

DIRECT SINGULARITY AVOIDANCE STRATEGY FOR THE HEXA PARALLEL ROBOT

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Abstract. *Machine supervision on parallel robots demand a powerful online singularity loci prediction. To detect the entrance of the end-effector on direct kinematic singularities that constrain the internal workspace is not trivial for parallel robots with five or six degrees of freedom. This paper presents a strategy to safely drive a Hexa parallel robot out of direct singularity regions based an index of power. The used index indicates the entrance of the Hexa parallel robot, considering its joint limits, to all its direct kinematic singularities. This index is calculated through the power inspired measure. The applied procedure uses screw theory to represent movements and actions in the robot. The use of this index allows the development of the presented strategy to safely drive the robot out of regions near singularities. This procedure is experimentaly verified on a Hexa parallel robot.*

Keywords: *Parallel Robots, Singularity detection, Singularity avoidance, Screw Theory.*

1. Introduction

A parallel robot (e.g. the Hexa robot in Fig. 1) typically consists of a moving platform, also called tool platform, connected to a fixed base by several serial chains, called legs (Hesselbach et al., 2004). The end effector is the tool that is positioned and orientated by the whole structure and is mounted in the moving platform. In general, the number of legs is equal to the number of degrees of freedom (DOF) such that each actuator controls each leg and all actuators can be mounted at or near the fixed base. Each leg is composed by active and passive joints and links. Active joints can be actuated by motors (electrical, hidraulic, pneumatic and so on). On the other hand, the passive ones have their movements defined by the configuration of the whole chain. An active link is mounted directly in an active joint, i.e. its movement is defined by the movement of this particular actuated joint. Finally, a passive link has its movement defined by the configuration of the whole chain.

This paper discusses some strategies to detect and avoid singularities for the Hexa parallel robot, which can be, under some circumstances, generalized to many kinds of parallel robots. Actually, this work represents an application of a power inspired measure, initially presented in (Voglewede, 2004b, Pottmann et al., 1998), which is a method to measure closeness to direct kinematics singularities in parallel structures.

Workspace problems, such as singularities, make the application of new trajectories more difficult. Frequently, the task must be adapted to take robot workspace constraints into account. Furthermore, these problems come out when new strategies are tested in the robot. Hence this operation is very dangerous and takes too much time if the whole system must be reinitialized every time the robot gets close to some problematic configuration. Furthermore, if the robot reaches

such configurations, its structure may be damaged. The purpose of the methods presented in this paper is to overcome some of these problems through the application of an online singularity supervision and an automatic method to drive the robot out of regions near singularities.

In fact, the generation of new trajectories represents one important characteristics of robot, which is their reprogrammability. Such characteristic makes robots suitable to flexible automation. Hence, new supervision approaches can improve even more the application of robot, since the process of generating new trajectories can be safer.

Parallel robots usually have high stiffness, good dynamic performance and large payload capacity, on the other hand, these kind of robots suffer from a small useful workspace (in comparison to the volume of the whole robot), design difficulties and singularities inside the workspace.

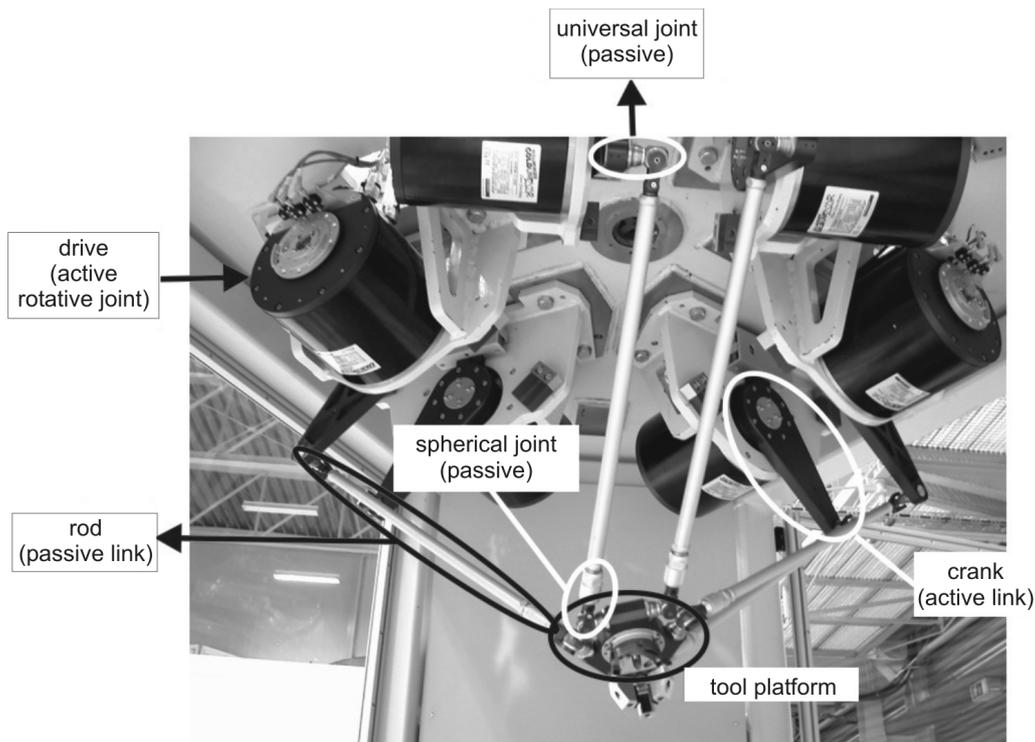


Figure 1. The Hexa parallel robot

Section 3 analyses the kinematic relations involving singularities in parallel robots. Section 2 presents some basic screw theory concepts, necessary to understand the power inspired measure method, which is described in Section 4. Later on, in Section 5, it is presented a strategy to drive the robot out of regions near singularities. This strategy is an application of the power inspired measure. The Hexa robot, where these techniques were implemented is briefly presented in Section 6. Finally, the obtained results are shown in Section 7 and conclusions about this work are presented in Section 8.

2. Screw Theory Basis

The screw is a geometric element composed by a directed line (axis) and by a scalar parameter h (length dimension) called pitch (Ball, 1900). If the directed line is represented by a normalised vector, the screw is called a normalised screw $\hat{\$}$. The screw theory is suitable to represent parallel robot end effector movements and actions which are used to detect singularities.

2.1 Differential kinematics

The Mozzi theorem (Ceccarelli, 2000) states that the velocities of the points of a rigid body with respect to an inertial reference frame $O(X, Y, Z)$ may be represented by a differential rotation ω about a certain fixed axis and a simultaneous differential translation κ along the same axis. The complete movement of the rigid body, combining rotation and translation, is called screw movement or twist $\$$. The body "twists" around an axis instantaneously fixed with respect of the inertial reference frame. This axis is called the screw axis and the rate of the translational velocity and the angular velocity is called the pitch of the screw $h = \|\kappa\|/\|\omega\|$.

The twist represents the differential movement of the body with respect to the inertial frame and may be expressed by

a pair of vectors, in ray order, as (Hunt, 2000)

$$\mathcal{S} = \begin{bmatrix} \omega \\ V_p \end{bmatrix} = [\mathcal{L} \mathcal{M} \mathcal{N} \mathcal{P}^* \mathcal{Q}^* \mathcal{R}^*]^T. \quad (1)$$

Here ω is the angular velocity of the body with respect to the inertial frame and V_p represents the linear velocity of a point P attached to the body, which is instantaneously coincident with frame O . It is possible to express the same twist in axis order $\mathcal{S} = [\mathcal{P}^* \mathcal{Q}^* \mathcal{R}^* \mathcal{L} \mathcal{M} \mathcal{N}]^T$.

A twist may be decomposed into its magnitude Ψ and its corresponding normalised screw $\hat{\mathcal{S}}$, *i.e.* $\mathcal{S} = \hat{\mathcal{S}}\Psi$ (Hunt, 2000).

2.2 Statics

In the same way, the Poinot theorem (Poinot, 1806, Hunt, 1978) states that a general action, *i.e.* a force and a couple, upon a rigid body may be carried by a screw, called wrench \mathcal{S}' (Ball, 1900, Hunt, 2000). In this case the wrench in ray order is

$$\mathcal{S}' = \begin{bmatrix} f \\ C_o \end{bmatrix} = [\mathcal{L}' \mathcal{M}' \mathcal{N}' \mathcal{P}'^* \mathcal{Q}'^* \mathcal{R}'^*]^T \quad (2)$$

where f is the resultant force and C_o is the resultant couple, around O , upon the body. The wrench may be decomposed as $\mathcal{S}' = \hat{\mathcal{S}}'\tau$ where τ is the wrench magnitude and the $\hat{\mathcal{S}}'$ is the normalised screw. The wrench pitch is determined by $h' = \|C_{||}\|/\|f\|$, being $C_{||}$ the couple component acting around the same screw axis.

2.3 Power

Consider a rigid body supporting a wrench $\mathcal{S}' = [f^T C_o^T]^T$ while it is moving around an instantaneous twist $\mathcal{S} = [\omega^T V_p^T]^T$. The power or rate of work δW carried out is given by (Ball, 1900, Hunt, 2000)

$$\delta W = C_o \cdot \omega + f \cdot V_p = \mathcal{S}'^T \mathcal{S} \quad (3)$$

where \mathcal{S}' and \mathcal{S} are given in axis and ray order, respectively.

3. Parallel robot Singularities

In this section, a differential kinematic relation for parallel robots, which allows the analysis of the singularities present in these machines, is described. Later on, the determinant analysis and grassmann geometry methods, usually used to detect direct singularities are mentioned. However, the strategy presented later to drive the robot out of singularity regions uses a third method, based on the power inspired measure, that is presented in Section 4.

In spatial parallel robots, the relationship between actuator coordinates q and end effector Cartesian coordinates x , may be stated as a function f

$$f(q, x) = \mathbf{0} \quad (4)$$

where $\mathbf{0}$ is the 6-dimensional null vector. Therefore, the differential kinematic relation may be determined as

$$J_q \dot{q} - J_x \mathcal{S} = \mathbf{0}; \quad \dot{q} = J \mathcal{S} \quad (5)$$

where \mathcal{S} is the end effector velocity twist in ray order and $J = J_q^{-1} J_x$ is the Jacobian matrix of the robot composed by direct J_x and inverse J_q Jacobian matrices.

Additionally, we may write Eq. (5) as a differential kinematic relationship between the end effector velocity \mathcal{S} and the vector $v = [v_1, \dots, v_n]^T$

$$v = J_x \mathcal{S} \quad (6)$$

where v is the component of the absolute linear velocity of the end effector connection point (C_i) (Fig. 2) in the direction of the passive link connected to the moving platform. Fig. 2a shows the wrenches acting upon the moving platform. Two examples of possible direct singularities are shown in Fig. 2b and Fig. 2c, these are two cases previewed by Grassmann analysis that are inside the robot workspace.

It is important to notice that the rows of the direct kinematic matrix J_x may correspond to the normalised wrenches, in axis order acting upon the end effector through the passive link, *i.e.* the distal link of each leg (serial chain between basis and end effector) (Davidson and Hunt, 2004). Therefore, a static relation may be stated as

$$J_x^T \tau_i = \mathcal{S}' \quad (7)$$

where \mathcal{S}' is the result wrench acting upon the end effector, in axis order, and τ_i are the input wrench magnitudes.

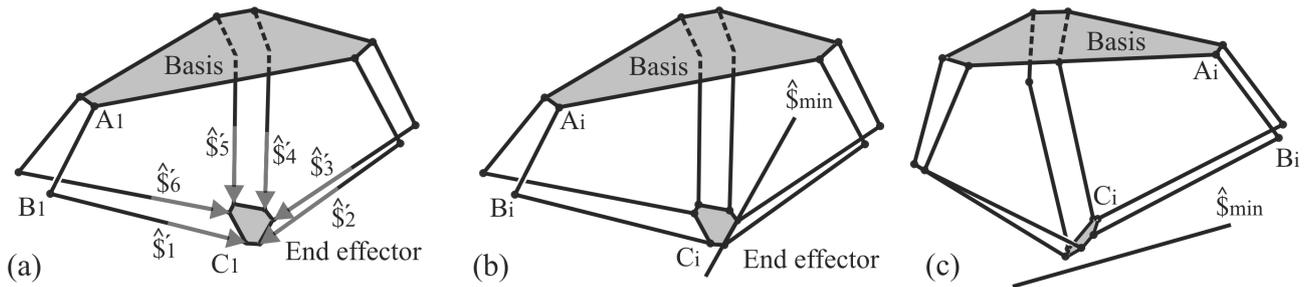


Figure 2. (a) The wrenches acting upon the moving platform. (b) and (c) Two direct singularities

Singular configurations appear if either J_x or J_q drops rank. If J_q drops rank, *i.e.* it is singular, an inverse kinematics singularity is encountered and the end effector is over-constrained (Tsai, 1999), *i.e.* it loses at least one DOF. The inverse kinematic singularity is found at the boundary of the workspace or when a leg folds upon itself. This type of singularity is caused due to the serial nature of the legs and is discussed in literature (Sciavicco and Siciliano, 1996, Tsai, 1999).

If J_x is singular, a direct singularity is encountered and the end effector can move even if all actuators are locked. At these configurations, the end effector gains one or more uncontrollable degrees of freedom. This type of singularity occurs within the workspace and the main goal of this paper is to detect and predict this kind of singularity. From now on, when the word singularity is used a direct singularity is taken into consideration.

There are three basic reasons why singularities become an issue in real life situations: reduced accuracy, large internal forces and loss of knowledge of solution tree, *i.e.* mechanically the robot is not where the control believes it is (Voglewede, 2004b), and the robot structure may be damaged.

Direct singular configurations are consequence of the direct kinematics Jacobian singularities, *i.e.* the roots of its Jacobian. However, the calculation of this determinant is a very complex task for most of robots (Merlet, 2000). Hence, it is not practicable to determine a relation among the joint variables that leads to the value of the determinant. Additionally, the value of the determinant cannot be physically interpreted, *i.e.* we do not know good values for it.

In most cases the wrenches acting upon the parallel robot end effector are pure forces (null pitch screw) and their screw components correspond to a Plücker components of a line, *i.e.* a Plücker vector (Davidson and Hunt, 2004). In these cases, the singular configurations of the robot are associated with linearly dependent set of lines (Fig. 2a), also called line based singularities (Hao and McCarthy, 1998). Grassmann studied the varieties of lines, *i.e.* the sets of linear dependent lines to n given independent lines, and characterised them geometrically (Merlet, 1989). These varieties were analysed and classified in order to study the singularities of spatial frameworks (Dandurand, 1984) and triangular simplified symmetric robot whose six legs connect to the end effector and to the basis only in three points, respectively (Merlet, 2000, Hao and McCarthy, 1998). In (Hesselbach et al., 2005), Grassmann geometry was applied on the Hexa parallel robot. The analysis proved that this robot cannot reach direct singularities whose gained uncontrollable degree of freedom is purely prismatic.

As it is explained in Section 5, the strategy presented in this paper is based on the idea of the gradient of an index that expresses the closeness of the robot to singularities. The determinant is not a good candidate for this index since this has no physical meaning and cannot be used as a measure of closeness to singularities. On the other hand, Grassmann geometry is a tool that allows the analysis of some robot singularities but it is not yet clear if it is possible to generate such index through this technique. Section 4 presents the power inspired measure method, which allows the calculation of this index.

4. The Power Inspired Measure Method

This technique is developed using the screw theory, specifically the power or rate of work, to determine how close a parallel robots is to a direct singularity (Pottmann et al., 1998). The power inspired measure determines closeness to singularity through an optimisation problem that results in a corresponding generalised eigenvalue problem. Using this methodology it is possible to describe the instantaneous behaviour of the end effector near singularities (Wolf and Shoham, 2003). Other measures were incorporated into a constrained optimisation framework, *e.g.* the natural frequency measure (Voglewede, 2004b), this method considers the stiffness matrix in its calculations and uses this to measure the natural frequency.

In this approach the objective function $F(\$, \$'_{i=1, \dots, n})$ to be optimised is the sum of the square of the power (Eq. (3)) of each leg ($\$'_i$) upon the end effector which is constrained to move on $\$(\|\omega\| = 1, h \neq \infty)$, a normalised finite pitch twist

$$F = \sum_{i=1}^n (\$'_i{}^T \$)^2 \quad (8)$$

with

$$\mathcal{S}'_i = [\mathcal{P}'_i \mathcal{Q}'_i \mathcal{R}'_i \mathcal{L}'_i \mathcal{M}'_i \mathcal{N}'_i]^T; \mathcal{S} = [\mathcal{L} \mathcal{M} \mathcal{N} \mathcal{P}^* \mathcal{Q}^* \mathcal{R}^*]^T \quad (9)$$

where \mathcal{S}'_i and \mathcal{S} are given in axis and ray order, respectively, and n is the number of legs.

Therefore, F may be expressed as

$$F = \mathcal{S}^T J_x^T J_x \mathcal{S} = \mathcal{S}^T M \mathcal{S}; \quad M = \sum_{i=1}^n \mathcal{S}'_i \mathcal{S}'_i{}^T \quad (10)$$

where M is called the Gramian matrix, and from (Eq. (6)) we may rewrite

$$F = (J_x \mathcal{S})^T (J_x \mathcal{S}) = v^T v = \|v\|^2 \quad (11)$$

being $\|v\|$ the Euclidean norm of the vector of linear velocities v (Eq. (6)).

Considering that the only unconstrained movements of the end effector, in a direct singularity, are finite pitch twists (no pure translational movements on singular configurations are permitted) with magnitude $\Psi = 1$, the unitary twist magnitude, which is the constraint of the optimisation method, is given through the invariant normalisation (Voglewede, 2004a)

$$\|\mathcal{S}\| = \sqrt{\omega \cdot \omega} = \sqrt{\mathcal{L}^2 + \mathcal{M}^2 + \mathcal{N}^2} = 1 \Rightarrow \|\mathcal{S}\|^2 = \mathcal{S}^T D \mathcal{S} = 1 \quad (12)$$

where $D = \text{diag}\{1, 1, 1, 0, 0, 0\}$.

Equation (10) under the constraint of Eq. (12) may be transformed to obtain the Lagrangian L ,

$$L = \mathcal{S}^T M \mathcal{S} - \lambda (\mathcal{S}^T D \mathcal{S} - 1) \quad (13)$$

where λ is the Lagrangian multiplier. The minimisation of the Lagrangian is performed by

$$\frac{\partial L}{\partial \lambda} = \mathcal{S}^T D \mathcal{S} - 1 = 0; \quad \frac{\partial L}{\partial \mathcal{S}} = (M - \lambda D) \mathcal{S} = 0. \quad (14)$$

The matrix expression in the parenthesis, for a nontrivial solution, has to be singular, *i.e.*

$$\det(M - \lambda D) = 0. \quad (15)$$

To transfer this into an eigenvalue problem we define $\xi = 1/\lambda$ and Eq. (15) becomes

$$\det(\xi M - D) = \det(\xi I - M^{-1} D) = 0 \quad (16)$$

which has only three roots, $\xi_i = 1/\lambda_i$, *i.e.* the eigenvalues of $[M^{-1} D]$, due to the three null elements of D diagonal. Each eigenvalue has its correspondent eigenvector \mathcal{S}_i that resolves Eq. (14).

Since the objective function is non-negative, given that M is a square symmetric positive semi-definite matrix (Voglewede, 2004b), the end effector normalised twist which minimises the supply power through the wrenches is the eigenvector \mathcal{S}_{min} associated to the smallest eigenvalue λ_{min} . In this case one gets the minimum of F upon the end effector moving on \mathcal{S}_{min} , see Eq. (12), as

$$F(\mathcal{S}_{min}, \mathcal{S}'_{i=1, \dots, 6}) = \mathcal{S}_{min}^T M \mathcal{S}_{min} = \lambda_{min} \mathcal{S}_{min}^T D \mathcal{S}_{min} = \lambda_{min}. \quad (17)$$

In direct singularity there is a twist \mathcal{S}_{min} for which none of the leg wrenches can do any work and then the minimum of F , *i.e.* λ_{min} , goes to zero. Out of a singularity, \mathcal{S}_{min} represents the less constrained twist and λ_{min} is a power function measure that indicates the robot singularity closeness.

The case of two or three similar or identical minimum eigenvalues means that wrenches are in the intersection of two or three linear complexes, *i.e.* a linear congruence or a regulus respectively, and the end effector gains two or three degrees of freedom.

Since a quadratic function is optimized, it is reasonable to define the index that measures the closeness of the robot to singular configurations as $\sqrt{\lambda_{min}}$, which will be called from now on as $I = \sqrt{\lambda_{min}}$.

5. A strategy to Drive the Robot Out of Singular Configurations

The power inspired measure can be used to provide a singularity online supervision. The implementation of this kind of supervision requires one consideration, that is what to do when the robot is close from some singularity. In this section, a strategy is presented to safely drive the robot out of singularities through an automatic operation.

Grassmann geometry analysis, briefly described in Section 3, applied to the Hexa Robot, shows that this robot cannot reach singularities whose uncontrollable DOF are purely prismatic (Hesselbach et al., 2005). This allows the use of the unique index $I = \sqrt{\lambda_{min}}$ to express the closeness of the robot to direct singularities.

The application of the power inspired measure requires the value of all $\$'_i$ from each leg, *i.e.* the wrenches acting upon the moving platform. For that, it is necessary, for most of parallel robot structures, the calculation of the current end effector position and orientation, *i.e.* the result of the direct kinematic problem (Tsai, 1999, Hesselbach et al., 2003). If this calculation is done by the robot controller, it is possible to apply this method in each controller cycle. Then, the robot can move normally over some defined limit for this index. The definition of this limit is normally experimental, and must consider joint clearances as well as the dimension of the forces and torques that act upon the moving platform. In fact, close to singular configurations, the actuators must perform great torques to compensate forces and torques acting upon the end effector.

When this limit is reached some action must be performed. The simplest approach is to brake the robot as fast as possible and then take it out from this position manually with the actuators powered off. This approach is not convenient because the whole system must be reinitialized and than the robot task must naturally be interrupted. Besides that, if the robot has great dimensions it is not feasible to move it manually.

On the other hand, if the robot is braked but the power of the actuators is not turned off, it is possible to drive the robot out of this dangerous position. However, this is not a safe operation since the robot is really close to the singularity. If the operator doesn't have a very detailed perception of the robot kinematics, these movements can lead the robot even more inside toward the singularity.

In practice, movements that decrease the value of the index must be avoided while the robot is driven out the singularity. If it is possible to calculate the value of the index gradient in the current position, it is possible to define a direction through which the index would mostly increase during a differential movement. If a velocity with this direction is applied and if this direction is updated every controller cycle, it is possible to drive the robot out of the singularity automatically. Following this approach, it is necessary sometimes to lock some end effector movements to avoid collision with surrounding objects during this operation. This could be easily done by setting zero to the values of velocity to these DOFs. In the case of the Hexa robot, for example, the value of this index normally increases importantly while the end effector moves in the positive z direction.

Hence, it is feasible to implement a strategy to drive the robot out of singularity regions if the value of the index gradient is available.

5.1 Estimation of the index gradient

As it is presented in Section 4, the computation of the index I involves the calculation of the maximum eigenvalue ξ of $(M^{-1}D)$ (Eq. 16). Due to that, an analytical solution to ∇I as a function of the current end effector position and orientation was not found. A possible approach to this estimation of the gradient value is presented here.

The index I is considered a function of the current robot position and orientation, *i.e.* $I = g(p)$, where $p = [x \ y \ z \ \psi \ \theta \ \phi]^T$, is the vector that contains the position $[x \ y \ z]^T$ and the orientation $[\psi \ \theta \ \phi]^T$ of the end effector, given by roll (ψ , that is the rotation around x), pitch (θ , rotation around y) and yaw (ϕ , rotation around z) angles. The function g means the application of the power inspired measure.

A possible approach to estimate the value of $\nabla I = \left[\frac{\partial I}{\partial x} \ \frac{\partial I}{\partial y} \ \frac{\partial I}{\partial z} \ \frac{\partial I}{\partial \psi} \ \frac{\partial I}{\partial \theta} \ \frac{\partial I}{\partial \phi} \right]^T$ is to estimate each one of the partial derivatives by using the property

$$\frac{\partial I}{\partial x} = \lim_{\Delta x \rightarrow 0} \frac{I^* - I}{\Delta x} \quad (18)$$

where $I^* = g([(x + \Delta x) \ y \ z \ \psi \ \theta \ \phi]^T)$ and $I = g([x \ y \ z \ \psi \ \theta \ \phi]^T)$. Hence, it is feasible to get a vector $v \approx \nabla I$. If the unitary vector is taken $\hat{v} = v/|v|$, the next setpoint p^* in cartesian coordinates can be calculated as

$$p^* = p + \hat{v} \cdot T_s \quad (19)$$

where T_s is the sample time of the controller. The p^* sequence obtained in this way is the path that drives the end effector out the singular configuration.

6. The Hexa Robot

The methods and strategies presented in this paper are implemented and tested in a parallel robot with a Hexa structure 1, originally presented by Pierrot (Pierrot et al., 1990), developed in the Institute of Machine Tools of the Technical University of Braunschweig, Germany. This is a 6 DOF parallel robot with electric actuators, which are six servo motors fixed on the base of the robot. Each leg contains an active rotative joint fixed to the basis, a passive universal joint (composed by two orthogonal rotative joints) and a passive spherical joint connected to the moving platform.

The six legs of the Hexa robot are arranged in three pairs of two active joints with collinear rotational axes, *i.e.* drive axes. The workspace is physically limited by a rotation constraint of the active rotative joints which avoid inverse kinematics singularities.

It can be demonstrated that Hexa robot cannot reach direct kinematics singularities where a pure prismatic extra degree of freedom is gained (Hesselbach et al., 2005). This means that the index calculated by the power inspired measure can give all the necessary information about the closeness to direct kinematics singularities.

7. Results

Power inspired measure was implemented as an online singularity detection for Hexa parallel robot, as well as the strategy presented in Section 5. Experimental tests defined that the robot could operate safely with $I > 0.029$, so the robot must be stopped when this limit is achieved during the operation, and then the robot can be automatically driven out this problematic configuration. This operation is performed until the index reaches the value of 0.031, when it is considered that the operator can take back the command of the robot.

It is important to elucidate that it is expected that the operator can not continue the exact task that was being attempted. In fact, if this were intended, the robot would probably reach nearly the same singular configuration. Hence the operator has to adjust the requirements of the task to avoid reaching this singularity.

Figure 3 shows the movement performed by each one of the end effector movements while it drives out a singularity. At instant $t = 1s$, the robot has just braked. Before $t = 2s$ the operation starts and lasts until $t \approx 5s$, when $I = 0.031$ is achieved. During the operation, x and y directions has not important influence during operation, as well as rotation around z (yaw angle). In fact, the important movements are the rotations around x and y (roll and pitch, respectively), as well as translation in positive z direction. As it is cited in Section 5, for the Hexa robot the component $\frac{\partial I}{\partial z}$ of vector v (the estimation of ∇I) has positive value and is responsible for an important part of the magnitude of v . In fact, during the operation showed in Fig. 3, there is a movement of almost 10mm in this direction, which can be dangerous if the robot manipulates objects that lay under it. Due to that, in this particular case, it should be usefull to lock this kind of movement through the definition of $\partial I / \partial z = 0$ in \hat{v} .

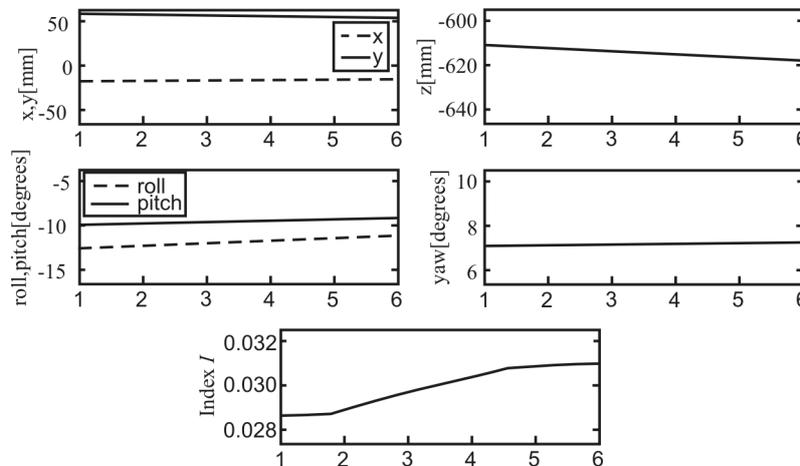


Figure 3. The automatic movement of the tool to escape the singularity (all x axes show time[s])

8. Conclusions

Parallel robots are known as the kind of robot whose workspace is limited and complex-shaped, and, due to that, whose movement is too constrained. One important reason for this problem is the fact that these robots have singularities inside its workspace and it is normally a complex task to determine the conditions that lead to singular configurations. This paper presents one possible approach, that has been implemented in a Hexa robot, to solve this problem. This approach is based on the power inspired measure, that generates an index related to the quantity of constraint imposed to the robot end effector by its legs, *i.e.* the closeness to direct singularities. Besides that, the application of the power inspired measure to the calculation of this index allows the development of the presented strategy to drive the robot out of singularity regions.

This method, however, requires considerable computational time to be computed due to the necessity of the solution of an eigenvalue problem and also because this method requires the output of the direct kinematics as an input. Due to that, the supervision of direct kinematics through this method is really efficient in small velocities, when the robot runs short distances during one controller cycle. Furthermore, when some singularity is detected, the robot need some time and space to be braked due to its inertia. Indeed, the purpose of this method is to improve robot safety when new trajectories or tasks are being tested. This is the kind of work that is done at low velocities until it is perceived that the new trajectory

is safe with respect to singularities, collisions and general workspace problems.

Furthermore, the strategy presented in Section 5 allows the robot to be driven out of dangerous regions, when a singularity is detected. However, if the robot has reached a singularity, the task probably has to be adapted taking this constraint into consideration. So, the purpose of this strategy is also to make the robot more practical while new trajectories or tasks are developed. In fact, in these situations it is common to get workspace problems, such as achievement of actuator limits, collisions and singularities. This last one is specially frequent in parallel robot.

Hence, the purpose of the methods presented in this paper is to allow that parallel robots be more suitable to one important property of robots, that is its reprogrammability. In fact, this is a characteristic that most distinguish robots from other machines used in hard automation. Workspace problems, such as collisions and singularities, as well as the shape and the volume of the robot workspace itself, has great influence to make robots reprogrammability safer and more practical.

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