ABCM Symposium Series in Mechatronics - Vol. 3 - pp.19-26

ROBUST CONSTRAINED PREDICTIVE CONTROL OF A 3DOF HELICOPTER MODEL WITH EXTERNAL DISTURBANCES

Marcelo Handro Maia, handro@ita.br
Roberto Kawakami Harrop Galvão, kawakami@ita.br
Instituto Tecnológico de Aeronáutica, Divisão de Engenharia Eletrônica
São José dos Campos - SP, 12228-900, Brazil

Abstract. *Predictive control strategies have been widely used in industry for their ability to handle operational constraints. It is known that the presence of disturbances may cause predictive controllers to lose feasibility and to violate system constraints. This work addresses the implementation of a state-space predictive control law with restricted constraints to ensure feasibility and constraint fulfillment in spite of the existence of unknown but bounded disturbances. The state regulation of a nonlinear, sixth order, three-degree-of-freedom helicopter model, subject to state and control polyhedral constraints, is considered. Simulation results show that, even though the prediction model is obtained by linearizing the nonlinear plant dynamics, the adopted robustness technique effectively ensures fulfillment of the safety and physical constraints when disturbances are present, while the nominal predictive control law fails to do so.*

Keywords: *predictive control, robustness, constraints, disturbances, multivariable systems*

1. INTRODUCTION

The control techniques known collectively as MPC (Model Predictive Control) essentially consist of applying the first element of the control sequence obtained as the solution of an optimal control problem which is solved at each sampling time. Due to its ability to deal with multivariable systems and transport delays, and to handle constraints by explicit including them in the optimization problem (Rossiter, 2003), MPC strategies have become widely employed in industry.

Stability requirements for predictive control laws have already been established when no uncertainties or disturbances are present (Mayne et al., 2000). However, predictive controllers may suffer from infeasibility problems in the presence of disturbances, possibly leading to violation of system constraints and system instability, even if the controller stabilizes the system in the nominal case (Chisci et al., 2001).

Among the possible approaches proposed to deal with this problem, one could cite min-max optimization (Lee and Yu, 1997), (Scokaert and Mayne, 1998), and constraint restriction (Gossner et al., 1997), (Chisci et al., 2001). (Badgwell, 1997) points out that min-max MPC has an increased computational burden associated with the usual optimization problem solved by MPC at each sampling time. This does not occur with the restricted constraint formulation, as the nominal optimization problem is solved considering modified constraints, which can be obtained *off-line*.

In this paper, a robust predictive state regulator is designed for a nonlinear, sixth-order model of a three-degree-of-freedom (3DOF) helicopter subject to bounded disturbances and physical restrictions on its maneuvering space. Constraints are assumed to be convex polyhedral sets and the robustness is achieved by the use of the restricted constraints formulation presented in (Chisci et al., 2001), which ensures feasibility and constraint fulfillment in spite of the existence of unknown but bounded disturbances. The computer routines used to calculate the modified restrictions were based on algorithms provided in (Kerrigan, 2000) and employed some operations on polyhedra already implemented in the Multi-Parametric Toolbox (MPT) for Matlab® (Kvasnica et al., 2004).

For comparison purposes, a nominal predictive control law is also considered. Simulations results are provided to illustrate that, in the presence of disturbances, while the robust predictive control law effectively guarantees that none of the system constraints is violated, the nominal predictive control law fails to do so.

2. DESCRIPTION OF THE 3DOF HELICOPTER

The system analyzed in this work is a simulation model of the Quanser© three-degree-of-freedom helicopter (Figure 1). The system is composed by the helicopter body, which is a small arm with one propeller at each end, and the helicopter arm, which connects the body to a fixed base. Although the system cannot exhibit translational motion, as it is fixed in a support, it can rotate freely about three axes. The helicopter position is characterized by the pitch, travel and elevation angles. The pitch movement corresponds to the rotation of the helicopter body about the helicopter arm, the travel movement corresponds to the rotation of the helicopter arm about the vertical axis and the elevation movement corresponds to the rotation of the helicopter arm about the horizontal axis. The control variables are the input voltages to the power amplifiers that drive each one of the two DC motors connected to the helicopter propellers. The maximum input voltage to the amplifiers is 5 V.

Three digital encoders provide measurements of the helicopter angles. Encoder resolution is about 0.044 degree for travel angle and 0.088 degree for pitch and elevation angles.

A nonlinear sixth-order model for the helicopter, which does not include the quantization effect due to the encoders, was derived in (Lopes, 2007). The model has the form shown in Eq. (1), where x_1 is the pitch angle (in rad), x_2 is the pitch rate (in rad/s), x_3 is the elevation angle (in rad), x_4 is the elevation rate (in rad/s), x_5 is the travel angle (in rad), x_6 is the travel rate (in rad/s), u_1 is the front motor amplifier input voltage (in V), u_2 is the back motor amplifier input voltage (in V), and the remaining terms represent constants determined experimentally.

$$\begin{bmatrix}\dot{x_1}\\ \dot{x_2}\\ \dot{x_3}\\ \dot{x_4}\\ \dot{x_5}\\ \dot{x_6}\end{bmatrix} = \begin{bmatrix} x_2 \\ \xi_1(u_1^2-u_2^2)+\xi_2(u_1-u_2) \\ x_4 \\ \xi_3\cdot x_6^2\cdot\sin(2x_3+\phi_1)+\xi_4\cdot\sin(x_3+\phi_2)+\{\xi_5(u_1^2+u_2^2)+\xi_6(u_1+u_2)\}\cos(x_1) \\ x_6 \\ \dfrac{\{\xi_7(u_1^2+u_2^2)+\xi_8(u_1+u_2)\}\sin(x_1)+\xi_9\cdot x_4\cdot x_6\cdot\sin(2x_3+\phi_3)}{\xi_{10}+\xi_{11}\cdot\sin(2x_3+\phi_4)}\end{bmatrix} \tag{1}$$

Figure 1. Quanser© 3DOF helicopter.

2.1 Linearized Model

The predictive control formulations adopted require a linearized model representing the system dynamics. One approximate linear model can be obtained by applying a first-order Taylor series expansion around a given equilibrium point for Eq. (1). In terms of deviations from the equilibrium point $\overline{x} = [0\ \ 0\ \ \text{-0.122}\ \ 0\ \ 0\ \ 0]^T$, which corresponds to the situation where the helicopter is hovering seven degrees below the horizontal, such model can be expressed as:

$$\dot{\tilde{x}} = \begin{bmatrix} 0 & 1.000 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.000 & 0 & 0 \\ 0 & 0 & -1.192 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.000 \\ -1.257 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}\tilde{x} + \begin{bmatrix} 0 & 0 \\ 2.806 & -2.806 \\ 0 & 0 \\ 0.395 & 0.395 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}\tilde{u}, \quad \overline{u} = \begin{bmatrix}2.804\\2.804\end{bmatrix} \tag{2}$$

3. PREDICTIVE CONTROL FORMULATION

The robust predictive control formulation adopted in this work was proposed by (Chisci et al., 2001). It concerns the regulation of time-invariant discrete-time linear systems subject to a disturbance input w, Eq. (3), in which the state and control variables are constrained (Eqs. (4) and (5), respectively). The disturbance is assumed to be unknown but must belong to a compact set, Eq. (6).

$$x[k+1] = Ax[k] + Bu[k] + Ew[k] \tag{3}$$

$$x[k] \in \mathcal{X} \subset \mathbb{R}^n, \forall k \geq 0 \tag{4}$$

$$u[k] \in \mathcal{U} \subset \mathbb{R}^p, \forall k \geq 0 \tag{5}$$

$$w[k] \in \mathcal{W} \subset \mathbb{R}^m, \forall k \geq 0 \tag{6}$$

It is also assumed that the pair (A,B) is stabilizable, $\mathcal{U}$ is compact and $\mathcal{X}, \mathcal{U}, \mathcal{W}$ contain the origin as an interior point.

Two predictive control laws were considered in this work: NPC (Nominal Predictive Control) and RPC (Robust Predictive Control). The NPC algorithm does not take into account the effects of the disturbances to which the system is subject. The RPC algorithm performs a nominal optimization but modifies the original constraints to ensure their fulfillment in spite of the unknown disturbances (Chisci et al., 2001). These modifications involve the use of the following set operations. Let $\mathcal{A}$, $\mathcal{B} \subset \mathbb{R}^n$, $\mathcal{F} \subset \mathbb{R}^p$ and $M \in \mathbb{R}^{p\text{x}n}$. Then $\mathcal{A} \sim \mathcal{B} \triangleq \{a \in \mathbb{R}^n | \; a + b \in \mathcal{A}, \forall b \in \mathcal{B}\}$ (Pontryagin Difference); $\mathcal{A} \oplus \mathcal{B} \triangleq \{a + b \in \mathbb{R}^n | \; a \in \mathcal{A}, b \in \mathcal{B}\}$ (Minkowski Sum); $\mathcal{LM}(M, \mathcal{A}) \triangleq \{Ma \in \mathbb{R}^p | \; a \in \mathcal{A}\}$; $\mathcal{LM}^{-1}(M, \mathcal{F}) \triangleq \{a \in \mathbb{R}^n | \; Ma \in \mathcal{F}\}$.

3.1 NPC algorithm

Let $x[k+i|k]$ represent the predicted system state at instant $k+i$, computed at instant k, based on the actual state $x[k]$ and on the future control moves. The nominal predictive control algorithm can be described by the following steps:

Step 1: Minimize the cost function (7), with $\Psi = \Psi^T > 0$, regarding the control sequence $C[k] = [c^T[k|k] \; \cdots \; c^T[k+N-1|k]]^T$, subject to the constraints defined in Eqs. (8) to (15).

$$J(C[k]) = \sum_{i=0}^{N-1} c^T[k+i|k]\Psi c[k+i|k] \tag{7}$$

$$x[k|k] = x[k] \tag{8}$$

$$x[k+i+1|k] = Ax[k+i|k] + Bu[k+i|k] \tag{9}$$

$$u[k+i|k] = Kx[k+i|k] + c[k+i|k] \tag{10}$$

$$c[k+i|k] = 0, \forall i \geq N \tag{11}$$

$$x[k+i|k] \in \mathcal{X}_i, \; 0 \leq i \leq N \tag{12}$$

$$u[k+i|k] \in \mathcal{U}_i, \; 0 \leq i \leq N-1 \tag{13}$$

$$\mathcal{X}_i = \mathcal{X}, \;\; 0 \leq i \leq N-1, \;\; \mathcal{X}_N = \mathcal{O}_\infty^h \tag{14}$$

$$\mathcal{U}_i = \mathcal{U}, \;\; 0 \leq i \leq N-1 \tag{15}$$

Step 2: Let $C^*[k] = [c^{*T}[k|k] \; \cdots \; c^{*T}[k+N-1|k]]^T$ be the optimal control sequence resulting from the optimization in step 1. Apply $u[k] = Kx[k] + c[k]$ to the plant, with $c[k] = c^*[k|k]$.

Step 3: Let $k = k+1$ and return to step 1.

The gain matrix K defines a nominal linear state feedback $u[k] = Kx[k]$. It can be arbitrarily chosen, as long as the system with this nominal control law is closed-loop stable. The set $\mathcal{O}_\infty^h$ is the maximal positively invariant subset of $\mathcal{X}^h = \mathcal{X} \cap \mathcal{LM}^{-1}(K, \mathcal{U})$ for the system under the nominal linear feedback, that is, the set of all states which satisfy state and control constraints (under nominal linear feedback) and for which the next state remains in such set.

If K is taken as the unconstrained LQR (Linear Quadratic Regulator) gain minimizing the cost (16), with $Q = Q^T \geq 0$ and $R = R^T > 0$, and the weight matrix Ψ is chosen as $\Psi = R + B^T PB$, where P is the unique symmetric positive definite solution of the discrete-time algebraic Riccati equation associated with the LQR problem, then it can be shown (Chisci et al., 2001) that the minimization of (7) subject to (8)-(15) is equivalent to the minimization of (16) subject to the same constraints.

$$\sum_{i=0}^{\infty} x^T[k]Qx[k] + u^T[k]Ru[k] \tag{16}$$

3.2 RPC algorithm

The robust predictive control algorithm is identical to the NPC algorithm, except by the replacement of constraints (14) and (15) by Eqs. (17) to (20):

$$\mathcal{X}_0 = \mathcal{X}, \; \mathcal{X}_N = \tilde{\mathcal{O}}_\infty^h \sim \mathcal{R}_N, \; \mathcal{U}_0 = \mathcal{U} \tag{17}$$

$$\mathcal{X}_i = \mathcal{X} \sim \mathcal{R}_i, \; 1 \leq i \leq N-1 \tag{18}$$

$$\mathcal{U}_i = \mathcal{U} \sim \mathcal{LM}(K, \mathcal{R}_i), \; 1 \leq i \leq N-1 \tag{19}$$

$$\mathcal{R}_i = \bigoplus_{j=0}^{i-1} \mathcal{LM}((A+BK)^j E, \mathcal{W}), \forall i \geq 1 \tag{20}$$

The set $\tilde{\mathcal{O}}_{\infty}^{h}$ is the maximal robust positively invariant subset of $\mathcal{X}^h = \mathcal{X} \cap \mathcal{LM}^{-1}(K, \mathcal{U})$ for the system under the nominal linear feedback, that is, the set of all states which satisfy state and control constraints (under nominal linear feedback) and for which the next state remains in such set, for all admissible disturbances.

The main property of the RPC algorithm, see (Chisci et al., 2001), is that, if the optimization problem has a solution for the initial state $x[0]$, then it will be feasible for all time, all state and control constraints will be fulfilled, the nonlinear predictive control law asymptotically approaches the nominal linear control law $u[k] = Kx[k]$, and the system state is asymptotically steered to a neighborhood of the origin $\mathcal{R}_{\infty} = \lim_{i \to \infty} \mathcal{R}_i$, for all admissible disturbances.

3.3 Implementation Details

If the constraints are defined by convex polyhedral sets, optimization (7)-(15) reduces to a quadratic programming problem:

$$C^*[k] = \arg\min_{C[k]} \{C^T[k] \boldsymbol{\Psi} C[k]\} \tag{21}$$

$$\text{subject to } \; S \cdot C[k] \leq r, \;\; \boldsymbol{\Psi} = \text{diag}_N\{\Psi\} \tag{22}$$

If the state and control constraints are defined, respectively, by $\mathcal{X}_i : \;\; S_{x[k+i]} \cdot x[k+i|k] \leq r_{x[k+i]}$ and $\mathcal{U}_i : \;\; S_{u[k+i]} \cdot u[k+i|k] \leq r_{u[k+i]}$, then matrices S and r can be obtained as follows:

$$S = \begin{bmatrix} S_X \cdot H_x(A+BK, B) \\ S_U \cdot H_u(K, A+BK, B) \end{bmatrix}, \;\; r = \begin{bmatrix} r_X \\ r_U \end{bmatrix} - \begin{bmatrix} S_X \cdot F_x(A+BK) \\ S_U \cdot F_u(K, A+BK) \end{bmatrix} x[k|k] \tag{23}$$

where:

$$S_X = \begin{bmatrix} S_{x[k+1]} & 0 & \cdots & 0 \\ 0 & S_{x[k+2]} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & S_{x[k+N]} \end{bmatrix}, \;\; S_U = \begin{bmatrix} S_{u[k]} & 0 & \cdots & 0 \\ 0 & S_{u[k+1]} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & S_{u[k+N-1]} \end{bmatrix} \tag{24}$$

$$H_x(\Phi, B) = \begin{bmatrix} B & 0 & \cdots & 0 \\ \Phi B & B & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \Phi^{N-1} B & \Phi^{N-2} B & \cdots & B \end{bmatrix}, \;\; H_u(K, \Phi, B) = \begin{bmatrix} I_p & \cdots & 0 & 0 \\ KB & \cdots & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots \\ K\Phi^{N-3} B & \cdots & I_p & 0 \\ K\Phi^{N-2} B & \cdots & KB & I_p \end{bmatrix} \tag{25}$$

$$r_X = \begin{bmatrix} r_{x[k+1]} \\ r_{x[k+2]} \\ \vdots \\ r_{x[k+N]} \end{bmatrix}, \;\; r_U = \begin{bmatrix} r_{u[k]} \\ r_{u[k+1]} \\ \vdots \\ r_{u[k+N-1]} \end{bmatrix}, \;\; F_x(\Phi) = \begin{bmatrix} \Phi \\ \Phi^2 \\ \vdots \\ \Phi^N \end{bmatrix}, \;\; F_u(K, \Phi) = \begin{bmatrix} K \\ K\Phi \\ K\Phi^2 \\ \vdots \\ K\Phi^{N-1} \end{bmatrix} \tag{26}$$

4. METHODOLOGY AND RESULTS

The predictive controllers require a discrete-time linear prediction model of the system dynamics. Assuming the control signals remain constant between sampling instants, a zero-order-hold discretization was performed on the model defined in Eq. (2), by using a 40 ms sampling time. This value was adopted because the control of the pitch dynamics is poor for larger sampling times (Lopes et al., 2006).

To eliminate steady-state regulation error on the elevation and travel dynamics, a model augmentation procedure was performed. Two artificial states were added to the plant dynamics, corresponding to the accumulated regulation error for the elevation and travel angles:

$$x_7[k+1] = x_3[k] + x_7[k] \tag{27}$$

$$x_8[k+1] = x_5[k] + x_8[k] \tag{28}$$

Assuming disturbances acting on the system inputs, the augmented discrete-time system matrices are those shown in

Eq. (29):

$$A = \begin{bmatrix} 1.000 & 0.040 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.000 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.999 & 0.040 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.048 & 0.999 & 0 & 0 & 0 & 0 \\ -0.001 & 0 & 0 & 0 & 1.000 & 0.040 & 0 & 0 \\ -0.050 & -0.001 & 0 & 0 & 0 & 1.000 & 0 & 0 \\ 0 & 0 & 1.000 & 0 & 0 & 0 & 1.000 & 0 \\ 0 & 0 & 0 & 0 & 1.000 & 0 & 0 & 1.000 \end{bmatrix}, \quad B = E = \begin{bmatrix} 0.002 & -0.002 \\ 0.112 & -0.112 \\ 0 & 0 \\ 0.016 & 0.016 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad (29)$$

The nominal feedback gain K, Eq. (30), and the cost weight matrix Ψ, Eq. (31), for both predictive controllers, were obtained as indicated in subsection 3.1, with the LQR weight matrices expressed in Eq. (32):

$$K = \begin{bmatrix} -2.063 & -0.848 & -2.159 & -2.570 & 1.803 & 2.369 & -0.068 & 0.020 \\ 2.063 & 0.848 & -2.159 & -2.570 & -1.803 & -2.369 & -0.068 & -0.020 \end{bmatrix} \quad (30)$$

$$\Psi = \begin{bmatrix} 115.400 & -6.732 \\ -6.732 & 115.400 \end{bmatrix} \quad (31)$$

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 500 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 250 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 200 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 100 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.1 \end{bmatrix}, \quad R = \begin{bmatrix} 100 & 0 \\ 0 & 100 \end{bmatrix} \quad (32)$$

Due to the facts that only the angles are measured in the helicopter and that the predictive formulation employs full-state feedback, it was necessary to develop a way to estimate the remaining system states, the angular velocities. As suggested by the manufacturer, this was accomplished by taking low-pass filtered derivatives of the angle signals. The following transfer functions were adopted for the filters: $20s/(s+20)$ for travel rate estimation and $15s/(s+15)$ for elevation and pitch rates estimation. Figure 2 shows a block diagram of the simulation model representing the helicopter dynamics.

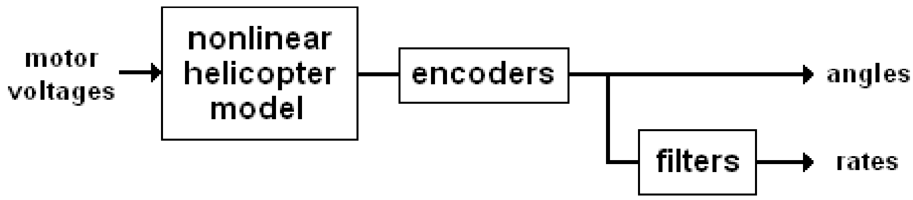

Figure 2. Block diagram of the helicopter simulation model.

4.1 Simulation results

All tests and simulations were executed in the Simulink and Matlab® 6.5.1 environments. A public domain C algorithm (Schittkowski and Powel, 1992) was used to solve the quadratic programming problem imposed by the predictive control law. The situation analyzed was a travel maneuver at constant elevation, with constraints on the states and control variables. It illustrates the advantages of using RPC over NPC as, in this situation, NPC loses feasibility and is not able to prevent the system from violating the travel constraint. The initial condition was $x[0] = [0 \;\; 0 \;\; \text{-}7 \;\; 0 \;\; \text{-}25 \;\; 0]^T$ degrees, corresponding to an initial travel displacement of -25 degrees from the equilibrium point.

Since the travel dynamics are somewhat slow (as compared to the pitch dynamics), the prediction horizon was chosen as $N = 200$, so that the system behavior during most part of the considered maneuver could be taken into account by the predictive control laws. The control horizon, which represents the number of independent control moves resulting from the optimization in the predictive control algorithms, was established as $M = 40$, so as to provide a good tradeoff between computational burden and control performance.

With the aim of allowing the predictive control laws to actuate during all of the prediction horizon, the control moves, for prediction purposes, were made constant during intervals of N/M steps. This distribution of the M independent control moves among those comprehended in the prediction horizon N can be achieved by multiplying the matrices H_x and H_u, defined in Eq. (25), on their right-hand side, by an appropriate NxM weight matrix W_e.

The procedure adopted to handle possible infeasibilities of the predictive controllers was to use only the nominal linear state feedback control law whenever infeasibilities arose.

System constraints included physical barriers limiting the maneuvering space of the helicopter and safety limits on the motor amplifiers input voltages. Pitch, elevation and travel constraints are expressed by Eqs. (33), (34) and (35), respectively:

$$x_1 \in [-20, +20] \text{ degrees} \tag{33}$$

$$x_3 \in [-15, +15] \text{ degrees} \tag{34}$$

$$x_5 \in [-60, +7] \text{ degrees} \tag{35}$$

Constraints on $\tilde{u}$ are indicated by Eq. (36). With this assumption, constraints on the power amplifier inputs are expressed by:

$$\tilde{u}_i \in [-1.5 \text{ V}, +1.5 \text{ V}], \;\; i = 1, 2 \tag{36}$$

$$-1.5 \text{ V} + \overline{u}_i \leq u_i \leq +1.5 \text{ V} + \overline{u}_i, \;\; i = 1, 2 \tag{37}$$

$$u_i \in [1.304 \text{ V}, +4.304 \text{ V}], \;\; i = 1, 2 \tag{38}$$

Disturbance bounds were assumed to be about 3% of the maximum allowed excursion for the control variables:

$$w_i \in [-0.05 \text{ V}, +0.05 \text{ V}], \;\; i = 1, 2 \tag{39}$$

Figures 4, 5 and 6 show the results of the simulation of the travel maneuver, when the system is subject to the extreme admissible disturbance depicted in Figure 3, for the situations where the system is controlled by the nominal linear state feedback law, by the NPC algorithm and by the RPC algorithm.

Figure 4 shows that the elevation angle is practically unaffected by the travel maneuver, as the system is already at the desired elevation position. Figure 6 shows that the input voltages are kept within the specified limits, defined by Eq. (38), by all the analyzed controllers.

The system behavior under nominal linear state feedback law $u[k] = Kx[k]$ was shown to illustrate how the predictive control laws deviate from it. Observation of Figures 4 and 5 reveal that the system, under the linear control law, violates the lower pitch constraint (at -20 degrees) and the upper travel constraint (at +7 degrees), exhibiting a considerable overshoot on the travel angle response.

Figure 5 also shows that NPC fails to guarantee constraint fulfillment, as it is unable to compensate, in the presence of the disturbance, the large overshoot induced by the linear law on the travel dynamics, resulting in the violation of the upper travel constraint. It is worth noting that NPC becomes infeasible during the time interval from 2 to 5.4 seconds (first due to the prediction of the violation and then due to the actual violation). Figures 4, 5 and 6, on the other hand, show that RPC, besides maintaining feasibility during the entire maneuver, is the only that succeeds in enforcing system constraints, despite the extreme disturbance acting on the system.

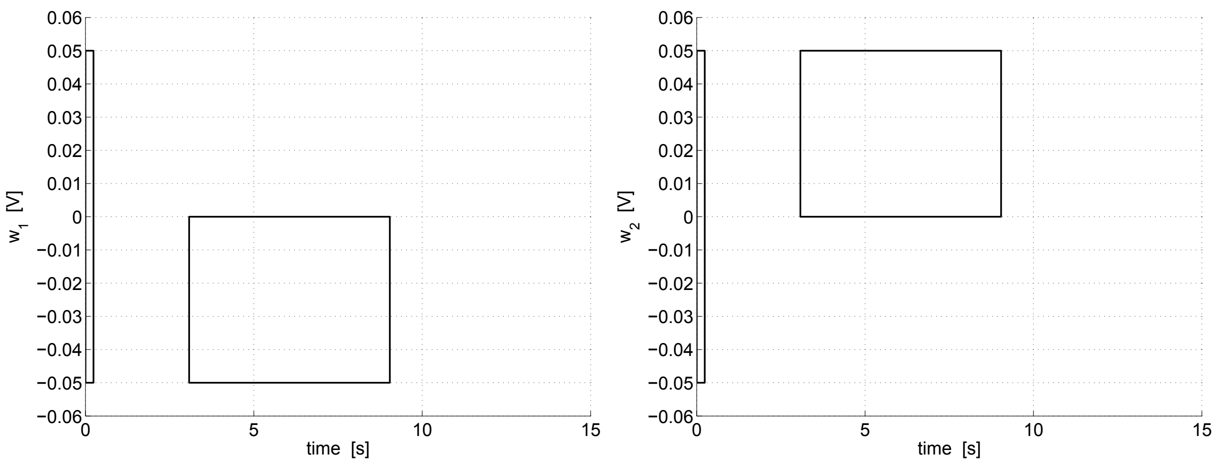

Figure 3. Extreme disturbance actuating on the system during maneuver.

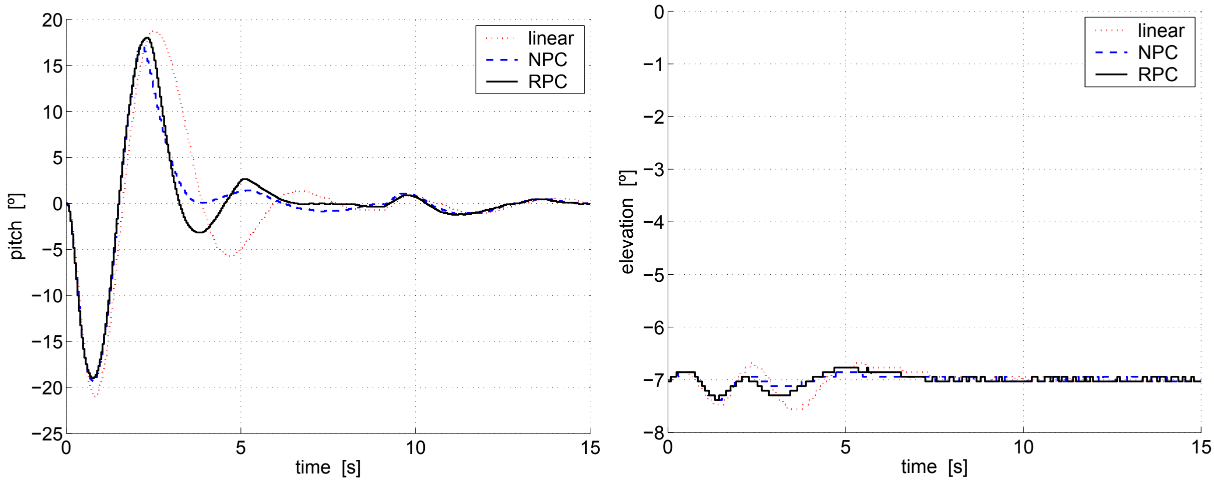

Figure 4. Pitch (x_1) and elevation (x_3) angles during maneuver.

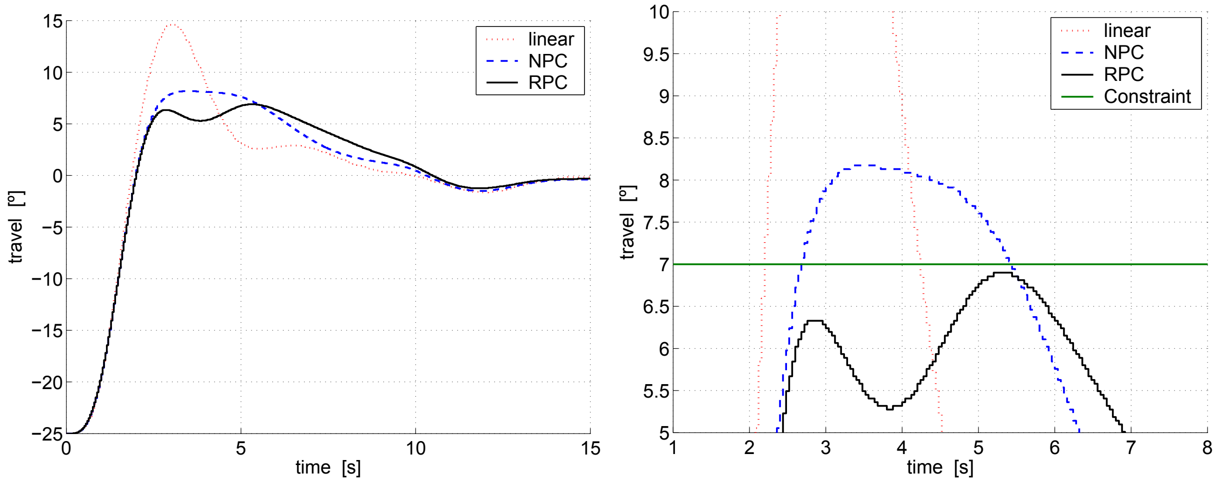

Figure 5. Travel angle (x_5) during maneuver and detail of travel constraint violation (at 7 degrees) by the nominal linear and NPC control laws.

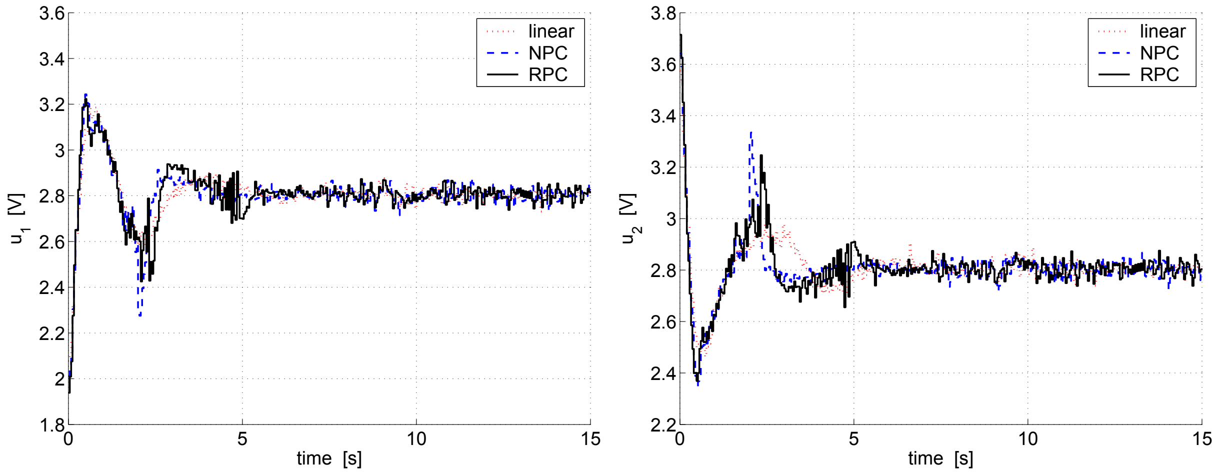

Figure 6. Control signals during maneuver.

5. CONCLUDING REMARKS

A robust predictive control law was designed for a 3DOF helicopter system represented by a sixth-order nonlinear model. The robustness technique enforces constraint fulfillment when unknown but bounded external disturbances are present, by modifying the original system constraints. To emphasize the importance of considering the effects of disturbances in the design of predictive control laws, a comparison was carried out with a nominal predictive controller. Simulation results were provided to show that infeasibility, leading to constraint violations, may occur in this latter case. It is worth noting that the robust predictive control law successfully maintained feasibility and guaranteed that none of the constraints were violated, even in the presence of factors not considered in its formulation, such as model nonlinearity, angle quantization and filter estimation of the angular velocities.

6. ACKNOWLEDGEMENTS

The authors acknowledge all support provided by FAPESP (grants 2003/09433-5, 2005/03210-0, 2006/58850-6) and CNPq (research fellowship).

7. REFERENCES

Badgwell, T. A. , 1997, "Robust Model Predictive Control", International Journal of Control 68, 797–818.

Chisci, L., Rossiter, J. A. and Zappa, G., 2001, "Systems with Persistent Disturbances: Predictive Control with Restricted Constraints", Automatica 37, 1019–1028.

Gossner, J. R., Kouvaritakis, B. and Rossiter, J. A., 1997, "Stable Generalized Predictive Control with Constraints and Bounded Disturbances", Automatica 33, 551–568.

Kerrigan, E. C., 2000, "Robust Constraint Satisfaction: Invariant Sets and Predictive Control", PhD thesis, St. John's College, Cambridge, UK.

Kvasnica, M., Grieder, P. and Baotić, M., 2004, "Multi-Parametric Toolbox (MPT)", http://control.ee.ethz.ch/∼mpt/.

Lee, J. H. and Yu, Z., 1997, "Worst Case Formulations of Model Predictive Control for Systems with Bounded Parameters", Automatica 33, 763–781.

Lopes, R. V., 2007, "Modelagem e Controle Preditivo de um Helicóptero com Três Graus de Liberdade", Master's thesis, ITA, São José dos Campos, Brazil.

Lopes, R. V., Galvão, R. K. H., Milhan, A. P., Becerra, V. M. and Yoneyama, T., 2006, "Modelling and Constrained Predictive Control of a 3DOF Helicopter", In: XVI Congresso Brasileiro de Automática, Salvador, Brazil, Paper 123, Vol. 1, pp. 429–434.

Mayne, D. Q., Rawlings, J. B., Rao, C. V. and Scokaert, P. O. M., 2000, "Constrained Model Predictive Control: Stability and Optimality.", Automatica 36(6), 789–814.

Rossiter, J. A., 2003, "Model-Based Predictive Control - A Practical Approach", CRC Press, Boca Raton, USA.

Schittkowski, K. and Powel, M. J. D., 1992, "C Translation of QLD: Fortran Code for Quadratic Programming".

Scokaert, P. O. M. and Mayne, D. Q., 1998, "Min-max Feedback Model Predictive Control for Constrained Linear Systems", IEEE Transactions on Automatic Control 43, 1136–1142.

8. RESPONSIBILITY NOTICE