

## Design of Piezoelectric Actuators Using the Topology Optimization Based on Density Method

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**Abstract.** *Flextensional Piezoelectric Actuator consists of a flexible structure actuated by piezoelectric ceramics. These actuators are applied to microelectromechanical systems (MEMS), nanotechnology equipment, microsurgery equipment, etc. However, due to the fact these actuators essentially consist of a compliant mechanism their design is complex. The compliant structure behaves as a mechanical transform by amplifying and changing the direction of small output displacements generated by piezoceramics. The flexible structure is designed by distributing flexibility and stiffness in the design domain, which can be achieved by using topology optimization. Therefore, the objective of this work is to implement topology optimization method (TOPOPT) to design flextensional piezoelectric actuators. Essentially, the TOPOPT consists of finding the optimal material distribution in a design domain. The implemented topology optimization method is based on the SIMP material model. To illustrate the method, examples presented herein are limited to two-dimensional models once in most part of applications of these actuators they are planar devices. These actuators are manufactured on a 150  $\mu\text{m}$  thickness copper plate through lithography method based on chemical corrosion, utilizing infrastructure of LNLS (Sincrotron Light National Laboratory).*

**Keywords.** *Topology Optimization, Flextensional Piezoelectric Actuators, Nanopositioners, MEMS*

### 1. Introduction

Piezoelectric materials possess the property of converting electric energy (electric field and electric load) into mechanical energy (force and displacement) and vice-versa. The piezoelectric materials are widely used in electromechanical sensors and actuators, accelerometers and ultrasonic transducers. In engineering, applied piezoelectric materials are in general ceramic (PZT) and polymers (PVDF). Flextensional piezoelectric actuators are composed of a flexible structure connected to the piezoelectric ceramics that amplifies and changes the direction of the small displacements generated by piezoceramic (or stack of ceramics) (Rolt, 1990). The design of the flexible structure is similar to the design of compliant mechanism, where the movement is given by the flexibility of the structure instead of the presence of joints. These flextensional piezoelectric actuators have traditional application in the actuation of fine movement of tools in CNC machines and large application in devices that involve precision mechanics, such as mechanisms of photographic machines, headstock reading device of a videocassette or a computer hard disk, where due to compact assembly in a small space, parts of small dimensions are demanded and the lowest possible number of components mounted (other wise the backlash in the assembly problem can make impracticable the functioning of the equipment) and whose operation must be commanded by electric signals. Moreover, the majority of the involved movements in this equipment must be carried through with small displacements. Another area of application of flextensional piezoelectric actuators is bioengineering, where the design of electromechanical surgical instruments applied to laparoscope surgeries. These mechanisms can be used in the construction of microclaws, microclamps or microshears set in motion for electric signals (commanded by physician) that can produce varied movements (Fukuda et al., 1995; Carrozza et al., 1998 and Mehta et al., 2002). Recently, another area of potential application of flextensional piezoelectric actuators that has emerged internationally is the area of microelectromechanical systems (MEMS).

In the past the development of these transducers was conducted using simple analytical models (Xu et al., 1991a, 1991b and Dogan et al., 1994) and experimental techniques (Dogan et al., 1994), in which, the engineers were limited to optimize specific dimensions (for example, the thickness of the piezoceramic). With the development of piezoelectric FEM (Finite Element Method) (Allik and Hughes, 1970) the parametric optimization was possible (Challande, 1990) through the construction of actuator performance curve considering as optimization parameters the resonance frequency, the generated displacement and the blocking force (Newnham et al., 1993 and Onitsuka et al., 1995), as a function of some dimensions of a specific topology, chosen for the connected structure and usually based on the physical intuition of the problem. The performance of the piezoelectric actuators is measured by generated displacements and the blocking force. Previous studies show that large displacements and high blocking force are commitment solutions, and depend on the stiffness and flexibility of the structure connected to piezoelectric ceramics. Thus, in the design of the flexible structure it must be considered the distribution of rigidity and flexibility, what can be obtained by using topological optimization.

The topology optimization method (TOPOPT) (Bendsøe and Kikuchi, 1988) originally developed for design of

maximum stiffness structures recently has been successfully used in the design of compliant mechanisms (Ananthasuresh et al., 1994). Frecker et al. (1997) and Nishiwaki et al. (1998) presented multiobjective formulation for compliant mechanism, in which the ratio between the mutual energy and the energy of deformation is maximized. Silva et al. (1999, 2000) expanded the concepts of mutual energy and mean compliance applied the compliant mechanisms to flextensional piezoelectric actuators, developing a method that allows the design of some types of non conventional piezoelectric actuators, based on the homogenization method (Murat and Tartar, 1985; Bendsøe and Kikuchi, 1988), such as, for example, claws, cramps, clamps, etc. Besides being generic and systematic the topology optimization has obtained great success and acceptance in the design of flextensional piezoelectric actuators (Frecker and Canfield, 2000; Lau et al., 2000).

The objective of this work is to implement a software for the design of flextensional piezoelectric actuators based on the TOPOPT using the SIMP material model (Simple Isotropic Material with Penalization) (Bendsøe, 1989), that is generic and systematic, being able to bring great contributions in this area of application. It makes possible to design flextensional piezoelectric actuators applied to precision mechanics, bioengineering, MEMS, microrobot, etc, by using implemented software. The work will be limited to the design of flextensional actuators in two dimensions, once most of the applications involve bidimensional devices, especially in the case of "MEMS" whose manufacturing techniques generally allow us to implement planar structures.

This paper is organized as follows. In section 2, a short description of TOPOPT and the SIMP material model is given. In section 3, the formulation of optimization problem for the piezoelectric medium is presented. In section 4, the multiobjective function, the optimization formulation of the problem, and the sensitivities of multiobjective function are described. In section 5, the numerical implementation of the optimization problem solution is presented. In section 6, flextensional piezoelectric actuator topologies resulting from the optimization are shown and a manufactured actuator is illustrated and results are discussed. In section 7, some conclusions are given.

## 2. The TOPOPT and the SIMP Material Model

The objective of the TOPOPT is to determine the spaces without material or void and the connectivity of the structure through the removal and addition of material in this fixed domain that will extremize an objective function. Therefore, the optimization problem consists in finding the optimal distribution of material properties in the fixed extended domain. The material in each point of the domain can vary from air ( $\rho = 0$ ) to solid ( $\rho = 1$ ) being able to assume intermediate densities between air and solid. The model material adopted in this work is the SIMP (Bendsøe, 1989; Zhou and Rozvany, 1991; Mlejnek, 1992). The SIMP or density method consists in a mathematical equation that defines the effective property of the base material used in the design, as a function of the density value (design variable that varies from zero to a maximum value) in each point of the fixed domain ( $\Omega$ ). The SIMP approach has the advantage that the material properties between solid and void are interpolated with a smooth continuous function which only depends on the material density. The SIMP interpolation of the effective property of the base material can be written as:

$$\mathbf{C}(x) = \rho(x)^p \mathbf{C}_0 \quad (1)$$

where  $\rho(x)$  it is interpreted as a function of distribution (continuous) of densities (variable of design), and  $x \in \Omega$ ,  $p$  is a penalization power that reduces the intermediate densities in the final result and typically chosen larger than 3. In this way, high values of the  $p$  result in "black and white" or "0/1" design which are easily manufacturable. To satisfy the Hashin-Shtrikman bounds, the power  $p$  for the conductivities must satisfy  $p \geq 2$ .  $\mathbf{C}_0$  is the tensor of the mechanical properties of the base material, which is isotropic and it depends on the Young's modulus of the material base ( $E_0$ ) and the Poisson's ratio.

## 3. Formulation of Optimization Problem for the Piezoelectric Medium

In the design of flextensional piezoelectric actuators, the objective is to maximize the displacements generated (or generated forces) in a region of the domain when it is excited with electrical loads (or electric potential). Thus, the actuator will have to combine a flexible structure to get a large deformation in one determined point with sufficient rigidity to support the external and internal loads. Then, the problem will be formulated in energy terms and it will be divided in to two parts: mean transduction and mean compliance. These formulations express the electrical or mechanical displacements in any region of the piezoelectric medium as a function of displacements (obtained through the FEM) and known electric field, caused by applied electrical loads to the piezoceramic electrodes.

### 3.1. Mean Transduction

The electromechanical function (mean transduction) is related to the electromechanical conversion between two regions (see Fig. 1),  $\Gamma_{d_1}$  (surface with electric loads) and  $\Gamma_{t_2}$  (surface with mechanical loads), of the design domain.

Thus, how larger this function, larger is the displacement generated in determined direction of the surface  $\Gamma_{t_2}$  due to applied electrical load to the surface  $\Gamma_{d_1}$ . Therefore, it is desired to maximize the mean transduction obtained through the load case illustrated in Fig. (1) case *a*, that it is divided in to two steps. The first step is related the actuator response due to application of the electrical load  $\mathbf{d}_1$  to the surface  $\Gamma_{d_1}$  of the transducer (in case a1 of Fig. 1) and second step is related to the simulation of an applied fictitious load  $\mathbf{t}_2$  to the surface  $\Gamma_{t_2}$  (in case a2 of Fig. 1). In each step it is indicated the direction and the point (or points) of desired actuation.

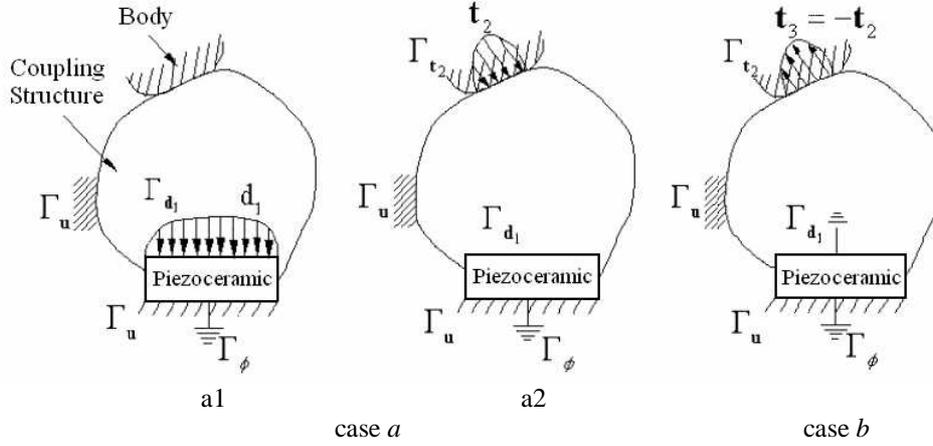


Figure 1. Load cases for calculation of the mean transduction (case *a*) and mean compliance (case *b*).

To illustrate this concept a two-dimensional body, whose elastic and piezoelectric properties are linear is considered. The mean transduction for the piezoelectric medium can be written in the following matrix form (Silva et al., 1999):

$$\begin{aligned}
 L_2(\mathbf{U}_1, \phi_1) &= \{\mathbf{U}_1\}' \{\mathbf{F}_2\} = \{\phi_2\}' \{\mathbf{Q}_1\} = \begin{Bmatrix} \mathbf{U}_2' \\ \phi_2' \end{Bmatrix}' \begin{bmatrix} \mathbf{K}_{uu} & \mathbf{K}_{u\phi} \\ \mathbf{K}_{u\phi} & -\mathbf{K}_{\phi\phi} \end{bmatrix} \begin{Bmatrix} \mathbf{U}_1 \\ \phi_1 \end{Bmatrix} \\
 &= \{\mathbf{U}_1\}' \begin{bmatrix} \mathbf{K}_{u\phi} \end{bmatrix} \{\phi_2\} - \{\phi_1\}' \begin{bmatrix} \mathbf{K}_{\phi\phi} \end{bmatrix} \{\phi_1\}
 \end{aligned} \tag{2}$$

where  $\mathbf{U}_i$ ,  $\phi_i$ ,  $\mathbf{F}_i$ , and  $\mathbf{Q}_i$ , ( $i = 1, 2$ ) are the nodal mechanical displacement, nodal electric potential, nodal mechanical forces and nodal electric loads, respectively. The matrixes  $\mathbf{K}_{uu}$ ,  $\mathbf{K}_{u\phi}$  and  $\mathbf{K}_{\phi\phi}$  are the elastic, piezoelectric and dielectric matrixes, respectively. The FEM equilibrium equation is given by following expression (Allik and Hughes, 1970):

$$\begin{bmatrix} \mathbf{K}_{uu} & \mathbf{K}_{u\phi} \\ \mathbf{K}_{u\phi} & -\mathbf{K}_{\phi\phi} \end{bmatrix} \begin{Bmatrix} \mathbf{u} \\ \phi \end{Bmatrix} = \begin{Bmatrix} \mathbf{F} \\ \mathbf{Q} \end{Bmatrix} \tag{3}$$

With the Eq. (2) the implementation of the concept of mean transduction for the piezoelectric medium using the FEM formulation (Eq. 3) becomes possible in a discretized domain. Thus, we can express the displacement or electric potential in some regions of the piezoelectric medium as function of its displacement and electric charges, both known, caused by applied electrical loads to surface.

### 3.2. Mean Compliance

However, if only the maximization of the mean transduction is considered, an “empty” structure can be obtained as optimal solution, that is, a structure with no rigidity. Therefore, the structural function must be defined to supply enough rigidity between  $\Gamma_{t_2}$  and  $\Gamma_{d_1}$ , what assures the existence of structure between the described regions. In addition, it is necessary for the actuator to generate force and to resist to the reaction of this force generated for any part of the surface while the actuator will be acting or moving a body. Thus, the actuator will have to keep the deflection in the region  $\Gamma_{t_2}$  while it is subjected to a superficial electrical load  $d_1$  in the region  $\Gamma_{d_1}$  (see Fig. 1 case *b*). These objectives are obtained by solving the problem of optimization related to the minimization of mean compliance in the region where it has the contact between the actuator and the body considering the surface of the piezoceramic where the electrodes are

grounded. The mean compliance for the piezoelectric medium is expressed by Silva et al. (1999):

$$L_3(\mathbf{u}_3, \phi_3) = \{\mathbf{U}_3\}' \{\mathbf{F}_3\} = \{\mathbf{U}_3\}' [\mathbf{K}_{uu}] \{\mathbf{U}_3\} + \{\mathbf{U}_3\}' [\mathbf{K}_{u\phi}] \{\phi_3\} = \begin{Bmatrix} \mathbf{U}_3' \\ \phi_3' \end{Bmatrix} \begin{bmatrix} \mathbf{K}_{uu} & \mathbf{K}_{u\phi} \\ \mathbf{K}_{u\phi} & -\mathbf{K}_{\phi\phi} \end{bmatrix} \begin{Bmatrix} \mathbf{U}_3 \\ \phi_3 \end{Bmatrix} \quad (4)$$

where  $\{\phi_3\}' \{\mathbf{Q}_3\} = \mathbf{0}$ . By changing the value of mean compliance we can control the blocking force value.

#### 4. Multiobjective function and problem formulation

To combine the mean compliance and mean transduction in a single optimization problem the following objective function is used (Silva et al., 1999):

$$F = \frac{L_2(\mathbf{u}_1, \phi_1)}{L_3(\mathbf{u}_3, \phi_3)} \quad (5)$$

and more generally

$$F = w * \ln(L_2(\mathbf{u}_1, \phi_1)) - (1 - w) * \ln(L_3(\mathbf{u}_3, \phi_3)) \quad (6)$$

$$0 \leq w \leq 1$$

where  $w$  is a weight coefficient. The objective function allows controlling the contribution of the mean transduction (Eq. 2) and mean compliance (Eq. 4) in the design. It was observed that any variation in the mean transduction has a small influence in the design of the actuator in relation to the mean compliance. In these problems, a large value of  $w$  must be considered to guarantee the maximization of the mean transduction  $L_2(\mathbf{u}_1, \phi_1)$ , and thus the generated displacement. The new problem of optimization (defined in the continuous form) can be expressed as:

$$\begin{aligned} & \text{Maximize: } F \\ & \text{subject to: } \mathbf{K}_{uu}\mathbf{u} + \mathbf{K}_{u\phi}\phi = \mathbf{F} \\ & \quad \mathbf{K}_{u\phi}\mathbf{u} + \mathbf{K}_{\phi\phi}\phi = \mathbf{Q} \\ & \quad \text{(FEM equilibrium equations)} \\ & \quad \int_{\Omega} \rho d\Omega \leq \Omega_s \\ & \quad 0 \leq \rho \leq 1 \end{aligned} \quad (7)$$

where  $\rho$  (design variable) is the pseudo-density function of the material used in the design, which can vary from void ( $\rho = 0$ ) to solid ( $\rho = 1$ ).

##### 4.1. Sensitivity Analysis

The gradients (or derivatives) of the objective function and restrictions are called sensitivities of the optimization problem. The calculation of these gradients is important due to the need of linearization of the objective function in relation to the design variable of the problem, as it will be presented ahead in the numerical implementation of sequential linear programming algorithm (SLP). Thus, in this item the defined gradients of the objective function for the topology optimization of the flextensional piezoelectric actuator are developed. The gradients of function  $F$  relative to the design variable  $\rho_n$  for the first objective function are given by:

$$\frac{\partial F}{\partial \rho_n} = \frac{1}{L_3(\mathbf{u}_3, \phi_3)} \left( \frac{\partial L_2(\mathbf{u}_1, \phi_1)}{\partial \rho_n} \right) - \frac{L_2(\mathbf{u}_1, \phi_1)}{(L_3(\mathbf{u}_3, \phi_3))^2} \left( \frac{\partial L_3(\mathbf{u}_3, \phi_3)}{\partial \rho_n} \right) \quad (8)$$

and, for the second objective function is:

$$\frac{\partial F}{\partial \rho_n} = \frac{w}{L_2(\mathbf{u}_1, \phi_1)} \left( \frac{\partial L_2(\mathbf{u}_1, \phi_1)}{\partial \rho_n} \right) - \frac{(1 - w)}{(L_3(\mathbf{u}_3, \phi_3))} \left( \frac{\partial L_3(\mathbf{u}_3, \phi_3)}{\partial \rho_n} \right) \quad (9)$$

where  $\partial L_2(\mathbf{u}_1, \phi_1) / \partial \rho_n$  and  $\partial L_3(\mathbf{u}_3, \phi_3) / \partial \rho_n$  are the derivatives of the mean transduction and compliance, respectively, in relation to the design variable. Considering the FEM formulation, the sensitivity analysis of the mean transduction can be expressed in the matrix form for the Eq. (10). In this equation, it is considered that the piezoelectric element is not optimized (does not depend on the design variable).

$$\frac{\partial L_2(\mathbf{U}_1, \phi_1)}{\partial \rho_n} = - \left\{ \begin{matrix} \mathbf{U}_2^t \\ \phi_2^t \end{matrix} \right\}^t \begin{bmatrix} \frac{\partial \mathbf{K}_{uu}}{\partial \rho_n} & 0 \\ 0 & 0 \end{bmatrix} \left\{ \begin{matrix} \mathbf{U}_1 \\ \phi_1 \end{matrix} \right\} = - \mathbf{U}_2^t \frac{\partial \mathbf{K}_{uu}}{\partial \rho_n} \mathbf{U}_1 \quad (10)$$

remembering, that for the piezoelectric element  $\frac{\partial \mathbf{K}_{uu}}{\partial \rho_n}$  is equal to the zero.

The sensitivity of mean compliance is expressed in the matrix form as described in the equation below (Eq. 11).

$$\frac{\partial L_3(\mathbf{U}_3, \phi_3)}{\partial \rho_n} = - \left\{ \begin{matrix} \mathbf{U}_3^t \\ \phi_3^t \end{matrix} \right\}^t \begin{bmatrix} \frac{\partial \mathbf{K}_{uu}}{\partial \rho_n} & 0 \\ 0 & 0 \end{bmatrix} \left\{ \begin{matrix} \mathbf{U}_3 \\ \phi_3 \end{matrix} \right\} = - \mathbf{U}_3^t \frac{\partial \mathbf{K}_{uu}}{\partial \rho_n} \mathbf{U}_3 \quad (11)$$

## 5. Numerical implementation

In this item the steps of the numerical implementation of the optimization problem solution are presented. Software was implemented using C language. Figure 2 describes the flowchart of the of topology optimization software. Initially the information (given initial) on geometry is supplied, such as, nodes coordinates, element connectivity and applied loads to the initial domain (extended fixed design domain).

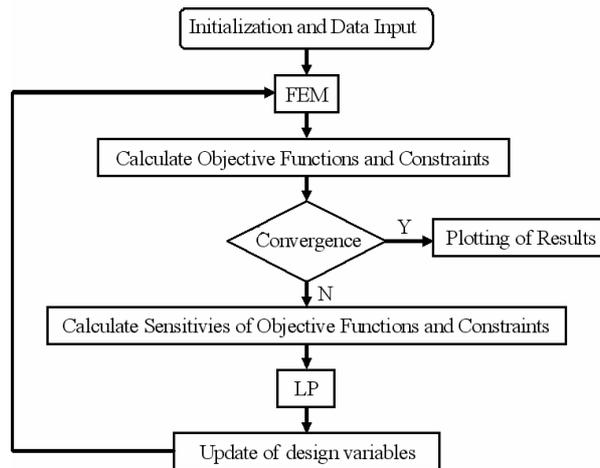


Figure 2. Flowchart of the optimization iterative process.

Software receives this information and through a FEM routine the nodal displacements for each load case are calculated, considering the same finite element mesh. With the nodal displacements and the matrix of global stiffness known, the mean transduction and compliance are calculated. In this way, it is also possible to calculate the value of the objective function. Through the SLP, the optimization problem is solved by supplying the gradients of the objective function, through a routine that mounts the vector of the derivative of the objective function in relation to the density (design variable) of each element of the domain. The LP routine also demands the information of the volume constraint ( $V^*$ ). The linear programming routine is based on the Kamarkar method (Haftka et al., 1996), which receives these data and through an iterative process, calculates the optimum values of design variables of the optimization problem that maximizes the objective function for each sub-problem.

Certain regions of the topology obtained from TOPOPT are characterized by the instability of presence and absence of material. This instability is denominated checkboard and it appears due to the fact the topology optimization problem contains a mixing functional, that involves the field of densities (design variable) and the field of displacements. To prevent the formation of the checkboard, a space filter developed by Cardoso and Fonseca (1999) has been adopted. This filter alleviates the space distribution of the design variable along the extended domain fixed, through a mathematical transformation of each variable of the optimization problem. The space filter with variable radius, adopted in this work, is applied to the moving limits of the design variable of the linearization optimization problem (see Fig. 2). The space filters minimize the dependence of the final result to the mesh refinement of finite elements and allows the

control of the topology complexity. In the linear space filter the neighboring elements are obtained by a fixed sweep radius ( $R_{max}$ ) around each element, making this filter spatial and mesh independent. The expression for this filter is presented in the Eq. (12).

$$\rho_i = \frac{\rho_i V_i + w^F \sum_{j=1}^{nv} \rho_j V_j}{V_i + w^F \sum_{j=1}^{nv} V_j} \quad (12)$$

thus,

$$w_i^F = \frac{R_{max} - R_{ij}}{R_{max}} \quad (13)$$

where  $V_i$  it is the volume of element  $i$  in the discrete problem,  $R_{ij}$  is the distance between centroids (less than  $R_{max}$ ) of central element  $i$  and of neighboring element  $j$ ,  $nv$  is the number of element neighbors inside sweep radius for the central element  $i$  and  $j = 1, 2, \dots, nv$ .

## 6. Results

Examples will be presented to illustrate the design of flexensional piezoelectric actuators using the proposed method. The initial design domain used is shown in Fig. (3) (unit in  $mm$ ) (Silva et al., 1999). It consists in a piezoceramic domain (material properties given in Tab. 1) and of the aluminum optimized domain (property given in Tab. 1). Only one quarter of the initial domain is considered, due to symmetry of the actuator (in relation to the horizontal and vertical direction). The electrical load, the desired displacement and the displacement constraint are shown in the same figure. The domain of Fig. (3) was discretized using rectangle elements in two different mesh sizes, as shown in Fig. (4). The intensity of the applied electric load is equal to  $-1 \mu C / m^2$ . The volume constraint of material  $V^*$  and the values of coefficient  $w$  of the objective function are indicated in Fig. (4).

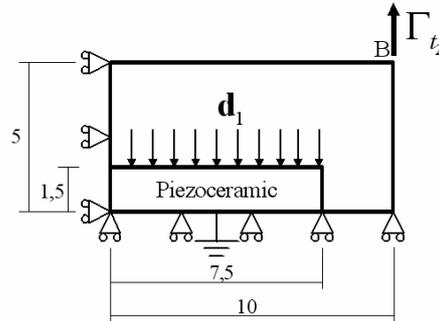


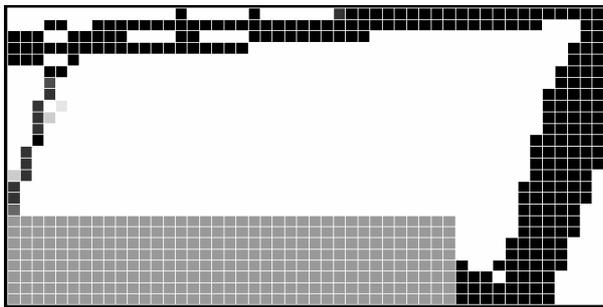
Figure 3. Initial design domain.

Table 1. Material properties.

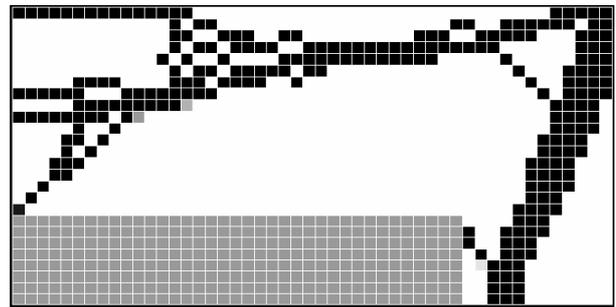
Piezoceramic	PZT5	Aluminum	
$c_{11}^E$ ( $10^{10} N/mm^2$ )	12,10	$E_0$ ( $10^9 N/m^2$ )	7,00
$c_{12}^E$ ( $10^{10} N/mm^2$ )	7,54	$\nu$	0,33
$c_{13}^E$ ( $10^{10} N/mm^2$ )	7,52	Copper	
$c_{33}^E$ ( $10^{10} N/mm^2$ )	11,10	$E_0$ ( $10^9 N/m^2$ )	11,50
$c_{44}^E$ ( $10^{10} N/mm^2$ )	2,11	$\nu$	0,31
$c_{66}^E$ ( $10^{10} N/mm^2$ )	2,26		
$e_{13}$ ( $C/m^2$ )	-5,40		
$e_{33}$ ( $C/m^2$ )	15,80		
$e_{15}$ ( $C/m^2$ )	12,30		
$\epsilon_{11}^S / \epsilon_0$	916		
$\epsilon_{11}^S / \epsilon_0$	830		

It can be verified from the Fig. (4a) to Fig. (4c) a change in the actuator topology when the value of  $w$  is modified. This can be explained due to the fact  $w$  is a coefficient of weight attribution that allows pondering the objective function, so that the actuator has more flexibility and less rigidity or vice-versa. Comparing the result presented in the Fig. (4d) (without application of the filter) with the results shown in the Fig. (4e) (with application of the filter), we can notice through the configurations of presented optimal topologies that when applying the space filter the phenomenon of the checkboard is eliminated, or the obtained optimal topology is exempt of some internal reinforcement and can easily be interpreted. However, an increase of the formation of gray scales is observed in the result presented in the Fig. (4e), once a soft transition is formed between solid material (dark region of the topology) and void (clear region of the topology), around the contour of the flexible structure. The filter makes the method to produce more densities with intermediate values to define the optimal topology.

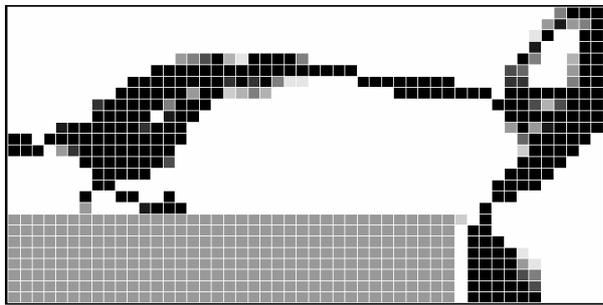
The application of the filter allows the method to search for the optimal global solution, but there is a divergence between objectives of factor  $p$  (Eq. 1) and the filter. Factor  $p$  penalizes the excess of intermediate densities (gray scales), while the filter tends to increase the gray scales reducing factor  $p$ . A solution to clean these gray scales is after to obtain the final topology, turn off the filter and to leave the TOPOPT to continue for more some iteration (Cardoso and Fonseca, 1999). When turning off the filter, the transition between solid material and intermediate materials in the interior of the initial domain are more abrupt due to penalty imposed to the intermediate values of density ( $p = 3$ ), as observed in the Fig. (4f) (final topology).



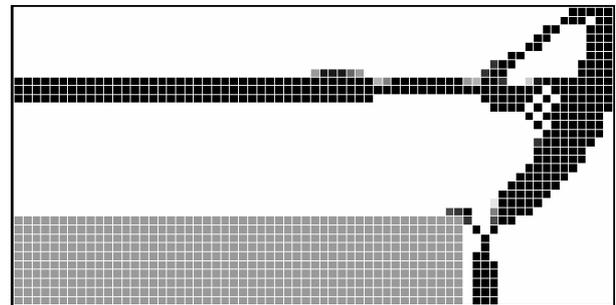
(a)  $w = 0,25$ ;  $V^* = 20\%$ ; 1200 elements and filter turn off



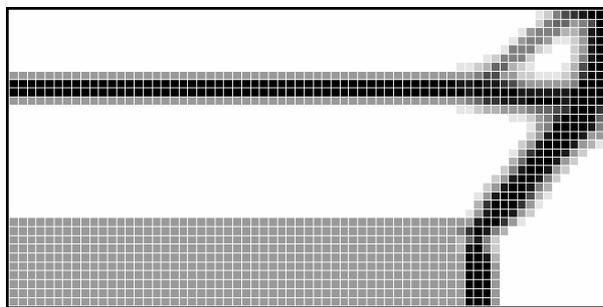
(b)  $w = 0,5$ ;  $V^* = 20\%$ ; 1200 elements and filter turn off



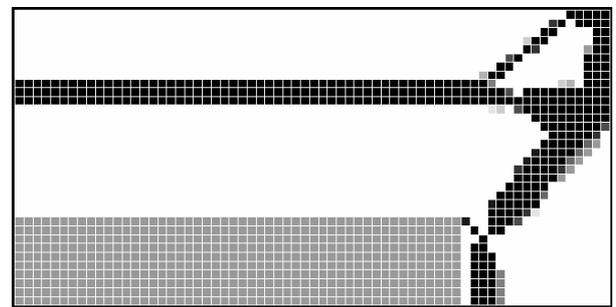
(c)  $w = 0,95$ ;  $V^* = 20\%$ ; 1200 elements and filter turn off



(d)  $w = 0,8$ ;  $V^* = 15\%$ ; 2500 elements and filter turn off



(e)  $w = 0,8$ ;  $V^* = 15\%$ ; 2500 elements and filter turn on



(f)  $w = 0,8$ ;  $V^* = 15\%$ ; 2500 elements

Figure 4. Result of the optimal Topologies.

Figure (5) shows the graphs of convergence for the obtained optimal topology with  $w$  equal to 0,5. It can be verified through the convergence graphs, that the mean transduction is maximized and mean compliance is minimized.

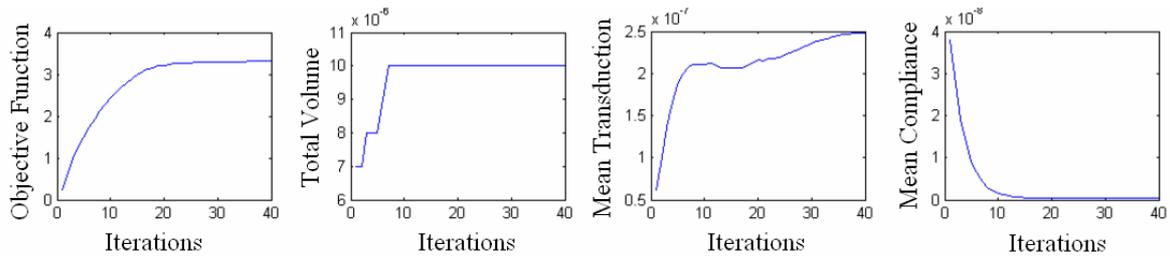


Figure 5. Graphs of convergence ( $w = 0,5$ ).

The second example presented in Fig. (6a) (unit in  $mm$ ) consists in a piezoceramic domain (material properties given in Tab. 1) and of a copper optimized domain (property given in Tab. 1). The actuator has symmetry in relation to the horizontal direction and only half of the domain is presented. The domain is discretized in a mesh with 2509 finite elements. The objective of this flexensional piezoelectric actuator is to obtain in the specified point the maximum displacement in the direction indicated in the Fig. (6a). Figure (6b) illustrates the configuration of the optimal topology, Fig. (6c) shows the optimal topology deformed and Fig. (6d) shows the prototype manufactured in copper obtained through lithography method based on chemical corrosion. Table (2) presents (Model 2D) the displacements obtained in the specified point through ANSYS software.

Some prototypes were manufactured to verify qualitatively the obtained designs. The applied manufacturing technique (Madou, 1997) consists of sensitizing by using UV-light a photopolymer layer deposited onto the material surface. The device geometry is defined by the lithographic mask. After sensitization, the mechanism is obtained by a single step wet-etching that depends on what basic material is used. This technique allows us to manufacture devices with  $50 \mu m$  resolution.

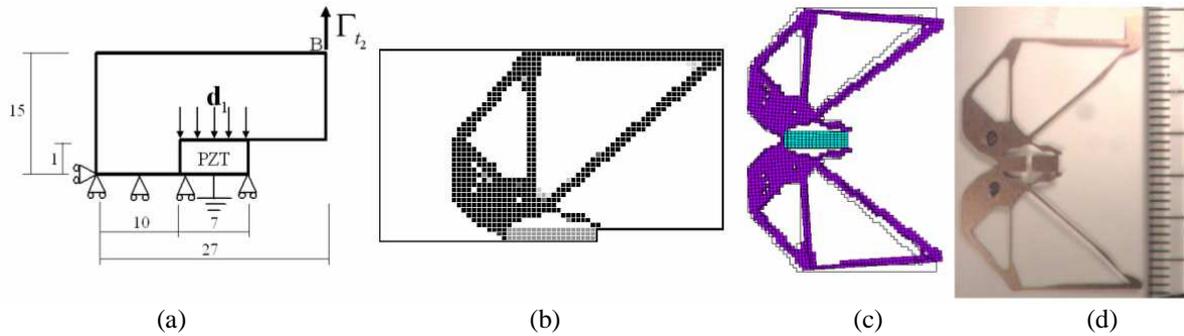


Figure 6. (a) initial design domain; (b) optimal topology; (c) deformed of the optimal topology and (d) manufactured actuator (1 division/mm).

A 3D model of the manufactured prototype was built considering the thickness of copper equal to  $60 \mu m$  and the piezoceramic dimension equal to  $7 \times 2 \times 1 \text{ mm}^3$ , thus this model allows to analyze the deformation out of the actuator plane (see Fig. 7a). The results analyzed (see Tab. 2 Model 3D-1) have the intend to verify the behavior of the actuator when deformed or to obtain a qualitative information of the deformation of the actuator for the design conditions.

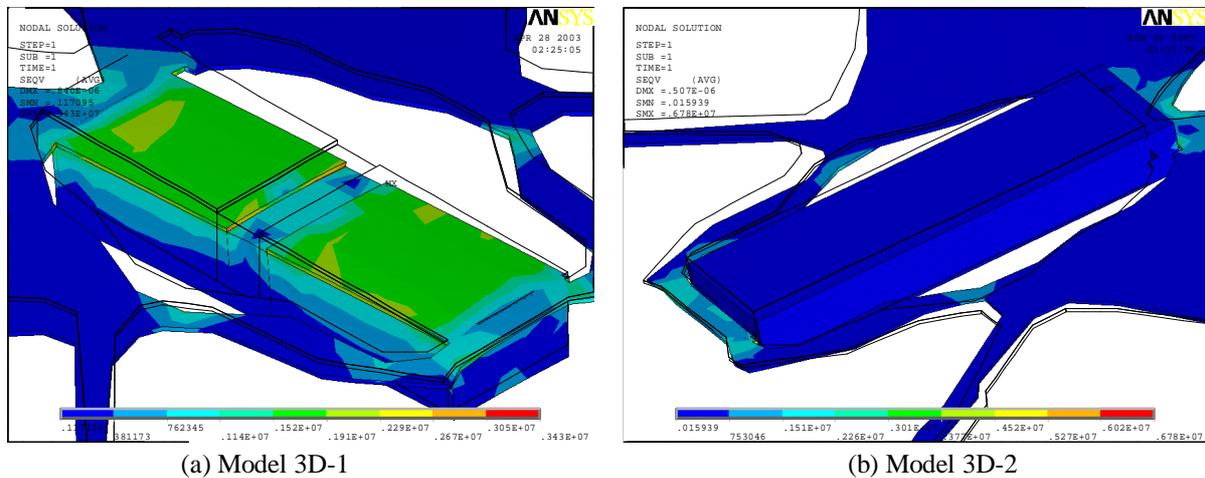


Figure 7. Von-Mises stress in  $Pa$  for the 3D models of manufactured prototype.

Analyzing the results obtained through this model 3D (Model 3D-1, Tab. 2) illustrated in Fig. (7a), using software ANSYS, it can be noticed that the displacements in direction  $y$  ( $x$  – actuation direction;  $y$  – direction of displacement out of plane and  $z$  – secondary direction) are of the order of magnitude of the main displacement ( $x$  direction). This is due to the contact piezoceramic / flexible structure that generate bending moment and buckling. Buckling is generated due to small thickness and thin structure. A suggestion to solve the problem of the buckling is to increase the thickness of the flexible structure, bonding flexible structures layer with epoxy, as a sandwich of layered structures.

A possible solution to solve the problem of the contact piezoceramic / flexible structure is to fit ceramics in the flexible structure by interference, thus the center lines of both structures coincide, eliminating the bending moment (see Fig. 7b). Through simulations made in ANSYS software, it is realized that the maximum out of plane displacement (see Tab. 2, Model 3D-2) has a reduction of 20 times.

Table 2. Values of displacements obtained through ANSYS software for an applied voltage equal to 100 volts.

Model	Direction $x$ ( $10^{-7} m$ )	Direction $y$ ( $10^{-7} m$ )	Direction $z$ ( $10^{-7} m$ )
2D	5,50	—	3,36
3D-1	4,05	8,06	2,56
3D-2	4,51	0,379	2,83

## 7. Conclusions

In this work, the TOPOPT based on the density method (SIMP) was implemented to design flextensional piezoelectric actuators. The software uses Sequential Linear Programming (SLP) that incorporates routines of mathematical programming and finite element method (FEM). A multiobjective function, that considers maximization of flexibility and minimization of compliance, was defined for the optimization problem subjected to material volume constraint of the structure. The control of parameters that influence in the topology optimization results (gray scales, checkboard and dependence of the increase of the initial domain discretization) becomes possible with the implementation of a space filter routine.

It was possible to design and to manufacture flextensional piezoelectric actuators in a mesoescala, using chemical corrosion through the lithography method. The material used in the manufacturing was the copper, with thickness equal to 60  $\mu m$ . The simulations had been carried through with ANSYS software, considering models 2D and 3D, to analyze the behavior of its deformation, when actuated. Thus, large displacements generated out of the actuator plane are due to the type of contact between the flexible structure and the piezoceramic, and due to the buckling.

As future work, a routine to eliminate the problems of the joints in the optimal topology can be implemented (Poulsen, 2002). Another suggestion is to develop the multiflexibility concept, to design actuators with multiple inputs and outputs. Moreover, other objective functions and initial domain can be considered and implemented depending on the desired specific application of the flextensional piezoelectric actuator.

## 8. Acknowledgments

The first author thanks the CNPq for the financial support through a master fellowship and all authors thank the micromanufacturing laboratory of Synchrotron Light National Laboratory (LNLS) for given technical support and facilities access.

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