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CONVENTIONAL AND LQG CONTROLLERS APPLIED TO AN ELECTROMAGNETIC DYNAMOMETER

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Abstract. *This paper shows an electromagnetic system, which aims at the simulation of loads through the torque control. The system is made of a disk, which rotates into a magnetic field, driven by the machine under test. The intensity of the magnetic field by a continuous current through which one can control the torque. The conventional (PID) and optimal (LQG) controlling techniques are used in a plant of linear model. The compensated system is analyzed through simulation where one can observe if the aims of the design are reached. Experimental results with PID and LQG controllers are shown.*

Keywords. *Systems identification, Dynamometer, Controllers*

1. Introduction

The magnetic coupling systems can be used to control the torque or the speed in loads such as sluice gates, slat conveyors and wire drawing in the textile industry. Depending on the system configuration, it can simulate loads to determine the mechanics characteristics of the driving motor. The variation of the torque in this kind of coupling is done by an electric current applied to a coil. This coil creates a magnetic field that induces an electric current to a disk or drum (depending on the equipment configuration), which interacts with the applied magnetic field and generates the necessary torque. In order to make sure that the applied torque remains steady and independent from external disturbances as: speed variation of the driving motor; voltage variation and magnetic field variation caused by the heating of the disk due to induced currents, it is necessary to use a control system.

In the 60’s, when the space-age technology became more competitive between United States of America and the Soviet Union, it was necessary to design machines that could fulfill certain criteria of the design, because, when their technology was based on the conventional control theory, it was not enough to solve the problems. This challenge has motivated the development of researches using techniques based on optimal theoretical control which began in 1940 with Wiener, as it is referred to by (Skogestad and Postlethwait, 1996). As a result of these researches there appeared the design procedure denominated LQG “Linear Quadratic Gausssiam”, as an alternative design technique, that has contributed to the development of the aircraft engineering.

This work has as objective to analyze the design of an electromagnetic dynamometer control using the LQG technique. First the dynamic model of the system is described, secondly the LQG technique is presented, and, finally, the controller is designed. Simulated and experimental results of the system performance are presented, and it also includes a comparative analysis with the conventional controller (PID), which was design through Ziegler and Nichols’ in 1942.

2. System and Plant Description

The electromagnetic dynamometer, made by Equacional, consists of a metallic disk driven by an electric induction asynchronous motor, as shown in Fig. (1). A group of four coils produces a transverse magnetic flow on the surface of the disk and it induces currents that create a torque that works in an opposite way to that of the motor torque. This brake torque depends on the magnetic flow generated by the coils; therefore, it is related to the voltage applied on them.

The controller was improved in the computer program Lab View installed in a computer Pentium 650 mHz. The interface between the computer and the system is done through an input and output data acquisition board, made by Quatech, model daq.801 and the computer program Lab View. Fig. (2) shows a block diagram of the control system.

Figure 1. Photographs of the plant.

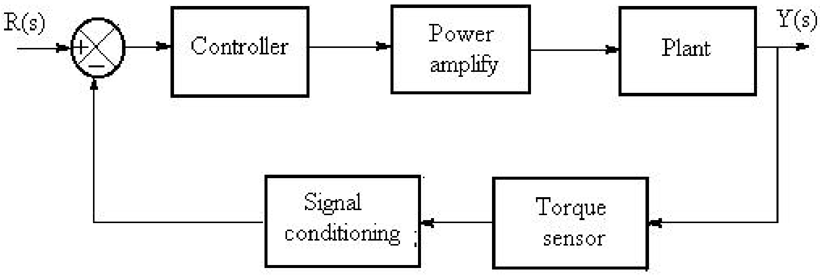


Figure 2. Block diagram of the system.

The torque sensor consists of a steel bar with four resistance strain gauge set up in bridge as shown in Fig. (3).

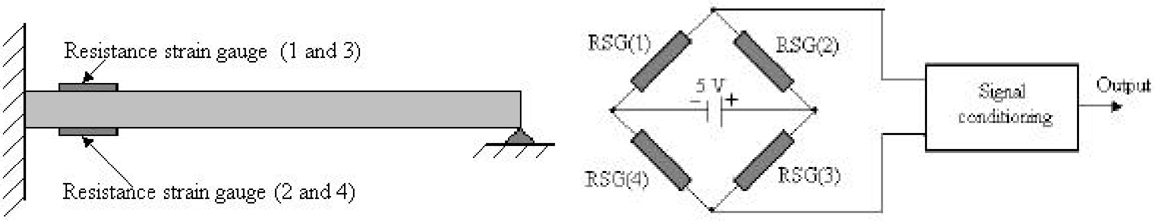


Figure 3. Torque sensor: a) mechanical assembly; b) electrical assembly.

The control variable (output of the computer), that varies between 0 and 5 V, is amplified to supply the coils. The output signal of the bridge is amplified and filtered by the signal conditioning circuit to provide an appropriate voltage (torque), for the computer. The controller, compares the values of the torque with the reference value, and it makes the necessary correction to reduce error.

3. Mathematical Modeling of System

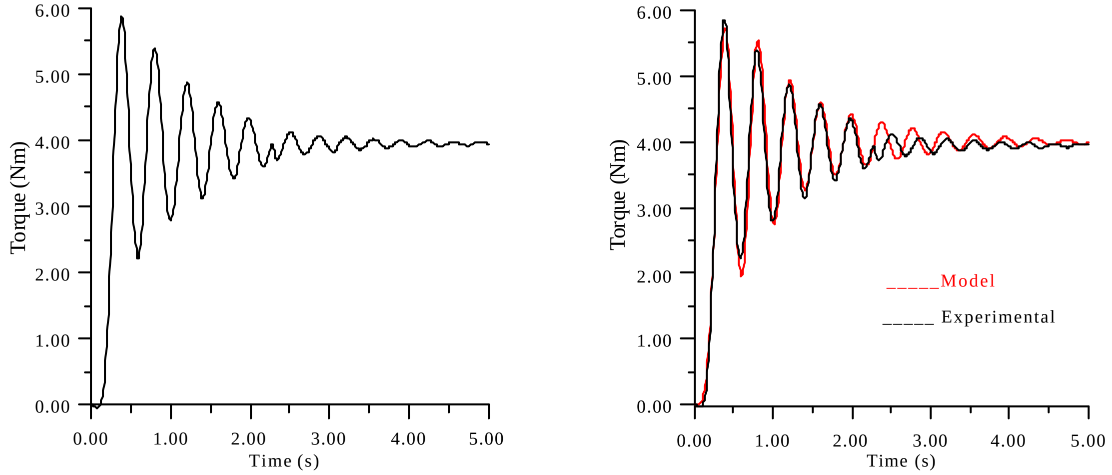
To obtain the mathematical model of system an input step was used to identify the plant (output voltage of the computer equal to 3.4 V) and a sample time of 50 ms, in open loop. Fig. (4a) shows a response curve in open loop, in the time.

Using the response curve identified, the transfer-function was determined through a parametric identification model BJ (Box Jekins model), (Ljung, 1987), inserted in the MATLAB. Fig. (4b) shows the response curves of the real system and simulated system.

The validation of the identification can be done if we compare a model response curve with an experimental response curve. An adjustment of parameters (syntony) should be done when the lining up of the curves is not considered satisfactory (Aguirre, 2000). Was not necessary to apply it in this case because of the similarity between the two curves as shown in Fig. (4b).

In this case, the model was used to represent the plant. Eq. (1) gives the identified transfer-function.

$$G_p = \frac{2.92.10^5}{s^5 + 12.36s^4 + 498.5s^3 + 4450s^2 + 5.86.10^4 s + 2.74.10^5} \qquad (1)$$



a) b)

Figure 4. Response curves in open loop of the system: a) experimental; b) experimental and model.

4. PID Controller Design

The conventional controller design (PID), by Ziegler/Nichols techniques, is done with root locus diagram of the plant (Fig. 5). The crossing point with imaginary axis gives the gain k_m and oscillation frequency ω_m.

According to Fig. (5) we got $k_m = 0.31$ and $\omega_m = 13.72$ rd/s. With these values it is possible to determine the controller parameters PID. These values are determined from the following equations:

$$K_p = 0{,}6k_m, \qquad K_d = \pi K_p/4\omega_m, \qquad K_i = K_p\,\omega_m/\pi \tag{2}$$

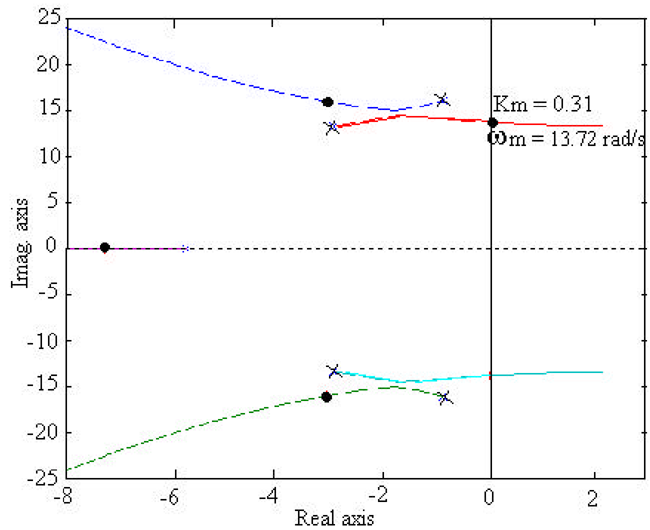


Figure 5. Root locus diagram.

Using the relation given by Eq. (2), we have the following values to the parameters: $K_p = 0.18$, $K_d = 0.011$ e $K_i = 0.81$. These values can be adjusted to meet some performance criteria expected for the system, according to Shahian and Hassull (1993).

5. LQG Controller Design

This technique is based on the stochastic optimum control and it was introduced in the 60's taking into consideration the linear system plant in the state space form to determine a control law to minimize previous values of a quadratic performance index through the state feedback. Considering that both, the states and output are affected by Gaussiam white noises, of null average and not correlated with one another (Cruz, 1996), the LQG regulator has the structure shown in Fig. (6). The equations that define the problem are:

$$\dot{x} = Ax + Bu + \Gamma w_x \tag{3}$$

$$\dot{\hat{x}} = A\hat{x} + Bu + Hv \quad (4)$$

$$y = Cx \quad (5)$$
$$e_y = y + \Phi w_y \quad (6)$$

$$v = -e_y - Cx \quad (7)$$

$$u = -G\hat{x} \quad (8)$$

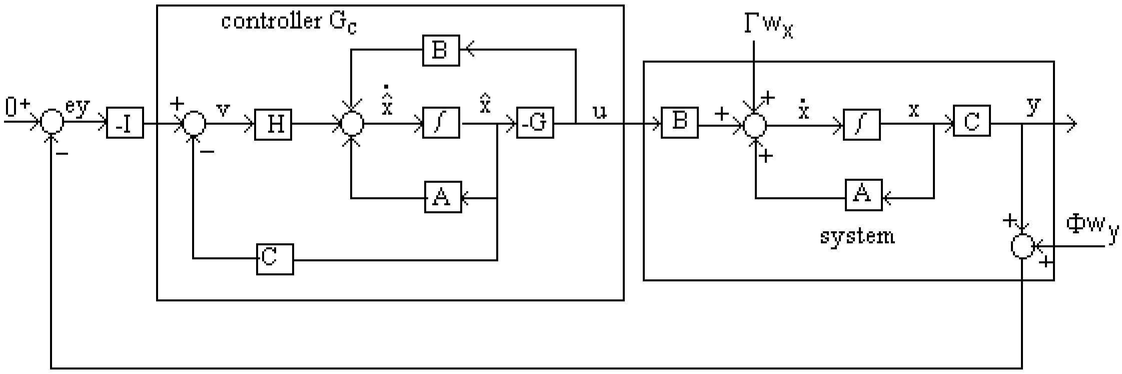


Figure 6. Block diagram of the controller and plant.

The $\hat{x}$ variable is estimated by the states x of the plant, they are determined by Riccati equation form:

$$A^t P + PA + Q - PBR^{-1}B^t P = 0 \quad (9)$$

wherein the R matrix is positive defined and the Q matrix is positive semi-defined to meet the specification of the design. To begin the design, the Q matrix can be selected as equal to:

$$Q = C^t.C \quad (10)$$

The G gain matrix is determined from the following equation:

$$G = R^{-1}B^t P \quad (11)$$

Then, we have a linear quadratic regulator LQR with feedback of the states in the controllers. This solution requires that the pair (A, B) should be controllable.

The H matrix is determined by similar form, but it must be considered that noises w_x in the states and w_y in the output of the plant are Gaussiam white noises, of null average and not correlated with one another (Shahian and Hassul, 1993), as been:

$$E[w_x(t)] = 0 \ \text{ e } \ E[w_y(t)] = 0 \quad (12)$$

$$E[w_x(t)w_x(t+\tau)] = Q_o\delta(t-\tau) \quad (13)$$

$$E[w_x(t)w_x(t+\tau)] = 0 \ \text{ para todo t e } \tau \quad (14)$$

$$E[w_y(t)w_y(t+\tau)] = R_o\delta(t-\tau)E[w_x(t)w_y(t+\tau)] = 0 \quad (15)$$

This problem consists in determining an optimum estimator so that the estimate error $e_x = x - \hat{x}$ is minimized. This requires that the pair (A, C) is observable.

$$\dot{\hat{x}} = A\hat{x} + Bu + H(y - c\hat{x}) \quad (16)$$

$$H = ØC^t R^{-1} \quad (17)$$

The Ψ matrix is determined from the following Riccati equation:

$$A\text{Ø} + \text{Ø}A^{t} + \tilde{A}Q_{o}\tilde{A}^{t} - \text{Ø}C^{t}R_{o}C\text{Ø} = 0 \qquad (18)$$

wherein the Q_o matrix is positive semi-defined and R_o matrix is positive.

To begin the calculation, firstly we consider $Q_o = I$, $\Gamma = B$, and vary R_o until that H matrix, that minimize the error becomes satisfactory. In the direct way (Fig. 6) the relation between ey and u is: u(s) = K(s).ey, being K(s) the transfer-matrix of the controller, that is given by:

$$K(s) = G(SI - A + BG + HC)^{-1}H \qquad (19)$$

To a reference-input yr $\neq$ 0, it is suggested that in Fig. (6), the estimates states $\hat{x}$ are replaced by z variable, that have the same dimensions of x of the nominal model. Making a similarity transformation of the form $\Omega = x - z$, the states equations of the system is defined in Eqs. (5,6,7,8,9 e 10) and can be written like that:

$$\begin{Bmatrix} \dot{x} \\ \dot{\text{Ù}} \end{Bmatrix} = \begin{bmatrix} (A - BG) & BG \\ 0 & (A - HC) \end{bmatrix} \begin{Bmatrix} x \\ \text{Ù} \end{Bmatrix} + \begin{Bmatrix} \tilde{A} \\ \tilde{A} \end{Bmatrix} w_x + \begin{Bmatrix} 0 \\ -H \end{Bmatrix} w_y + \begin{Bmatrix} 0 \\ H \end{Bmatrix} yr \qquad (20)$$

$$\begin{Bmatrix} y \\ u \end{Bmatrix} = \begin{bmatrix} C & 0 \\ -G & G \end{bmatrix} \begin{Bmatrix} x \\ \text{Ù} \end{Bmatrix} \qquad (21)$$

The stability of the nominal system consists of assuring that: Re [λ_i (A –BG)] < 0 and Re [λ_i (A –HC)] < 0.

As the mathematical model of the dynamometer is of 5^{th} order and of the type zero, for LQG controller design it is necessary to introduce an integrator in the input of the system so that the same has null error for an input step type.

In this way, the design is developed considering the original model to which of the integrator was added. Fig. (7) shows the LQG controller $G_c(s)$ in series with a pure integrator and the original system Gp(s). After the $G_c(s)$ controller design, the integrator is removed of the original system and incorporated to the controller design.

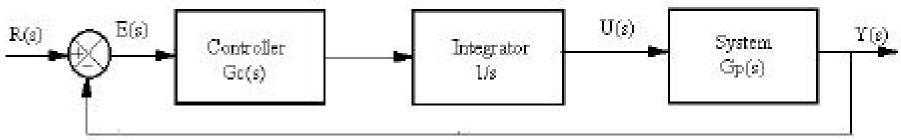


Figure 7. Block diagram of the LQG controller in series with the original system $G_p(s)$.

The gain matrices G and H can be determined apart due to the separation principle (Cruz, 1996; Shahian and Hassul, 1993). In this design the H matrix was determined by considering a reference input yr =0, and the noises w_x and w_y were generated in the MATLAB so the result was white Gaussiam with spectral density Q_o = 1 e R_o = 0.01. The state penalty matrix Q and control R were considered equal to I (identity matrix) and [1], respectively. In this design $\Gamma = [0\ 0\ 0\ 0\ 0\ 1]^t$ and $\Phi = [1]$, was considered.

The gain matrix G was determined considering a reference input step yr equal to 4 Nm maintaining Q_o and R_o constant and the noises w_x and w_y were ignored. The best result to the gain G matrix was obtained considering $Q = 0.8 x C_i^t x C_i$ e R=0.1. C_i = [0 0 0 0 292000 0] is the output transfer-function matrix of the original model to which a pure integrator was added. With theses values we obtained the transfer-function of the LQG controller that is given by Eq. (22).

$$G_C(s) = \frac{27.689468s^5 + 4.416156.10^2 s^4 + 1.497139.10^4 s^3 + 1.448609.10^5 s^2 + 1.808876.10^6 s + 8.259009.10^6}{s^6 + 28.841629s^5 + 8.251750.10^2 s^4 + 1.315978.10^4 s^3 + 1.624917.10^5 s^2 + 1.295632.10^6 s + 5.524165.10^6} \qquad (22)$$

6. Results Obtained from Simulation of the PID and LQG Controllers

The simulation was done into MATLAB without adjustment of the parameters obtained in the design of the PID and LQG controllers. Fig. (8) shows the simulation response curves with a step input of amplitude equal to 4 Nm.

It was verified in Fig. (8) that the PID controller is very oscillatory and the LQG presented a small overshoot with smaller settling time .

7. Experimental Results

To implement the controllers in the Lab View program it is necessary to transform the transfer-function (Eq. 1) to discrete form (Hemerly, 1996). Using the MATLAB program was obtained the transfer-function to the PID and LQG controllers in the discrete form were obtained (Eq. 23 and 24), respectively. In this transformation a sample time of 50 ms was used.

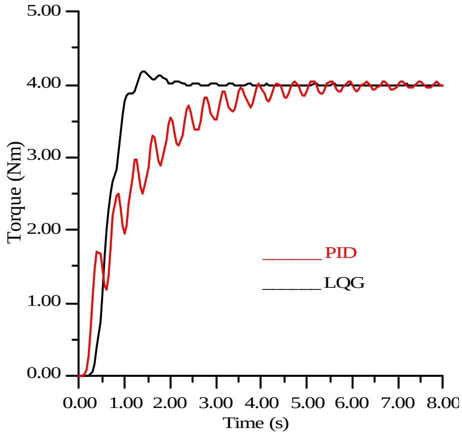


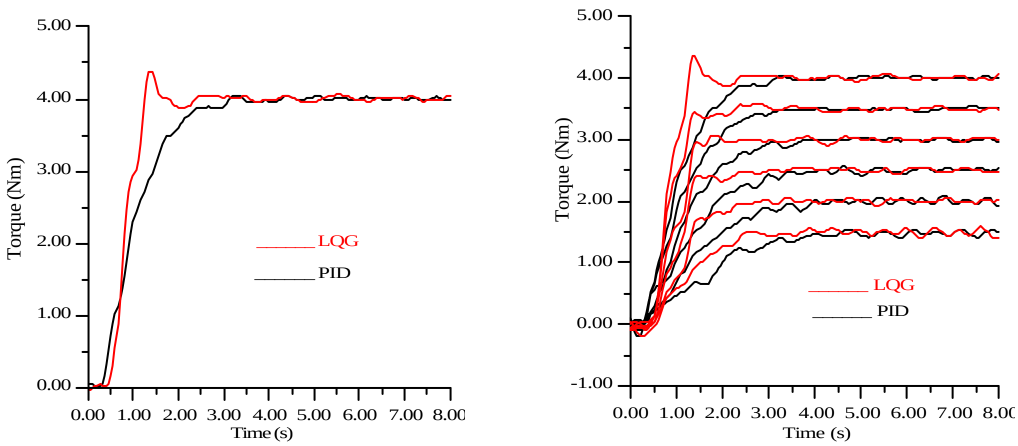
Figure. 8 Simulated response curves of the PID and LQG controllers.

$$G_c(z) = \frac{0.211z^2 + 0.178z + 0.011}{z^2 - z} \tag{23}$$

$$G_c(z) = \frac{0.989602z^5 - 3.293581z^4 + 5.095045z^3 - 4.370843z^2 + 2.061731z - 4.443664}{z^6 - 3.738311z^5 + 6.574557z^4 - 6.728277z^3 + 4.185012z^2 - 1.490890z + 0.236435} \tag{24}$$

The response curves to a step of 4 Nm, to the PID and LQG controllers, are shown in Fig. (9a). The same way that was done in the simulation, the experimental results were obtained without making adjustment in the parameters. To verify the behavior of the controllers to a certain torque range, trials were carried out with a sequence of references, as is shown in Fig. (9b).

Is observed in Fig. (9a) that the LQG controller presented an overshoot of 9% and the PID wasn't presenting that, but the settling time was bigger. Fig. (9b) shows the response curves to several reference step, wherein it is observed that the response of the controllers are slower as soon as the reference step decrease and the overshoot of the LQG controller decrease and it is null starting from 2.5 Nm.

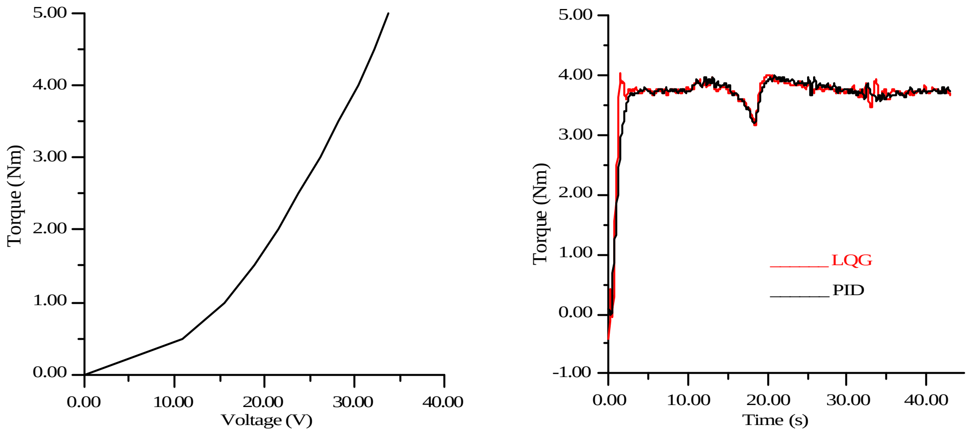


a) b)

Figure 9. Experimental response curves for step input: a) 4 Nm; b) 1.4, 2.0, 2.5, 3.0, 3.5 e 4.0 Nm.

This happened because the plant is nonlinear, as shows Fig. (10a), and the techniques used in the identication and design of the controllers are used in linear systems.

To verify the robustness of the controllers, and the motor was turned off a pre-determined period of time and turned on soon after for a period of time enough for the system to stabilize. The result is shown in Fig. (10b). In this case it was verified that the response of the system to the PID and LQG controllers were practically the same ones.



a) b)

Figure 10. a) Torque x voltage in the coils curve; b) response to the disturbance curve.

8. Conclusions

The simulated response to the PID controller with the obtained parameters using the Ziegler/Nichols technique was very oscillatory and the experimental response didn't show any overshoot. The LQG controller instead shows overshoot but the settling time was smaller in all references. The step response curves presented different performance to different reference values, mainly in the settling time for the two controllers, due to the nonlinear characteristics of the plant. This behavior was already expected because the used techniques are for linear systems, where the best performance of the controllers is verified in small closed region around the identification point was done. It is also observed that the performance of the PID controller improved when adjustment in its parameters, but the purpose of the comparison was to implement the controllers with the parameters of the design, since that adjustment for LQG controller is very laborious due to the great number of variables. Although the PID controller has presented a step inferior response, its robustness for a disturbance was similar.

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