ABCM Symposium Series in Mechatronics - Vol. 4 - pp.264-269

FINITE ELEMENT MODELING OF EDDY-CURRENT COUPLERS

Joaquim Girardello Detoni, joaquim.detoni@polito.it
Politecnico di Torino, Mechatronics Laboratory, Corso Duca degli Abruzzi 24, Torino (TO), CAP 10129, Italy

Walter Jesus Paucar Casas, walter.paucar.casas@ufrgs.br
Universidade Federeal do Rio Grande do Sul, Departamento de Engenharia Mecânica, Rua Sarmento Leite 425, Porto Alegre - RS, CEP 90050-170, Brazil

Abstract. *Eddy-current systems have been employed in many industrial applications such as in braking, transmission or damping systems. For coupling applications, the main advantage with respect to other devices is the possibility of working in complete absence of mechanical contact, eliminating wear problems and reducing the need for high accuracy alignment techniques, typically required in mechanical coupling systems. In this work, the analysis of an eddy-current coupler is performed using both analytical and numerical methodologies. The analytical solution is developed using a lumped parameter model of a conductor moving in a magnetic field. The construction and analysis of the numerical model is done in a partial differential equation solver, and the results are obtained by the insertion of the Maxwell's equations in terms of the magnetic vector potential directly into the solver. The studies are developed by means of simplified models, and the validity of the approximations is shown by comparison of the results with experimental data obtained from bibliography.*

Keywords: *Eddy-current, coupling systems, finite element, electromagnetism*

1. INTRODUCTION

The working principle of eddy-current couplers relies on the interaction between a conductor in motion with respect to a magnetic field. This motion determines an electric field in the conductor that induces eddy currents. The eddy-currents interact with the same magnetic field that induced them to create the coupling effect. Usually a coupler is constituted by a magnetic circuit with several magnetic pole pairs oriented in attraction, and a conductor disk that spins in the air gap. Figure 1 shows a possible configuration for the coupler with 4 pole pairs, where R_m is the radial position of the center of the magnets and R_d is the conductor disk radius. The main advantage of this kind of configuration is the possibility of working without mechanical contact, eliminating wear problems and reducing the need for high accuracy alignment techniques typically required in mechanical coupling systems. Furthermore, the employment of high energy product rare earth permanent magnets and high conductivity materials allow good coupling performance.

The design techniques and analysis of eddy-current couplers are already well consolidated, and many works on the subject can be found in the literature. The solution approach to the problem can be purely numerical (Nehl *et al.,* 1994), purely analytical (Tonoli, 2007) or hybrid (Canova and Vusini, 2003). In order to get deeper insight, experimental characterization can be carried out, and in a recent work Amati and Tonoli (2008) showed the procedure to obtain design parameters from experimental tests and the importance of these parameters for design of eddy-current dampers and couplers in dynamic conditions.

In this work a finite element scheme is used to solve a 3D model of an eddy current coupler. The analytical solution of a conductor in linear motion in a magnetic field is developed based on the equivalent mechanical model developed by Amati *et al.* (2008) in order to obtain the design parameters of the eddy current coupler. Next, the governing equations are directly introduced in a partial differential equation solver.

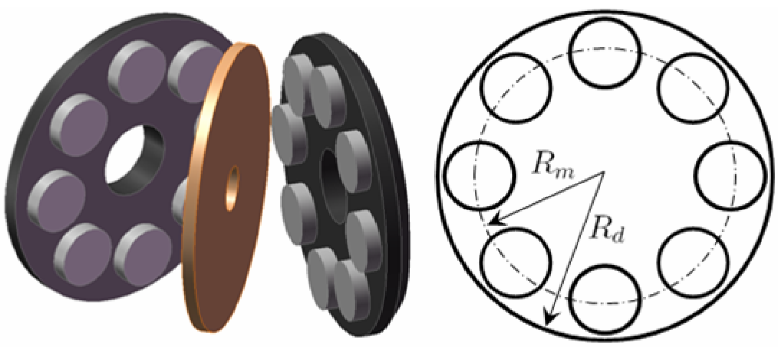

Figure 1. Typical configuration of an eddy current coupler.

ABCM Symposium Series in Mechatronics - Vol. 4 - pp.264-269

2. LUMPED PARAMETER MODEL OF A LINEAR EDDY-CURRENT COUPLER

The relative motion between a conductor and a magnetic field generates an electric field that is governed by Faraday's law and is oriented according to Lenz law. In its simpler form an eddy-current coupler can be represented as a lumped parameter electric system of a N turns coil shunt-connected to a resistance R and an inductance L, with velocity v relative to a perpendicular magnetic field B, as shown in Fig. 2. For this system, the induced electric field is proportional to the velocity of the relative motion and results in an induced electromotive force ε_{emf} between the circuit terminals.

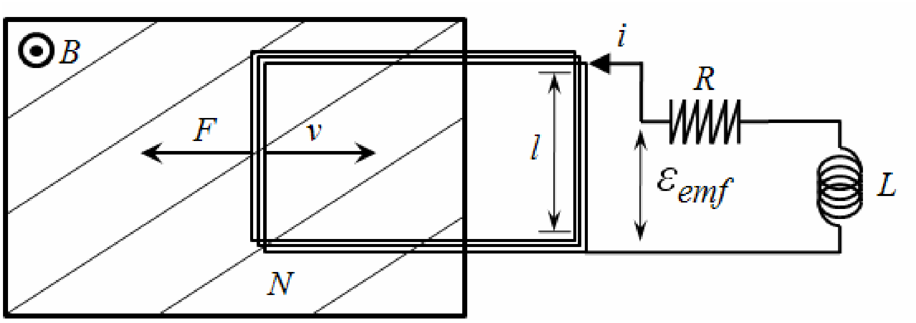

Figure 2. Lumped parameter model of an eddy-current coupler

As the circuit is closed an electric current i will flow, and its interaction with the constant magnetic field generates a force F proportional to the current and in opposite direction with respect to the velocity. This phenomenon is governed trough the state equation given by Eq. (1) and the output expression is specified by Eq. (2).

$$\frac{di}{dt} = \omega_{RL}\ i + \frac{K_m}{L} v \tag{1}$$

$$F = K_m\ i \tag{2}$$

where the electromechanical constant K_m is equal to NBl, and the frequency of the electric pole of the system ω_{RL} is equal to R/L.

Equations (1) and (2) correspond to the same formalism of a mechanical system formed by a spring in series with a viscous damper. This analogy allows to obtain more straightforward results because of the coupler can be best described as a mechanical system , and Eqs. (3) and (4) show the new set of equations to be solved.

$$\dot{x} = \omega_{RL}\ x + v \tag{3}$$

$$F = k\ x \tag{4}$$

where the natural frequency ω_{RL} is equal to k/c, the equivalent stiffness k is equal to K_m/L and the damping coefficient c is equal to K_m/R, all corresponding to the mechanical representation.

In the eddy-current coupler, the relative velocity between the conductor and the magnetic field can be approximated as the tangential speed of the conducting disk at the mean radius of the magnets R_m. When the conductor is in motion with constant slip speed Ω, the magnetic field seen by each point of it at radius R_m is approximately sinusoidal. This allows that Eq. (1) can be solved considering that the coil in Fig. 2 moves at a velocity v with harmonic profile and angular frequency Ω, in the magnetic field frame of reference.

Then, Eq. (5) gives the torque obtained from the coupler as function of the slip speed. This expression has the classic form of the curve torque versus slip speed of an induction machine running at constant speed, and it is obtained by multiplying Eq. (2) times R_m.

$$T = \frac{c\ \Omega\ R_m^2}{1 + \left(\dfrac{\Omega}{\omega_{RL}}\right)^2} \tag{5}$$

Two design parameters are necessary to build the torque - slip speed curve of an eddy current coupler using Eq. (5). They are the angular frequency ω_{RL} of the electric pole and the equivalent damping coefficient c, which can be obtained by either numerical simulations or experimental tests.

3. FINITE ELEMENT SCHEME

For a given coupler, a finite element model can be used for obtaining more approximate design parameters. The eddy-current coupler, as seen before, behaves as a typical induction machine and for simulating this kind of behavior a finite element scheme using a partial differential equation (PDE) solver is used. In this section, an approaching method using the direct insertion of Maxwell equations into the solver is presented.

3.1. Fomulation of the eddy-current problem

An induction problem can be characterized by the solution of Eqs. (6) and (7), that represent the laws of Faraday and Ampere respectively.

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{6}$$

$$\nabla \times \vec{B} = \mu\ \vec{J} \tag{7}$$

These two equations show that time-varying solution of electromagnetic problems with finite elements require the discretization of both the problem regions and the time domain, using time-stepping methods (Henneberger *et al*, 2000). According to Rodger *et al*. (2002), moving conductor eddy-current problems can be solved in steady state conditions by means of a Minkowski transformation provided that the conductor has invariant cross-section at right angles to the direction of motion (*smooth* conductor). This approach uses the quasi-static approximation, and Lorentz current density terms accounts for the motional eddy-currents. This procedure eliminates the need for time-stepping solutions.

The utilization of a magnetic vector potential $\vec{A}$ formulation for writing Eqs. (6) and (7) gives the governing equations for a moving smooth conductor eddy-current problem in a formalism more adequate to our purposes. If only linear materials are used, Eq. (8) gives the equation to be solved in the conductor disk and Eq. (9) gives the equation to be solved in all other domains of the problem.

$$-\frac{1}{\mu}\nabla^2 \vec{A} = -\sigma\,\vec{v}\cdot\nabla\vec{A} + \vec{A}\times\omega \tag{8}$$

$$-\frac{1}{\mu}\nabla^2 \vec{A} = 0 \tag{9}$$

where $\vec{v}$ is the conductor's speed and ω is its angular velocity.

3.2. Setting the PDE problem

To solve the eddy-current coupler problem, a finite element solver for partial differential equations was chosen. In this kind of approach a general form equation is given, and many types of physical problems can be solved directly by introducing the properties of the problem in the general equation. Equation (10) shows the expression to be solved in each region of the coupler problem. In this case, the representative coefficients c, γ, a and β must be obtained by manipulation of Eqs. (8) and (9), resulting in a direct insertion of Maxwell's equations into the solver.

$$\nabla\cdot\left(-c\ \nabla\vec{u} + \gamma\right) + a\vec{u} + \beta\cdot\nabla\vec{u} = 0 \tag{10}$$

From the conceptual point of view, the eddy-current coupler contains only three different regions. These are the conductor disk, where the induction phenomenon takes place; the permanent magnets that generate the magnetic field;

ABCM Symposium Series in Mechatronics - Vol. 4 - pp.264-269

and the back iron and surrounding air, where the last two (back iron and surrounding air) can be represented using the same coefficients. Equations (11), (12), (13) and (14) give the values of the coefficients that characterize the governing equations for each region in terms of classical electric and magnetic quantities for coordinates x, y and z. Using these coefficients it's possible to do modeling in two or three dimensions.

$$c_{x,y,z} = \frac{1}{\mu} \tag{11}$$

$$\begin{cases} \gamma_x = \{ \quad 0 & -\mu_0 M_z & \mu_0 M_y \ \} \\ \gamma_y = \{ \ \mu_0 M_z & 0 & -\mu_0 M_x \} \\ \gamma_z = \{ -\mu_0 M_y & \mu_0 M_x & 0 \quad \} \end{cases} \tag{12}$$

$$\begin{cases} a_x = \{ \quad 0 & \omega_z & -\omega_y \} \\ a_y = \{ -\omega_z & 0 & \omega_x \ \} \\ a_z = \{ \ \omega_y & -\omega_x & 0 \quad \} \end{cases} \tag{13}$$

$$\beta_{x,y,z} = -\sigma \vec{v} \tag{14}$$

From this setting the solution for the magnetic vector potential is obtained. More meaningful quantities such as the magnetic flux density $\vec{B}$, the electric current density $\vec{J}$ and the Lorentz force $\vec{F} = \int \vec{J} \times \vec{B}\, dv$ can be calculated through post-processing.

3.3. Finite element model

The finite element model of the eddy-current coupler is constructed using the anti-symmetry plane in the middle of the conductor and a sector of 90° is considered to take advantage of the model's cyclic symmetry, resulting in a simulation model of one eighth of the total problem geometry.

The geometric dimensions and properties of the problem are given in Tab. 1. All characteristics were considered to be linear. No thermal or saturation effects are taken into account as they are considered to be irrelevant in this case.

Table 1. Geometric and physical parameters of the eddy current coupler

Symbol	Geometric parameter	Value [mm]
g	Air gap	1
S_d	Disk thickness	7
S_m	Magnet thickness	6
M_r	Magnet radius	30
R_d	Conductor radius	80
R_m	Radial position of the magnets	55
	Physical parameter	Value
σ	Electric conductivity of copper	5.998E7 [AV^{-1}]
B_r	Remanent magnetic flux density	1.22 [T]
μ_0	Magnetic permeability of vacuum	4πE-7
μ_r	Relative magnetic permeability of iron	4000

Figure 3 shows the eddy-current distribution in the conductor disk as a consequence of its relative motion in the magnetic field. It is true that the proposed scheme is capable of taking into account the coupling between the two directions of motion and the static magnetic field. Also the cyclic boundary conditions ensure the continuity of the quantities thru them.

As the angular velocity grows, the velocity currents increase and their distribution in the conductor change due to the *RL* dynamics. Figures 4a and 4b show the Lorentz forces and magnetic flux density distribution at low frequency and at the electric pole frequency respectively.

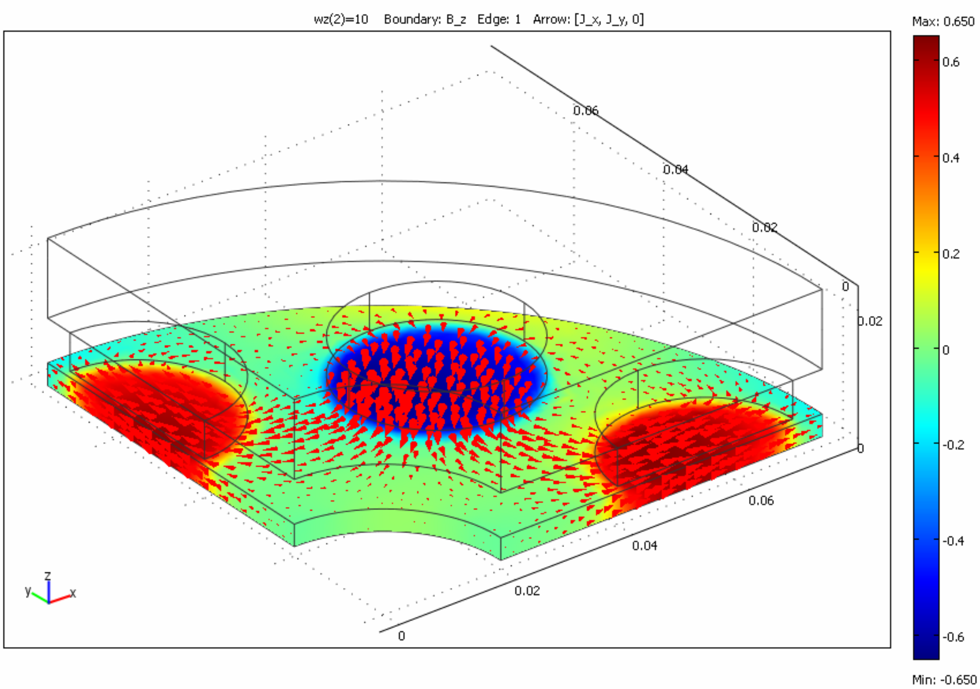

Figure 3. Three dimensional magnetic flux density and eddy-current distribution showed as arrows at 10 rpm

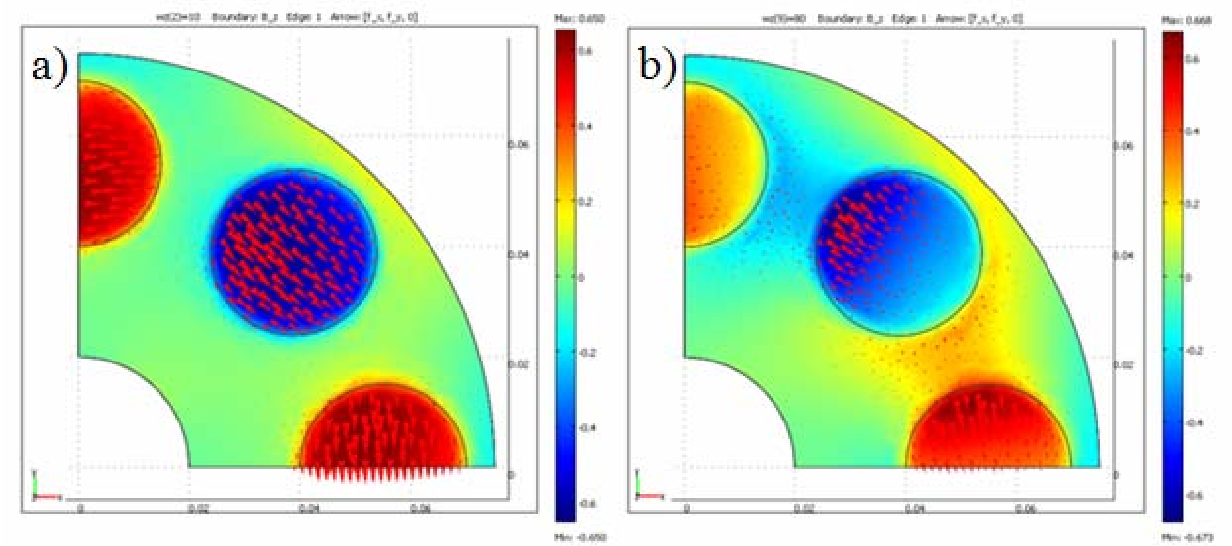

Figure 4. Lorentz force distribution showed as arrows in the conductor, a) at low frequency (10 rpm), b) at electric pole frequency (860 rpm)

To obtain the coupling torque, the integration of the Lorentz force over the disk domain is carried out and Fig. 5a shows the comparison of the obtained results in the numerical model with experimental data obtained from the bibliography (Amati and Tonoli, 2008). The perfect correspondence between the two sets of data confirms the validity of the hypothesis assumed and the accuracy of the finite element scheme used to solve Maxwell's equations. The equivalent mechanical design parameters can be obtained from the analytical model. A curve fitting between the numerical data and the analytical curve is done and Fig. 5b shows the comparison between these curves. An equivalent viscous damping of $c = 392.5\,[\mathrm{Nsm}^{-1}]$ and a pole frequency of $\omega_{RL} = 860\,[\mathrm{rpm}]$ is obtained.

At higher frequencies the analytic and numerical curves deviate a little probably due to the bi-dimensional nature of the analytical model that is not capable of taking into account some of the effects that are inherently three-dimensional.

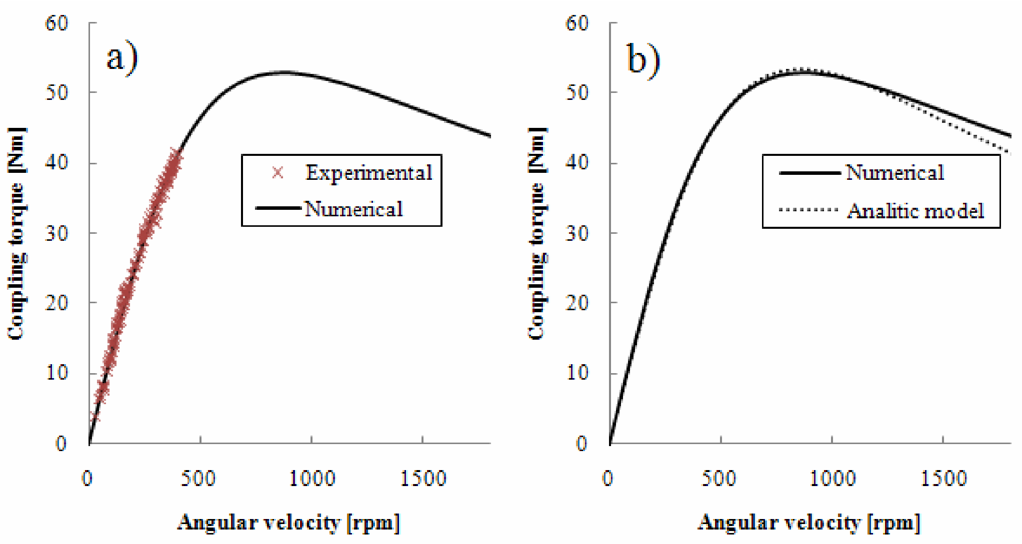

Figure 5. Comparison of results obtained by analytical, experimental and numerical modeling.

4. CONCLUSIONS

In this work a finite element scheme using the direct insertion of the governing equations in a partial differential equation solver is presented and used to solve a 3D model of an eddy current coupler. The procedure is validated for the simulation of such components by comparison of the obtained results with experimental data obtained from literature.

The analytical solution of a conductor in linear motion in a magnetic field is developed as an equivalent lumped parameters model of the eddy-current coupler and a curve fitting of the analytical curve is used to obtain the mechanical equivalent design parameters.

Concluding, both the analytical model and the numerical scheme allow for good results and can be used as design tools for eddy-current couplers and similar devices.

5. ACKNOWLEDGEMENTS

The authors gratefully acknowledge the support given to this activity, developed as part of the master thesis of JGD at Politecnico di Torino, the Mechatronics Laboratory and its staff. The results wouldn't be possible without the help from Professors Andrea Tonoli and Nicola Amati, and from Xavier De Lépine who gave many contributions to the understanding of the principles and concepts existent in the problem.

6. REFERENCES

Amati, N. and Tonoli, A., 2008, "Dynamic Modeling and Experimental Validation of Eddy-Current Dampers and Couplers", Journal of Vibration and Acoustics, Vol. 130, Issue 2, pp. 021011-1-021011-9.

Amati, N., De Lépine, X. and Tonoli, A., 2008, "Modeling of Electrodynamic Bearings", Journal of Vibration and Acoustics, Vol. 130, Issue 6, pp.061007-1-061007-9.

Canova, A. and Vusini, B., 2003, "Design of Axial Eddy-Current Couplers", IEEE Transactions on Industry Applications, Vol. 39, Issue 3, pp.725-733.

Hennenberger, G., Arians, G. and Albertz, D., 2000, "Comparison Between Transient and Quasi-Stationary Calculations of Eddy-Current Field Problems with Moving Conductors", Compel: International Journal for Computation and Mathematics in Electrical and Electronic Engineering, Vol. 19, Issue 2, pp. 173-179.

Nehl, T. W., Lequesne, B., Gangla, V., Gutkowski, S. A., Robinson, M. J. and Sebastian, T., 1994, "Nonlinear Two-Dimensional Finite Element Modeling of Permanent Magnet Eddy Current Couplings and Brakes", IEEE Transactions on Magnetics, Vol. 30, Issue 5, pp. 3000-3003.

Rodger, D., Lai, H. C. and Leonard, P. J., 2002, "A Comparison of Finite-Element Models of 3-D Rotating Conductors", IEEE Transactions on Magnetics, Vol. 38, Issue 2, pp. 537-540.

Tonoli, A., 2007, "Dynamic Characteristics of Eddy-Current Dampers and Couplers", Journal of Sound and Vibration, Vol. 301, pp. 576-591.

5. RESPONSIBILITY NOTICE

The authors are the only responsible for the printed material included in this paper.