

# SIMULATION OF AN OMNIDIRECTIONAL CATADIOPTIC VISION SYSTEM WITH HYPERBOLIC DOUBLE LOBED MIRROR FOR ROBOT NAVIGATION

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**Abstract** *An Omnidirectional Catadioptric Vision System consists of a convex mirror in the front of a camera to get a 360° view of the observed environment. The association of a hyperbolic mirror with a camera of perspective projection produces only one center of projection, generating images free of distortions. Geometric information from an unknown environment can be obtained with stereo vision techniques and one of the limitations of the stereo omnidirectional vision is the formation of an invisible region of the space in the motion direction. A proposed solution to this problem is the use of a hyperbolic double lobed mirror. With this type of mirror the pair of stereo images is acquired at the same instant without the need of known relative displacements. More than the solution of this problem is the possibility of working with dynamic environment mapping and the reduction of the processing time at the stage of the correspondence between image pairs, since the search of corresponding points is limited to a radial line. In this work it is presented a model for simulation of the image projections from a virtual hyperbolic double lobed mirror, aiming at the determination of the ideal profile of the mirror and the simulation of its use in specific applications as robot navigation.*

**Keywords:** *Omnidirectional Vision, Hyperbolic Double Lobed Mirrors, Machine Vision, Robot Vision*

## 1. INTRODUCTION

An Omnidirectional Catadioptric vision system consists of a convex mirror fixed in the front of a camera to make possible the acquisition of an image around 360 degrees of the surrounding environment. This wide angle image has the potential to be used in diverse areas, such as robot navigation, visual servo control and teleoperation. The navigation of mobile robots using this system allows a high degree of autonomy and therefore has been the focus of many researchers. (Nayar, 1997, Winters, 2001, Deccó, 2004, Svoboda *et al.*, 1995).

The mirrors used by most of researchers in omnidirectional vision up to date can acquire only a single image for each robot position and therefore the robot has to move to other location to get a pair of stereo images to construct a 3D map of the environment. One limitation of this strategy is the existence of an invisible region of the environment in the direction of the robot motion, turning the system to be improper for navigation in environments with many obstacles and in dynamic environments.

An alternative to overcome the limitations commented above in robotic navigation is the use of a double lobed mirror in such a way that one lobe is displaced vertically relative to the other. This type of mirror can acquire stereo images of the environment overcoming the need to move the robot to various locations, making possible the use of single camera to fulfill the mapping of a dynamic environment. The map can be generated from images acquired at the same instant, reducing the computational cost and speeding up the process.

The proposal of this article is to describe the main issues involving the project of a double lobed mirror profile and to carry out a computer simulation to test the mirror performance. In section 2 the catadioptric system is described with its advantages and disadvantages, and also the equations that make possible the accomplishment of the project of a hyperbolic double lobed mirror. In section 3 it is described how to extract geometry information from an unknown environment by using stereo vision, as well as its limitations. Section 4 explains how a double lobed mirror can solve the limitations described in section 3. In section 5 the double lobed mirror parameters are defined and explained and how it was designed. In section 6 the computer simulation results of the image acquisition process with the double lobed hyperbolic mirror is shown to check the system and the parameters calculated. The conclusion is presented in section 7.

## 2. OMNIDIRECIONAL CATADIOPTIC VISION SYSTEM

An Omnidirectional Catadioptric Vision System can be constructed with a camera with its field of vision directed to a mirror that reflects the surrounding environment. Among these systems one can find several setups, but the most important differences between them are the mirror profile, the camera model and the type of lens.

The mirror profile can vary depending on the application and the available technology to manufacture it. Amongst the various mirror profiles that had been studied so far, it can be cited the spherical mirror (Gaspar *et al.*, 2001), (Vassallo, 2004, Grassi-Jr, 2002), the conical mirror (Yagi, 1999), mirrors designed to preserve in the image the

geometry of a perpendicular plan to the mirror symmetry axis (Hicks and Bajcsy, 1999), mirrors designed to preserve in the image the object dimensions when it moves in a vertical plan (GÄCHTER et al, 2001), the parabolic mirror (Nayar, 1997) and the hyperbolic mirror (Corrêa, 2002, Grassi-Jr, 2004).

The mirror profile can also be designed in association with other profiles, such as the triple lobed mirror (CASPAR et al, 2002). This type of mirror can be seen in (Fig. 1). The superior profile represented by M3 reflects light on the camera CCD in the R3 region, projecting the area along the vertical line L2. The profiles represented by M1 and M2 reflect light on the camera CCD in the regions R1 and R2 respectively, projecting the region along the horizontal line L1.

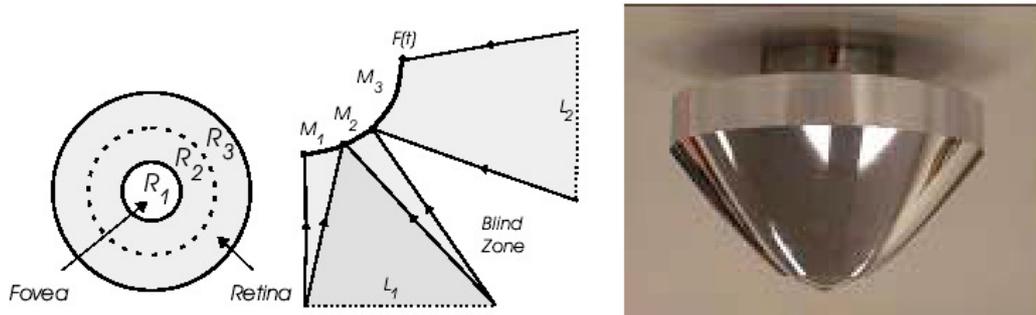


Figure 1. A triple lobed mirror (Gaspar et al., 2002)

## 2.1. Single Projection Centre

For one to get a clear image from a spherical mirror with radius,  $R$ , as shown in Fig. (2), it is necessary that the incident ray,  $F$ , from the light source  $P$  reaches the surface  $S$  of the mirror parallel or lightly inclined relative to the mirror main axis,  $K$ , that means, the incidence angle,  $i$ , has to be small. Thus, for the mirror to reflect a clear image, the mirror aperture angle,  $j$ , has to be smaller than  $10^\circ$ . This is a characteristic of the Gauss' spherical mirror.

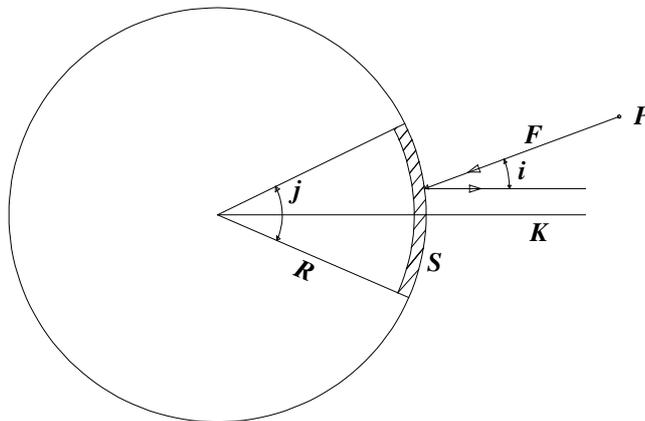


Figure 2. Spherical mirror.

When the mirror aperture angle increases, the light rays do not intersect the mirror focus, producing various projection points and turning the image to be blurred. When an omnidirectional vision system has only a single projection center, each point of the three-dimensional space has only one projection on the camera CCD, generating an image free of distortion and blurring.

One of the ways to get only one projection center is by using a mirror with a parabolic profile,  $S_p$ , associated with a camera with orthographic projection (Fig. 3a) (Nayar, 1997, Winters, 2001). The light beams originated from the points  $a1$  and  $b1$  are reflected on the  $S_p$  mirror surface at the points  $a2$  and  $b2$  and projected to the camera CCD on the points  $a3$  and  $b3$  respectively. Since the mirror profile is parabolic the reflected rays  $a2-a3$  and  $b2-b3$  will also be parallel. This is a characteristic of the orthographic projection camera, turning this type of camera the proper for parabolic mirrors.

Another way to get only one center of projection is the association of a mirror with hyperbolic profile,  $Sh$ , with a camera with perspective projection Fig. (3, b) (Deccó, 2004), (Winters 2001) e (Grassi-Jr., 2002). The light beams originated from the points  $a1$  and  $b1$  are reflected by the  $Sh$  mirror surface at the points  $a2$  and  $b2$  and projected to camera CCD on the points  $a3$  and  $b3$  respectively. Since the mirror profile is hyperbolic the reflected rays  $a2-a3$  and  $b2-b3$  are directed to the focus  $F2$ . This is a characteristic of the perspective projection camera, turning this type of camera suitable for hyperbolic mirrors.

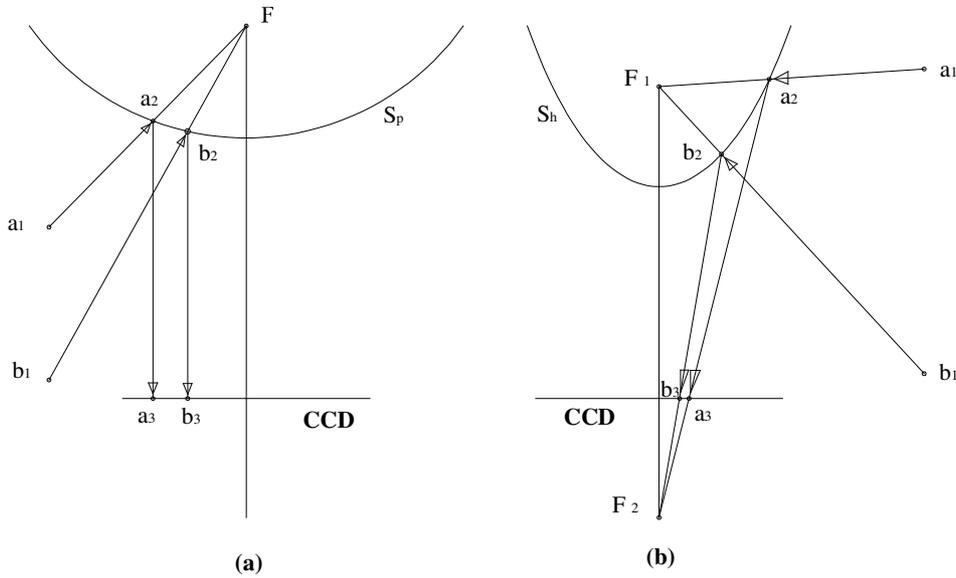


Figure 3. Hyperbolic (a) and parabolic (b) mirror profiles

## 2.2. Hyperbolic Mirror Profile

A hyperbole is defined as the intersection between a conical surface and a plane. The curve generated in the intersection is the set of all the  $C_n$  points for which the difference between their distance from two focus  $F1$  and  $F2$  is constant, as shown in Fig. (4).

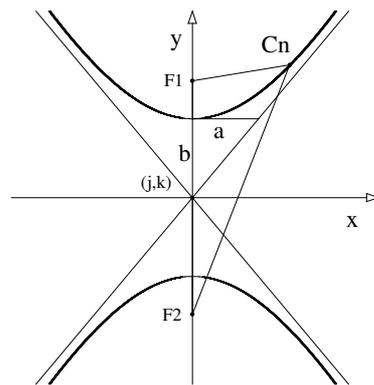


Figure 4. Hyperbole

The hyperbole of Fig. (2) is defined by Eq. (1):

$$\frac{(y-k)^2}{a^2} - \frac{(x-j)^2}{b^2} = 1 \tag{1}$$

such that (j,k) it is the hyperbole center and the parameters  $a$  and  $b$  are the semi-axis. If it is considered that the hyperbole center is located at the origin (j=0 , k=0) one can simplify the equation above as:

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1 \quad (2)$$

If the origin of the coordinate frame is moved from the point (j, k) (Fig. 4) to the mirror focus F1 (Fig. 5) and the camera focus coincides with the mirror focus F2, one can have the geometry of the association between a hyperbolic mirror with a camera of perspective projection.

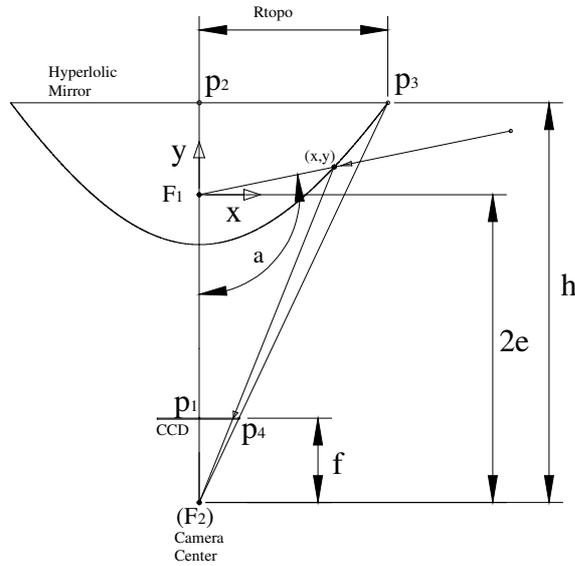


Figure 5. Hyperbolic mirror

In Fig. (5)  $f$  is the camera focal length,  $h$  is in the distance between the mirror surface and the camera center, so called mirror eccentricity,  $R_{topo}$  is the coordinate  $x$  of the top of the mirror and  $\alpha$  is the angle of vision. Eqs. (3, 4, and 5) relate these parameters.

$$e = \sqrt{a^2 + b^2} \quad (3)$$

$$y = \sqrt{a^2 \cdot \left(1 + \frac{x^2}{b^2}\right)} - e \quad (4)$$

$$\alpha = \frac{\pi}{2} + a \tan\left(\frac{h - 2e}{R_{topo}}\right) \quad (5)$$

For the omnidirectional vision system to use all the CCD capacity and to be compact, a good relation between parameters  $h$  and  $R_{topo}$  must be found empirically. A relation between the similar triangles  $F_1p_1p_4$  and  $F_1p_2p_3$ , in Fig. (5), can be formulated as in Eq. (6).

$$h = \frac{f \cdot R_{topo}}{T_{pixel} \cdot R_{pixel}} \quad (6)$$

, where  $R_{pixel}$  is the ray of the largest circumference that can be enrolled in the camera CCD, measured in pixel and  $T_{pixel}$  is the dimension of each pixel of the camera CCD, measured in millimeters. If the pixel cell is not squared it must be considered the dimension that is in the direction of the shorter width of the CCD rectangle.

The relation  $a/b$  of the semi-axis must be determined according to the mirror profile and angle of vision (Grassi Jr., 2002). In this work the relation  $a=2b$  was adopted.

The project of a hyperbolic mirror consists in the determination of a hyperbolic curve that in revolution generates the mirror surface (Fig. 6). The use of this profile in the construction of an omnidirectional catadioptric vision system has the advantage of having only one center of projection  $F_1$ , thus generating images free of distortions, where two points  $a_1$  and  $b_1$  of the  $L_1$  straight line generate only two corresponding points  $a_2$  and  $b_2$  respectively on the mirror surface and other two points  $a_3$  and  $b_3$  on the camera CCD respectively. Another issue that is very important is that a vertical straight line  $L_1$  in the environment generates a radial straight line  $L_2$  on the camera CCD, such as the extremities are the points  $a_3$  and  $b_3$ , making the identification of a straight line an easy task (Deccó, 2004).

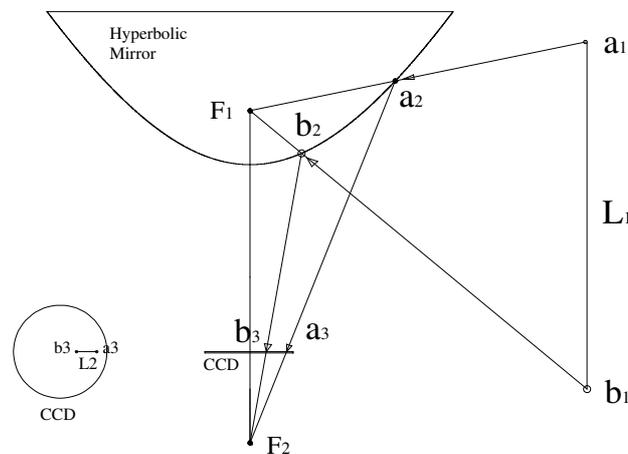


Figure 6. Projection of a straight line on the camera CCD with a hyperbolic mirror

For one to get an image of the environment by using a hyperbolic mirror associated to a camera of perspective projection it is necessary to transform the world coordinates into the coordinates of the mirror surface, that will be transformed into camera coordinates and then transformed to image coordinates in pixels (Deccó, 2004).

### 3. OMNIDIRECTIONAL STEREO VISION

One way to extract information from the geometry of an unknown environment is by using stereo vision. This technique reconstructs the geometry of three-dimensional objects from a pair of images acquired from different positions. The spatial displacement between the two cameras produces small differences in the location of the objects on the image, so called disparities. Using specific algorithms and knowing the camera parameters and the position of one camera relative to the other it is possible calculate the three-dimensional coordinates of an object using the disparities between the images on the two cameras.

A stereo algorithm must work basically with the extraction of specific image features, correspondences between similar features and further with triangulation between those corresponding features.

#### 3.1. Image features extraction

An image feature is an element of the environment that is projected onto the image and its position must be defined such that the three-dimensional reconstruction of the environment is possible. The feature can be points, such as the corners of a box, straight lines, such as the edges of a table and planes that can be represented by areas with sharp contours. The choice of the features to be searched is an important step of the work and for good results it must be considered the type of environment and vision system used. To locate a feature on an image, specific techniques for image processing must be used.

### 3.2. Correspondence between similar features

The correspondence between features in a pair of stereo images consists in identifying pairs of existing similar features in the two images. So, it is necessary to define a feature in the first image and to use a sweeping algorithm that finds the equivalent feature in the second image. This task is generally responsible for the largest span of time for processing in stereo vision. So, good algorithms and techniques must be used to reduce the computational cost.

### 3.3. Triangulation using similar features

With the previous knowledge of geometric parameters and using the pair of similar features it is possible to identify the distance between the features searched and the mirror focus by using a triangulation scheme. The triangulation process must follow the inverse direction of the light rays that generate the image. With the pair of identified similar features, it is possible to calculate two straight lines that have their origin in the mirror focus and the other extremity on the surface of the mirror, where the light ray from the environment is reflected. In the intersection of the prolongation of these two straight lines the origin of the generated images is located. From then on, these data can be used to generate the environment map, which can be of two or three dimensions.

### 3.4. Omnidirectional Stereo Vision System limitations

The techniques used to reconstruct a scene from a pair of omnidirectional stereo images restrict the effectiveness of the system. One of these limitations is the existence of an invisible space region, in the motion direction (Corrêa, 2004). The availability of only one camera and the need to capture at least one pair of stereo images requires the location of the system at two places to capture the two images. As the map of the environment is still not available, the motion to the second location may cause crashes with existing objects in the environment. Besides, it is necessary to measure the motion in the environment. In Fig. (7), the resultant optic flow from a translation motion (Vassallo, 2004) is used to estimate the position of selected features.

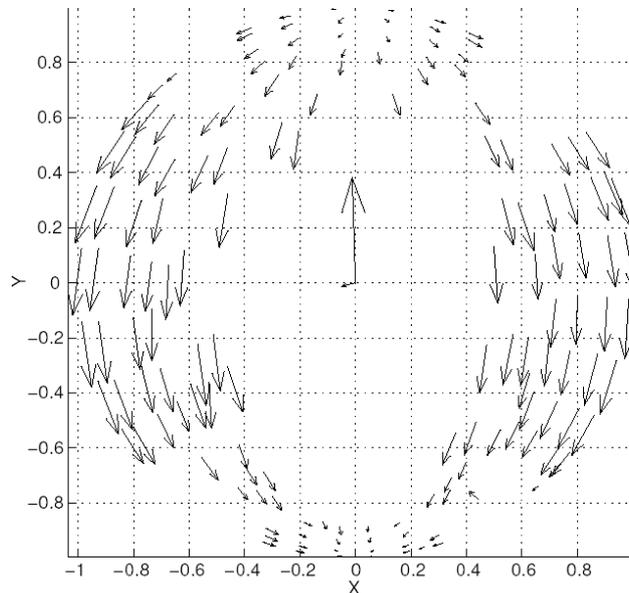


Figure 7. Estimation of the optical flux from motion (Vassallo, 2004).

From Fig. (7) one can notice that the optic flow vectors are shorter as much as they are closer to the central vertical line ( $x=0$ ), where the vector representing the motion of the system is. This vector is calculated by the vector addition of two vectors located at the central part of the map. When the optical flow vectors have a very small length it is impossible to calculate the distance between the mirror focus and the point that generated the optical flow. This explains the existence of an invisible region of the space, in the direction of the motion.

As the system needs move to acquire the second image of the pair of stereo images, mapping dynamic environments turns to be impossible. This can be explained by the loss of information caused by the simultaneous displacement of the system and the object.

In stereo vision, one of the most difficult steps and with higher computational cost is the correspondence between similar features. In this step it is necessary to sweep a large part of the second image to search for the correspondent image feature in the first image.

#### 4. PROPOSED SOLUTION

To reduce the limitations of the omnidirectional stereo vision system as explained in the previous section, a mirror with two hyperbolic lobes is proposed which can acquire simultaneously both images of a pair of stereo images, using a camera of perspective projection.

With the existence of two centered mirrors vertically, one above of the other, there is no need of displacements for the acquisition of the second image of the pair of stereo images, reducing the probability of crashing with existing objects in the environment and makes possible the use of the system in dynamic environments.

As the displacement between the images is vertical, the optic flow is radial and there will not be the formation of an invisible region of the space.

The processing time during the step of correspondence will be reduced, since the search of the corresponding point is limited to a radial line..

In this work the calculation of the double lobed mirror profile will be carried out along with a simulation of a mirror application. A real mirror will be manufactured from these calculations.

#### 5. DESIGN OF A DOUBLE LOBED HYPERBOLIC MIRROR

The double lobed mirror profile will be designed using the equations showed in section 2.2. Each profile corresponds to different parameters, but with the same sequence of application of the equations. The mirror profile had been calculated in a computer program developed in C language.

In Fig. 8 the inferior lobe is shown as the profile 1 mirror surface, and it is responsible for the visualization of region A. The superior lobe is shown as the profile 2 mirror surface which is responsible for the visualization of region B. The region of interest is the area of intersection AB, since all the objects that are within this area will be reflected by the two lobes and will compose a pair of stereo images. The regions that are not part of the intersection will be discarded during the image processing.

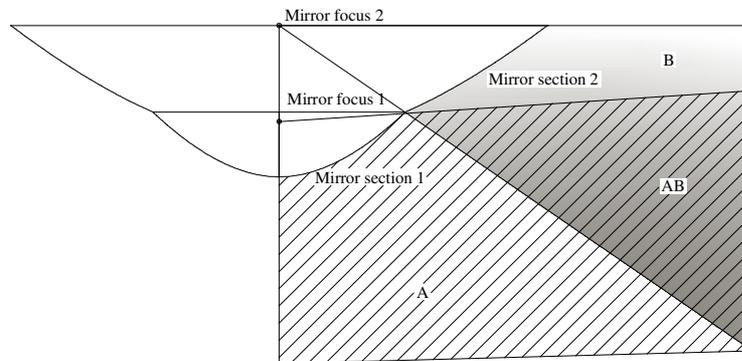


Figure 8. Double lobed mirror profile designed

For the vision system to be compact and be installed in the superior part of the robot it will initially be established the values of the  $R_{topo\ 1} = 20\text{ mm}$  and  $R_{topo\ 2} = 35\text{ mm}$ .

The lens have  $f = 16\text{ mm}$  and  $T_{pixel} = 0,009\text{mm}$ . The camera CCD is a rectangle of  $1008 \times 1018$  pixels, resulting in  $R_{pixel} = 504$  pixels.

With all the fixed parameters defined, the equations of item 2.2 will be empirically used to define the all the other necessary parameters to design the mirror and the catadioptric omnidirectional vision system, described in the Tab. (1).

Table 1. Vision System Parameters

CONSTANT PARAMETERS					CALCULATED PARAMETERS				
MIRROR	$R_{topo}$ (mm)	$f$ (mm)	$T_{pixel}$ (mm)	$R_{pixel}$ (pixel)	$c$ (adim)	$h$ (mm)	$a$ (mm)	$b$ (mm)	$\alpha$ (graus)
01	20	16	0,009	504	110,64	115,607	48,358	26,866	13,948
02	35	16	0,009	307	123,44	123,457	46,661	40,399	0,028

Using the mirror profile in Fig. (8) a 3D model can be generated (Fig. 9). This image was generated with AutoCAD™ representing precisely the hyperbolic double lobed mirror, whose parameters are expressed in the Tab. (1).

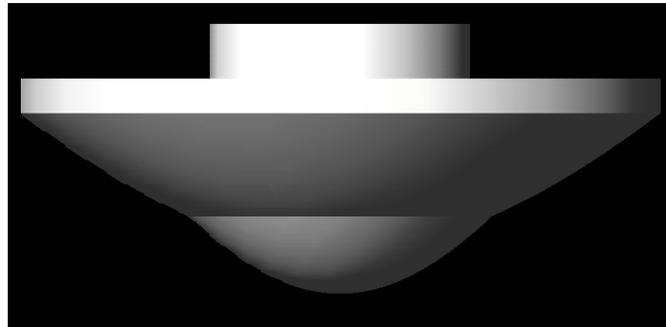


Figure 9. Solid generated from the revolution of the double lobed mirror designed.

## 6. SIMULATION OF IMAGE ACQUISITION WITH THE DOUBLE LOBED HYPERBOLIC MIRROR

The image acquisition using an omnidirectional catadioptric vision system with a hyperbolic mirror is complex and requires precision in the modeling of the mirror and the assembly of parts. A wrong choice of the mirror parameters can produce the visualization an undesirable environment region. When the mirror has two lobes this is more evident, since the two lobes must have only one visualization region. The images must form a stereo pair to allow depth estimation.

The image acquisition process will be simulated by using a hyperbolic double lobed mirror in a virtual environment within which objects have known positions and dimensions. The program used for this simulation belongs to a group of software whose objective is the creation of drawings in three dimensions. The POV-Ray is a free software and allows the creation of realistic drawings and has sufficient precision.

A block diagram with all steps of the simulation process is shown in Fig. (10).

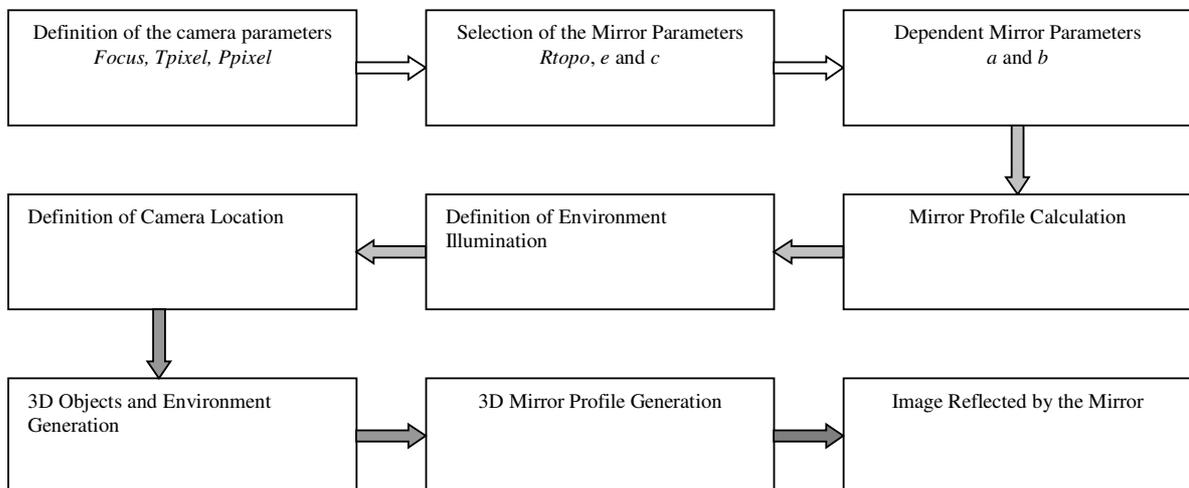


Figure 10. Block diagram showing the simulation process

The simulation with the double lobed mirror was carried out considering a square room with squared floor patterns in white and black, with square wall patterns in black and red or black and green. Three blue boxes of different sizes were located on the room floor (Fig.11).

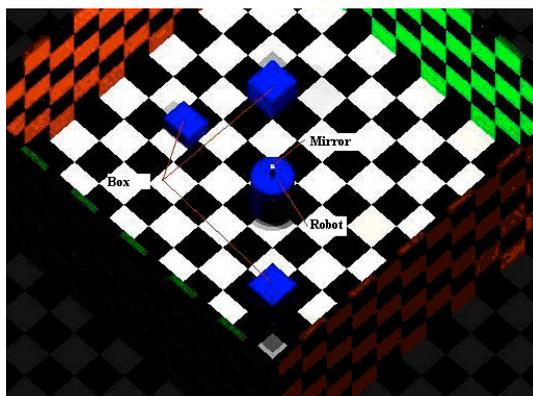


Figure 11. Simulated environment

The simulation results showed a clear image of the environment, where almost all objects and patterns were present on the two images (Fig. 12). There was a small difference between the images on each lobe, due to the fact that there are some regions that do not belong to the intersection of the images, fact that have already been commented before.

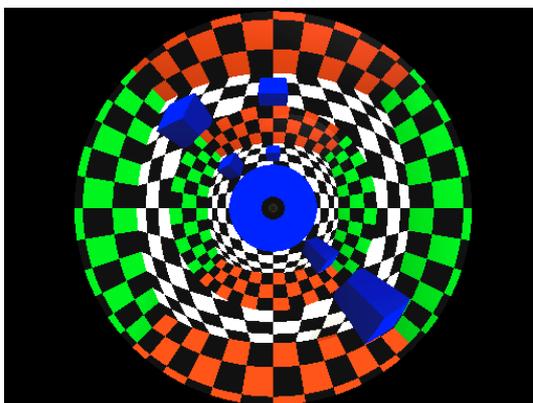


Figure 14. Simulation of the image acquisition process using the double lobed hyperbolic mirror

The simulation was carried out using coordinates with a resolution in the order of  $10^{-6}$ . The real mirror will be manufactured from an aluminum cylinder in a CNC lathe with resolution of 0,005 mm. Thus, aiming at assessing the surface quality of the machined mirror, the resolution and number of recorded points to simulate the new image were the same as those that will be used in the CNC, with a coarser resolution. For this simulation, 528 points were used, which are approximately the number of coordinates that the lathe is capable to record in its memory. The result is shown in Fig. (15).

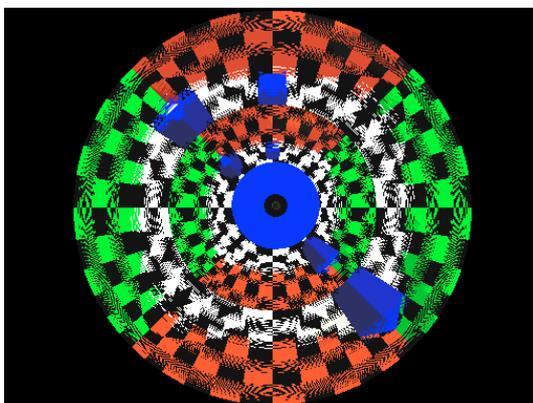


Figure 15. Simulation of the image with a coarse resolution

It is clearly observed that there are distortions in the image, and such optic effects are generated by the errors of rounding and truncations of the coordinates. The results suggest a manufacturing strategy, either by machining the mirror with a higher precision lathe or polishing the surface as long as the process does not deform the mirror profile.

## 7. CONCLUSIONS

In this article it was shown and discussed procedures to design a double lobed mirror specifically for omnidirectional catadioptric vision systems aiming at overcoming the main problem of robot navigation, which is the difficulty to acquire and process stereo images. A computer simulation to generate synthetic images was also described and results shown. The simulation process used a free software of high geometric precision as a platform to construct synthetic images, as long as the mirror profile is previously calculated and the environment conceived.

The parameters used for the development of the model had been chosen in order to allow the construction of a small size system, that can be used in mobile robot navigation.

The results obtained from the simulation using real coordinates showed to be satisfactory, making possible the use of this type of mirror in the construction of a real system. The simulation with a reduced number of points showed the necessity of using a polishing process to keep the desired mirror profile.

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