors (sliding control with integral surface, considering parametric variation).

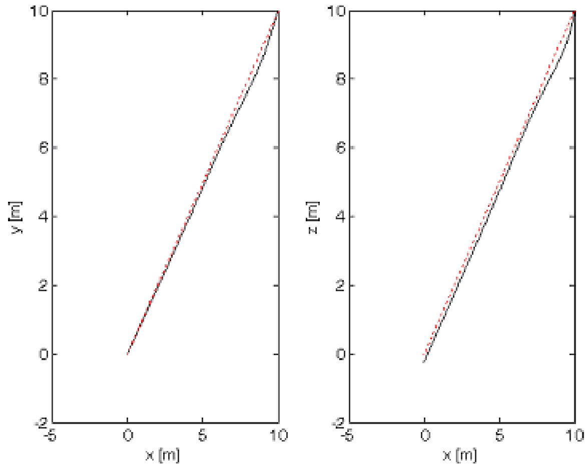


Figure 6 – Trajectories in the planes XY e XZ (sliding control with integral surface, considering parametric variation).

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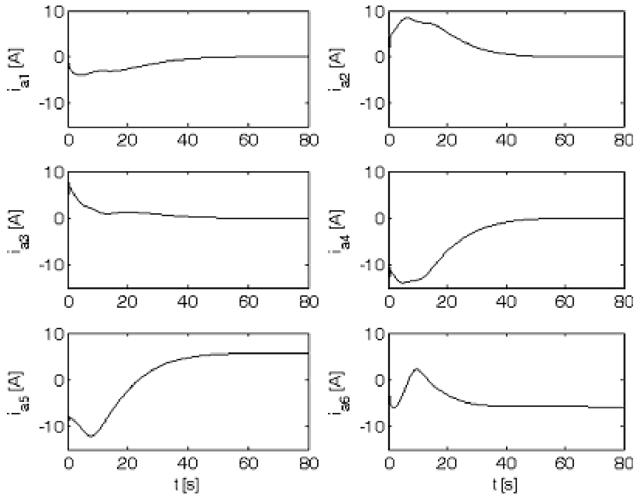


Figure 7 – Electric current signal at each thruster (sliding control with integral surface, considering parametric variation).

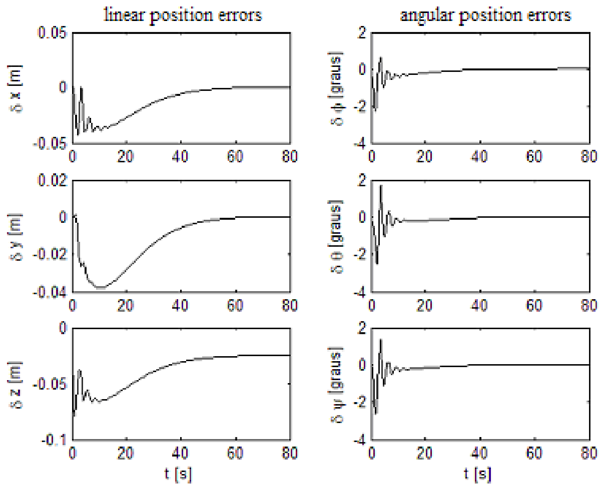


Figure 8 - Linear and angular position errors (PID control, considering parametric variation).

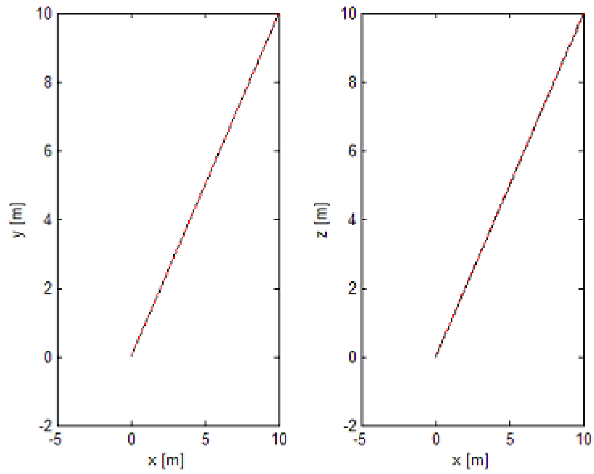


Figure 9 – Trajectories in the planes XY e XZ (PID control, considering parametric variation).

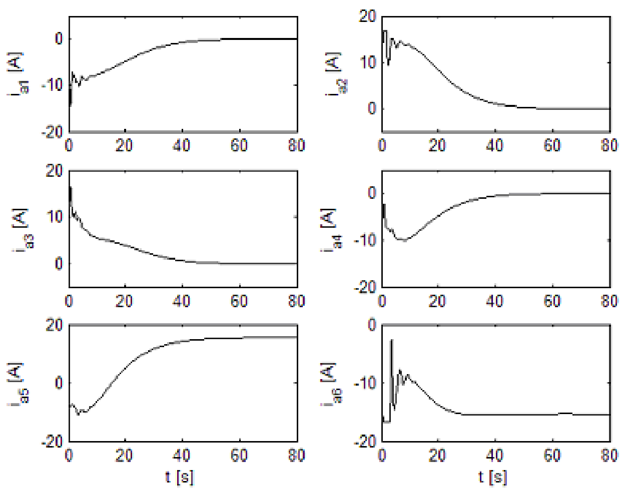


Figure 10 – Electric current signal at each thruster (PID control, considering parametric variation).

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