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ROBUST MODEL PREDICTIVE CONTROL FOR A MAGNETIC LEVITATION SYSTEM EMPLOYING LINEAR MATRIX INEQUALITIES

Mariana Santos Matos, mariana.smatos@yahoo.com.br
Roberto Kawakami Harrop Galvão, kawakami@ita.br
Takashi Yoneyama, takashi@ita.br
Instituto Tecnológico de Aeronáutica, Divisão de Engenharia Eletrônica, 12228-900, São José dos Campos - SP, Brazil

Abstract. *With the evolution of computational technology, the implementation of predictive control techniques in systems with fast dynamic became viable. In this formulation, the control action is obtained on line by the minimization of a constrained quadratic performance index within a receding horizon. The main advantage of these techniques is the possibility to deal with input/output constraints explicitly. The standard model predictive control (MPC) scheme does not consider model uncertainties, which can be caused by modeling simplifications or system faults. Among the formulations proposed to address this problem, linear matrix inequalities (LMI) techniques have become very popular. This work employs an LMI-based robust MPC approach for regulation of a magnetic levitation system with gain uncertainty. In order to achieve offset-free regulation, the plant model was augmented with an additional state associated to the accumulated error. However, such modification, which amounts to including integral control action, can cause windup problems that substantially increase the settling time. To circumvent this inconvenience, an anti-windup scheme was proposed. For this purpose, the error integrator is reset when the plant state approaches the boundaries of the feasible region. A modification was also introduced to allow the inclusion of an overshoot constraint in the MPC formulation. This is not possible in the basic LMI technique, which only handles symmetric output constraints. The proposed approach was evaluated by using a simulation model of a magnetic levitation system. The results show that the use of a robust MPC formulation may be indeed necessary to prevent constraint violations in the presence of uncertainties such as gain mismatch between the design model and the plant. Moreover, it was notice that the proposed anti-windup scheme is capable reducing the settling time without introducing steady-state errors.*

Keywords: *LMI-based predictive control, gain uncertainty, magnetic levitation*

1. INTRODUCTION

Magnetic levitation technology has found increased applicability in various industrial and scientific sectors. The main motivation for use of this technology has been the advantages of magnetic actuators, such as their non-contact actuation (Rocha et al., 2008) and possibility of handling large loads. Applications can be found, for example in high-speed transportation (Holmer, 2003), self-bearing blood pumps (Masuzawa et al., 2003) and micro robots (Khamesse et al., 2002).

In the control of such systems, it is necessary to consider the plant operation constraints, for instance the controller must avoid collision between the attraction system and the suspended object, while the gap must be the smallest possible to obtain a good electromechanical conversion.

The application of predictive control strategies allows the explicit treatment of physical and operational constraints (Maciejowski, 2002) (Rossiter, 2003), which makes it an attractive solution for the magnetic levitation control problems. Indeed, such approach was used in Miura (2003) for the didactic magnetic levitation system control shown in Fig. 1. This work was complemented by Fama et al. (2005), whose paper included integral control action and a noise sensitivity analysis, by Afonso and Galvão (2007), with the handling of non-feasibility problem, and more recently, by Matos, Galvão and Yoneyama (2008) who dealt with uncertainties caused by gain mismatches between the design model and the actual plant.

In the present work, an LMI-based robust MPC approach is employed for regulation of a magnetic levitation system under uncertainty in gain. The design model is augmented with an additional state associated to the accumulated error to achieve offset-free regulation. An anti-windup mechanism is proposed to circumvent the increase in settling time caused by the integral control action. A scheme is also proposed to allow the inclusion of an overshoot constraint in the LMI-based MPC formulation. Simulation results show that the use of the proposed robust MPC formulation can succeed preventing constraint violations in the presence of gain mismatches between the nominal model and the actual plant. The LMI-based approach was found to be favorably comparable with a finite-horizon robust formulation.

2. DESCRIPTION AND MODELING OF THE PROCESS

The plant used in this work is the magnetic levitation system produced by Feedback$^{\mathrm{TM}}$. It consists of an electromagnet, a current driver, a photo-emitter, a photo-receiver and a control system, as shown in Fig. 2 (Fama et al., 2005). The photo-sensor is designed to provide an output signal y that is linearly related to the distance h, as $y = \gamma h + y_0$, where the gain

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Figure 1. Feedback^{TM}magnetic levitation system.

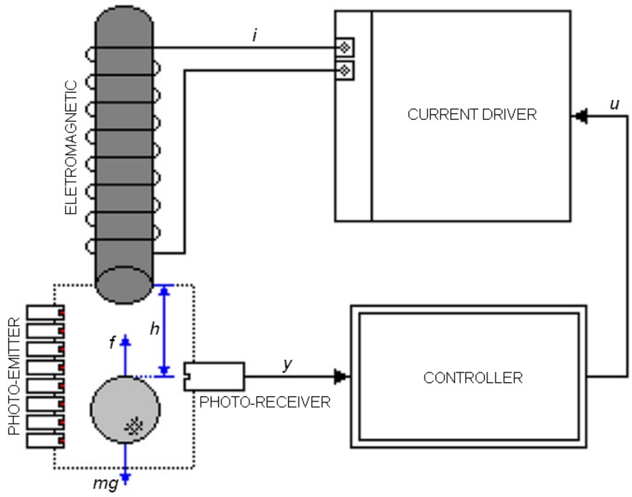

Figure 2. Control loop schematic. f denotes the attraction force related to i and h.

$\gamma > 0$ and the offset y_0 are constants. Similarly, the electromagnet current i is given by $i = ru + i_0$, with $r > 0$ and $i_0 > 0$, constants. There are physical constraints to the control actions, so that $u \in [-3V, 5V]$ and $|\dot{u}| \leq 50V/s$. The system state equations, with the state $x = [y, dy/dt]$, linearized using the equilibrium point $\bar{x} = [\bar{y}, 0]$, are given by (Fama et al., 2005) and by (Matos, Galvão and Yoneyama, 2008):

$$\dot{\tilde{x}} = \begin{bmatrix} 0 & 1 \\ \eta & 0 \end{bmatrix} \tilde{x} + \begin{bmatrix} 0 \\ -\beta \end{bmatrix} \tilde{u} = A\tilde{x} + B\tilde{u} \tag{1}$$

where the tilde notation indicate small variations around the point of equilibrium and

$$\eta = \frac{2\gamma g}{(\bar{y} - y_0)} \tag{2}$$

$$\beta = \frac{2r\gamma^3}{(\bar{y} - y_0)} \sqrt{\frac{Kg}{m}} \tag{3}$$

where g is the acceleration of gravity, m denotes the mass of the sphere and K is the electromechanic conversion gain.

The constant values previously cited were taken from Fama et al. (2005), with: $m = 2.12 \times 10^{-2} kg$, $g = 9.8 m/s^2$, $y_0 = -7.47V$, $\gamma = 328V/m$, $i_0 = 0.514A$, $r = 0.166A/V$ and $K = 1.2 \times 10^{-4} Nm^2/A^2$.

3. CONTROL STRATEGY

3.1 Finite-horizon robust model predictive control

In the finite-horizon predictive control scheme using the state feedback strategy, the future outputs up to a certain receding horizon N are predicted at each sampling time k using the process mathematical model, under a given sequence

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of future control. The optimal future control sequence is obtained by minimizing a cost function over a control horizon M subject to constraints on the input control range and system output range. Other constraints are also possible, such as state range (Kothare, Balakrishnan and Morari, 1996). However, only the first step of the optimal control sequence is applied, and this process is repeated at each new sample time, in order to take into account the new feedback information.

Consider a time-invariant linear system model of the form:

$$x(k+1) = Ax(k) + Bu(k) \tag{4}$$

$$y(k) = Cx(k) \tag{5}$$

where for each k, $x(k) \in \Re^n$, $y(k) \in \Re$, $u(k) \in \Re$ and A, B and C are matrices of appropriate dimensions.

Given a sequence of inputs u, Maciejowski (2002) shows that the predicted outputs are given by

$$\hat{Y} = H\hat{U} + Q_F x(k) \tag{6}$$

in which

$$\hat{Y} = \begin{bmatrix} \hat{y}(k+1|k) \\ \hat{y}(k+2|k) \\ \vdots \\ \hat{y}(k+N|k) \end{bmatrix}, \ \hat{U} = \begin{bmatrix} \hat{u}(k|k) \\ \hat{u}(k+1|k) \\ \vdots \\ \hat{u}(k+M-1|k) \end{bmatrix} \tag{7}$$

$$H = \begin{bmatrix} CB & 0 & \dots & 0 \\ CAB & CB & \dots & 0 \\ \vdots & \ddots & \dots & \vdots \\ CA^{N-1}B & \cdots & CA^{N-M-1}B & \sum_{n=M}^{N} CA^{N-n}B \end{bmatrix} \tag{8}$$

$$Q_F = \begin{bmatrix} CA \\ CA^2 \\ \vdots \\ CA^N \end{bmatrix} \tag{9}$$

In order to obtain an offset-free regulation, an additional state w (Matos, Galvão and Yoneyama, 2008) can be included:

$$w(k+1) = w(k) - y(k) \tag{10}$$

Then, the augmented model Eq. (4) + (5) + (10) can be written in the form

$$\xi(k+1) = A_\xi \xi(k) + B_\xi u(k) \tag{11}$$

$$y(k) = C_\xi \xi(k) \tag{12}$$

with

$$\xi(k) = \begin{bmatrix} x(k) \\ w(k) \end{bmatrix}, \ A_\xi = \begin{bmatrix} A & 0 \\ -C & 1 \end{bmatrix}, \ B_\xi = \begin{bmatrix} B \\ 0 \end{bmatrix}, \ C_\xi = \begin{bmatrix} C & 0 \end{bmatrix} \tag{13}$$

and the predicted values of w in $\hat{W}$ can be obtained by the equation

$$\hat{W} = H_w \hat{U} + Q_w \xi(k) \tag{14}$$

where

$$\hat{W} = \begin{bmatrix} \hat{w}(k) \\ \hat{w}(k+1) \\ \vdots \\ \hat{w}(k+N-1) \end{bmatrix}, \ Q_w = \begin{bmatrix} C_w A_\xi \\ C_w A_\xi^2 \\ \vdots \\ C_w A_\xi^N \end{bmatrix} \tag{15}$$

$$H_w = \begin{bmatrix} C_w B_\xi & 0 & \dots & 0 \\ C_w A_\xi B_\xi & C_w B_\xi & \dots & 0 \\ \vdots & \ddots & \dots & \vdots \\ C_w A_\xi^{N-1} B_\xi & \cdots & C_w A_\xi^{N-M-1} B_\xi & \sum_{n=M}^{N} C_w A_\xi^{N-n} B_\xi \end{bmatrix} \tag{16}$$

The cost function is given by

$$J(\hat{U}) = \hat{Y}^T\hat{Y} + \mu \hat{W}^T\hat{W} + \rho \hat{U}^T\hat{U} = (H\hat{U} + F)^T(H\hat{U} + F) + \mu(H_w\hat{U} + F_w')^T(H_w\hat{U} + F_w') \tag{17}$$

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with ρ and μ are design parameters,

$$F = Q_F x(k) \tag{18}$$

$$F'_w = Q_W \xi(k) \tag{19}$$

and N the prediction horizon and M the degrees of freedom of the control action u. Moreover, let $\hat{u}(k+\lambda-1|k) = \hat{u}(k+M-1|k)$ for $M < \lambda \leq N$.

The output and input range constraints must be considered in the cost function minimization. The gain K is assumed to belong to the interval $[\epsilon_1 K_{nom}, \epsilon_2 K_{nom}]$. Matos, Galvão and Yoneyama (2008) presents an analysis about the influence of the conversion gain. It is shown that reductions of K can cause violations in output constraints. Therefore, this study considers only the particular case of $\epsilon_1 = \epsilon$ and $\epsilon_2 = 1$:

$$\begin{bmatrix} \epsilon H \\ H \\ -\epsilon H \\ -H \\ I_M \\ -I_M \end{bmatrix} \hat{U} \leq \begin{bmatrix} \Gamma_N y_{max} - F \\ \Gamma_N y_{max} - F \\ F - \Gamma_N y_{min} \\ F - \Gamma_N y_{min} \\ \Gamma_M u_{max} \\ -\Gamma_M u_{min} \end{bmatrix} \tag{20}$$

where Γ_N and Γ_M are a $(N \times 1)$ and $(M \times 1)$ matrix of unit elements, respectively.

It is worth noting that the integral control action may cause an increase in settling time. This problem is due to the minimization of the $\hat{W}^T\hat{W}$ cost function term, which is a sum of errors values during the prediction horizon (an error integrator). Therefore, if the integrator state w is not null when the output variable reaches the origin (in a regulation problem) the controller will keep the error different from zero, with an appropriate sign in order to minimize $\hat{W}^T\hat{W}$. In order to reduce this undesirable effect an anti-windup scheme is devised. It consists of reseting the error integrator when the system state approaches, for the first time, the boundaries of the feasible region.

3.2 LMI-based robust predictive control strategy

The analysis that follows is based on Maciejowski (2002). Consider a time-invariant linear system described by Eq. (4) and (5).

Assumed that the matrices A and B belong to a convex polytope Ω with L "corners" (A_i, B_i) $(i = 1, 2, ..., L)$. Then, $(A, B) = \sum_{i=1}^{L} \lambda_i (A_i, B_i)$ and $\sum_{i=1}^{L} \lambda_i = 1$. Let:

$$\bar{J}(\hat{U}_\infty) = \sum_{j=0}^{\infty} \{\|\hat{x}(k+j|k)\|^2_{Q1} + \|\hat{u}(k+j|k)\|^2_R\} \tag{21}$$

where $\hat{U}_\infty = \{\hat{u}(k|k), \hat{u}(k+1|k), \hat{u}(k+2|k), \ldots\}$, $Q_1 > 0$ and $R > 0$ are symmetric weighting matrices.

The robust control problem can be formulated as

$$\min_{\hat{U}_\infty} \max_{[A,B]\epsilon\Omega} \bar{J}(\hat{U}_\infty) \tag{22}$$

It is possible to shown that this problem is equivalent to the LMI-based robust control problem represented by

$$\min_{\gamma, Q, \Sigma} \gamma \; s.t. \tag{23}$$

$$\begin{bmatrix} Q & x(k|k) \\ x(k|k)^T & 1 \end{bmatrix} \geq 0 \tag{24}$$

$$\begin{bmatrix} Q & 0 & 0 & A_i Q + B_i \Sigma \\ 0 & \gamma I & 0 & Q_1^{1/2} Q \\ 0 & 0 & \gamma I & R^{1/2}\Sigma \\ Q A_i^T + \Sigma^T B_i^T & Q Q_1^{T/2} & \Sigma^T R^{T/2} & Q \end{bmatrix} \geq 0 \textit{ for every } i = 1, \ldots, L \tag{25}$$

If this convex problem has a solution γ, Q, Σ, then let $K_k = \Sigma Q^{-1}$ and the control optimal sequence is given by $\hat{u}(k+j)|k) = K_k \hat{x}(k+j)|k)$.

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By further adding LMIs, input and output constraints can be taken into account. Maciejowski (2002) and Kothare, Balakrishnan and Morari (1996) presents the details and the feasibility of the result problem. Range constraints for input and output signals are satisfied if it is possible to find a solution for the convex problem (Eq. (4) and (5)) with the additional LMIs:

$$\begin{bmatrix} X & \rho\Sigma \\ \rho\Sigma^T & Q \end{bmatrix} \geq 0 \text{ with } X \leq u_{max}^2 \tag{26}$$

$$\begin{bmatrix} Q & \rho\left[A_j Q + B_j \Sigma\right]^T C^T \\ \rho C\left[A_j Q + B_j \Sigma\right] & y_{max}^2 \end{bmatrix} \geq 0 \text{ for } j = 1, \ldots, L \tag{27}$$

where y_{max} and u_{max} are symmetric constraints about the output and input variable, respectively.

Asymmetric constraints are incorporated into the robust formulation as in Rodrigues and Odloak (2000) paper:

$$X \leq u_{min}^2 \tag{28}$$

$$X \leq u_{max}^2 \tag{29}$$

This formulation corresponds to the application of the most severe constraint at each sampling time.

Again, it is possible to include integral control action by adding a $w(k)$ state (Eq. (10)) and the proposed anti-windup scheme.

The inclusion of an overshoot constraint in the LMI-based formulation requires the treatment of different constraints in different operation points. The approach used for the asymmetrical control constraint is not applicable because if, for example, the maximum overshoot y_{max} is lower than the initial condition constraint y_{min}, then the y_{max} will be applied and problem will not be feasible. The solution to this problem is divided in three stages, according to the present output $y(k)$ and for the hypothesis of $y_{min} < 0$ and $y_{max} > 0$. This analysis considers $|y_{min}| > |y_{max}|$, but the symmetrical case is analogue. The proposed strategy uses the following rule:

If $y(k) > y_{min}$ then

If $|y(k)| < |y_{max}|$ then a moving pseudo-reference $r(k)$ can be defined as the middle point between the current output value and y_{max}, i.e. $r(k) = (y(k) + y_{max})/2$. The output constraint is defined symmetrical with the value of the distance between $r(k)$ and y_{max} (see Fig. 3).

Else the symmetric constraint is defined as y_{max}.

Else the symmetric constraint is defined as y_{min}.

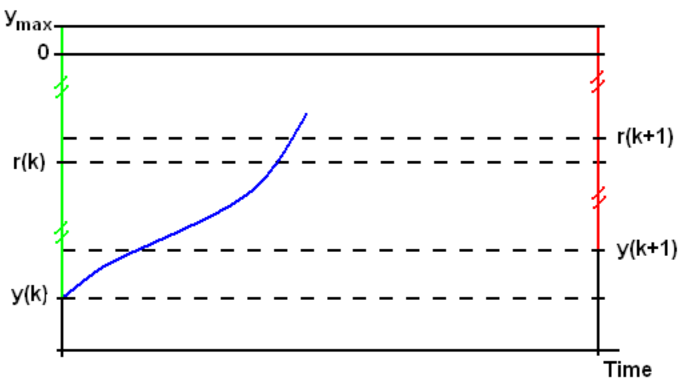

Figure 3. Moving pseudo-reference strategy.

The asymmetric constraint problem was defined in terms of local symmetric constraints.

4. Methodology

The predictive controller for the maglev plant was designed using the sampling time $T = 5ms$, as in Fama et al. (2005) and Afonso and Galvão (2007). The initial condition is $x(0) = [-0.2, 0]$ and the system constraints are: $u_{min} = -3V$,

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$u_{max} = 5V$, $y_{min} = -0.02V$ and $y_{max} = 0.01V$. The model matrices resulting after linearization and discretization are:

$$A = \begin{bmatrix} 1.0108 & 0.0050 \\ 4.3185 & 1.0108 \end{bmatrix}, \; B = \begin{bmatrix} -0.0142 \\ -5.6779 \end{bmatrix}, \; C = \begin{bmatrix} 1 & 0 \end{bmatrix} \tag{30}$$

The finite-horizon formulation was employed with: $N = 20$, $M = 4$, $\rho = 1$ and $\mu = 0.05$. In both cases, the output is considered to be in the vicinity of the boundary of the feasible region if approaches a distance 10% of the range. When infeasibility occurs, the control action is set to zero, which is corresponding to the equilibrium control value in the non-linear case. The simulations are made using the discrete model and the only mismach between the design model and the real plant are caused by the gain uncertainty. In this case, all states are considered available. All simulations are carried out using MatLab 6.5 software.

5. RESULTS AND DISCUSSIONS

For the nominal case, Fig. 4 shows the output variable in the both formulation, finite-horizon and lmi-based, firstly without anti-windup scheme (a) and then with the proposed scheme (b). It is possible to verify that the settling-time decreases significantly.

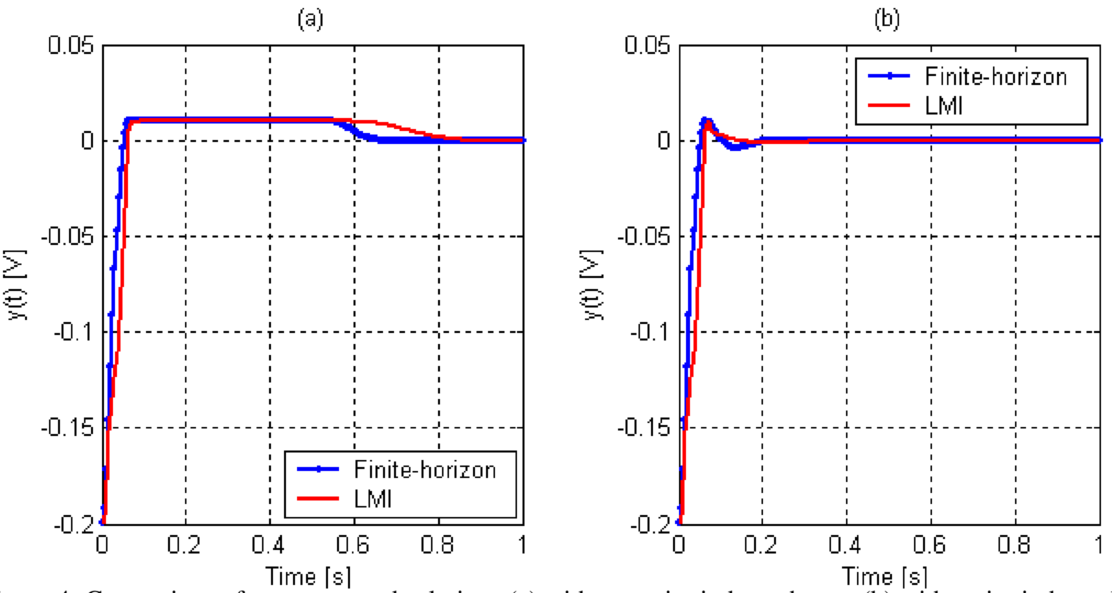

Figure 4. Comparison of two presented solutions:(a) without anti-windup scheme. (b) with anti-windup scheme.

As mentioned in Matos, Galvão and Yoneyama (2008) paper, reductions in the value of the process gain may cause violations of the system output constraints. For the same initial condition, $x(0)$, comparisons were made between the finite-horizon and the LMI-based robust predictive control solutions. Initially, the behavior for a 0.90 nominal gain was analyzed, with the two formulations without taking into account the robustness (with $\epsilon = 1$ and $\Omega = [(A, B)]$). In this case, both solutions have not been able to satisfy the constraint representing the maximum system output value $y_{max} = 0.01$ (see Fig. 5).

The comparison using the robust versions ($\epsilon = 0.90$ and $\Omega = [(A, B), (A, 0.90B)]$) it was found that the two solution respect the output constraints (Fig. 6).

Finally, an uncertainty of $K_{real} = 0.85K_{nominal}$ was imposed. Again, for the case without robust treatment, the two solutions violated the upper y constraint as shown in Fig. 7.

To $\epsilon \leq 0.88$, the Finite-horizon formulation presents infeasibility of the optimizing problem beyond constraints violations. On the other hand, the LMI-based robust approach was able to obtain a solution without constraints violations up to $\epsilon = 0.26$.

6. CONCLUSIONS

The case study presented in this work indicates that robust formulations of MPC may be of paramount importance in plants such in magnetic levitations systems. The advantage of the LMI-based formulation over the finite-horizon problem became evident when the uncertainty range is trivial. An anti-windup scheme was conceived to circumvent the

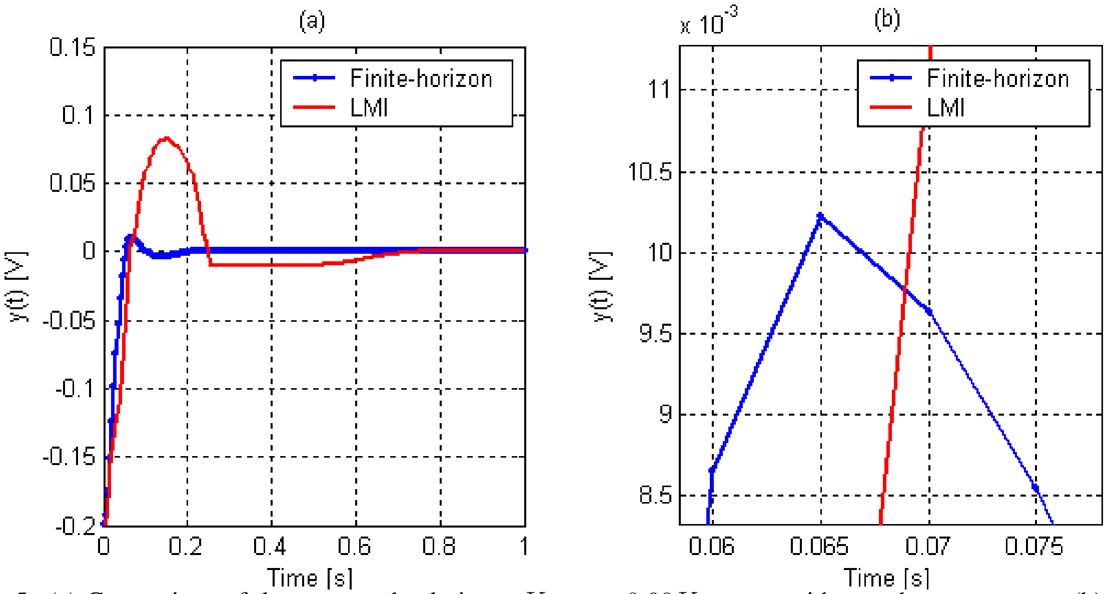

Figure 5. (a) Comparison of the presented solutions: $K_{real} = 0.90K_{nominal}$ without robust treatment. (b) Constraint violation detail.

inconvenience of the increase in setting time caused by the integral control action inclusion. The introduced modification allowed the inclusion of an overshoot constraint in the LMI-based MPC formulation. It is noted that LMI formulations have high computational processing time so that it is interesting to study methods to mitigate it. Wan and Kothare (2003) paper presents an off-line approach faster than on-line LMI-based MPC technique, which determines an sequence of control laws corresponding to a sequence of asymptotically stable invariant ellipsoids.

7. ACKNOWLEDGEMENTS

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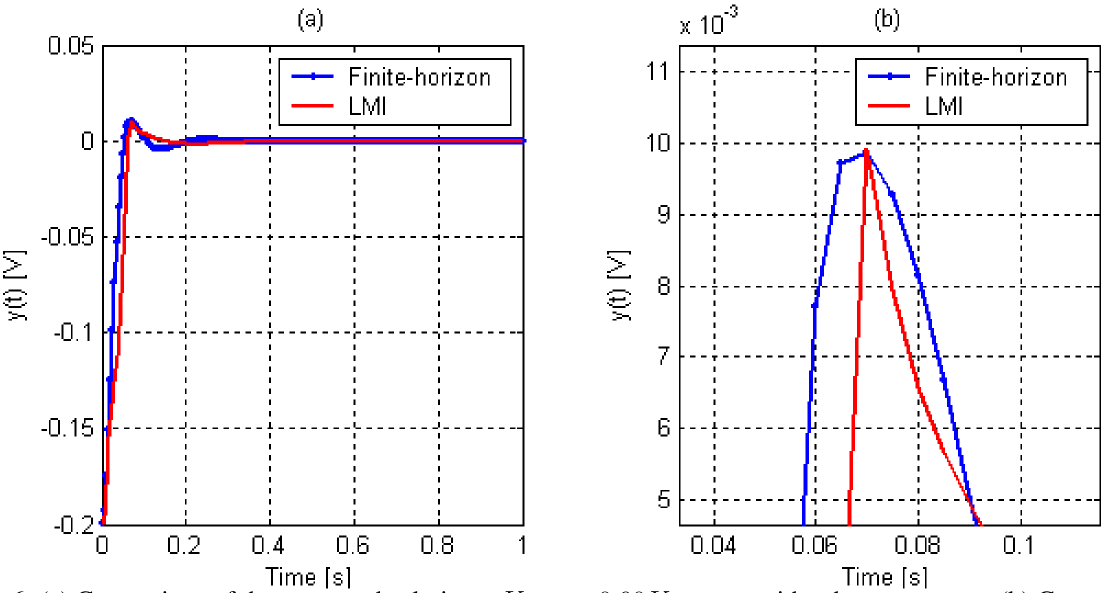

Figure 6. (a) Comparison of the presented solutions: $K_{real} = 0.90K_{nominal}$ with robust treatment. (b) Constraint detail.

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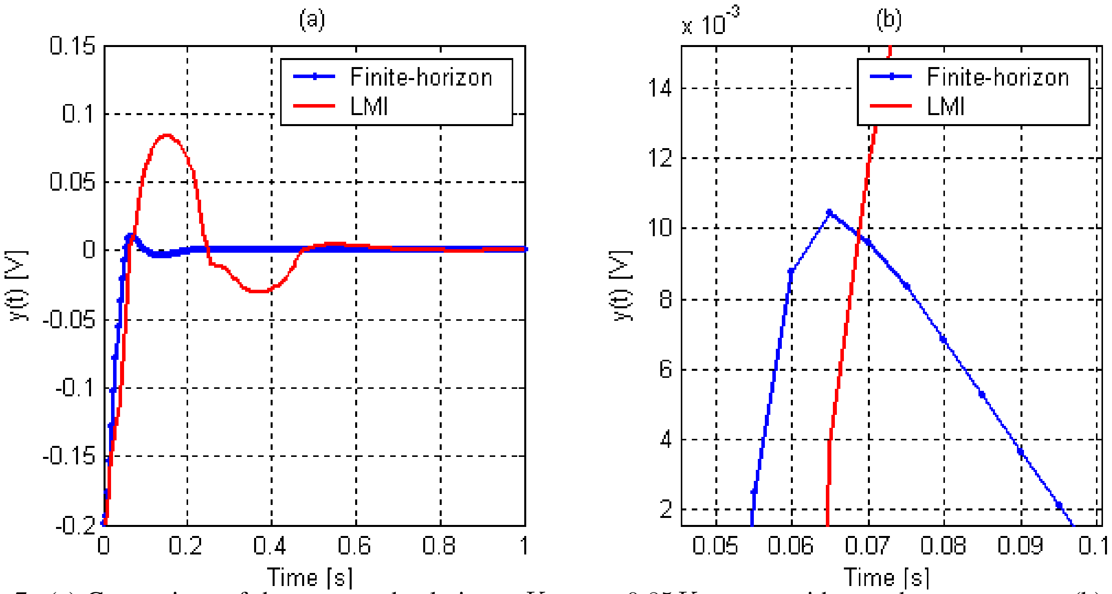

Figure 7. (a) Comparison of the presented solutions: $K_{real} = 0.85K_{nominal}$ without robust treatment. (b) Constraint violation detail.

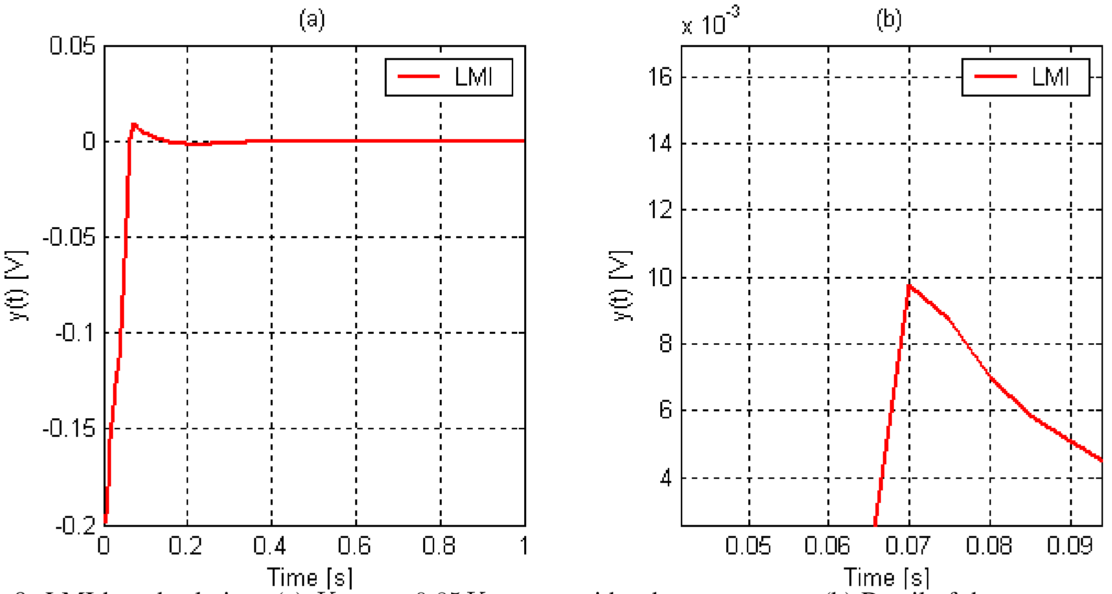

Figure 8. LMI-based solution: (a) $K_{real} = 0.85K_{nominal}$ with robust treatment. (b) Detail of the system output in the vicinity of the constraint.