MERCOFRIO2000 – CONGRESSO DE AR CONDUCIONADO, REFRIGERAÇÃO, AQUECIMENTO E VENTILAÇÃO DO MERCOSUL

SIMULATION OF A RESIDENTIAL THERMAL SOLAR SYSTEM PERFORMANCE

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***Abstract**. This paper presents the thermal performance of a residential water heating thermal storage system in function of design and operating parameters. A mathematical model is obtained by an energy balance around the thermal storage water tank. The obtained model describes the conversion of the solar radiation absorbed by the collector into useful energy gain, and thermal losses. The obtained mathematical expression is used to investigate the sensitivity of the stored water temperature to the collector plate absorber area, water volume, global heat transfer coefficient, type of collector, maximum solar radiation and water consumption. The last is considered to be in a steady state flow. The daily solar radiation distribution is estimated by an analytical model and validated using an experimental data obtained from the scientific literature. This analysis is useful in the prediction of the thermal performance and the optimisation of the design and operational parameters of thermal solar systems.*

***KEY-WORDS**: Simulation, Thermal storage, MAPLE_V, Solar energy*

1. INTRODUCTION

Man has appreciated for thousands of years that life and energy flow from the sun. Socrates (470-399 B. C.) is believed to have been the earliest philosopher to describe some of the fundamental principals governing the use of solar energy in applications to buildings as the following passage from Xenophon's Memorablia indicates (Mcviegh 1976): *In houses with a southerly aspect, the rays of the sun penetrate into the porticos during the winter, but in the summer the sun's path is directly over our heads and above the roofs, so that there is shade. If, therefore, this is the best arrangement, we should build the south side higher, to catch the winter sun, and the north side lower to exclude the cold winds.* Solar energy is one of the most adequate sources of energy used in residential water heating. It is inexhaustible, easily converted to heat and its use does not produce pollution.

The design of a residential thermal solar collector is concerned with obtaining minimum cost energy. Thus, it may be desirable to design a collector with an efficiency lower than is technologically possible if the cost is significantly reduced. It is also important to specify the quantity and temperature of the storage tank heated water which depends on the users number. In any event, it is necessary to be able to predict the performance of the thermal solar system.

The residential water heating solar system consists as shown in Fig. 1 of a thermal solar collector, a water storage tank, and piping. The solar collector consists of a flat plate, painted black to enhance solar energy absorption, insulation to reduce the bottom heat losses by conduction and convection., one or more transparent covers to reduce the top losses to atmosphere above the collector by radiation and convection. It is recommended for the absorbing plate to have a high thermal conductivity and low emittance and reduced weight, and the cover, or covers, to have a high transmittance and to be opaque a infrared radiation.

The major part of the industrial sectors fabricate solar collectors without any analysis of their cost and thermal performance. The aim of this work is to provide a mathematical model to evaluate the heated water temperature distribution during the day time period. To establish this aim a mathematical model is developed through an energy balance around the water storage tank. MAPLE-V code is used to solve the obtained differential equations that describe the thermal performance of the system set up The model is used to investigate the heat stored in function of various design and operational parameters. The results are presented in a graphical form representing the influence of various parameters on the performance of thermal solar systems.

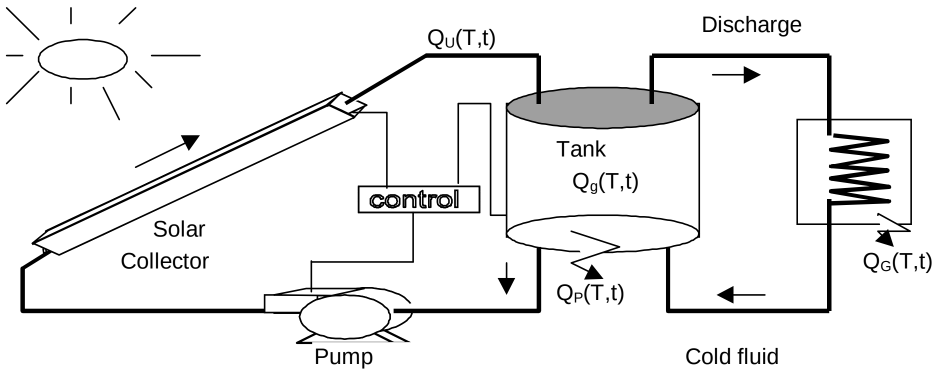

Figure 1- Schematic diagram of a solar collector system

2. MATHEMATICAL MODELING

To obtain the temperature curve of the hot fluid in the storage tank in function of time, physical properties of the working fluid (water), design parameters and fluid volume in the heat storage tank, a macroscopic transient state energy balance about the storage tank is conducted as follows:

$$\left\{\begin{matrix} \text{Heat accumulation} \\ \text{by working fluid} \\ \text{in storage tank} \end{matrix}\right\} = \left\{\begin{matrix} \text{Useful heat} \\ \text{obtained by} \\ \text{solar energy} \end{matrix}\right\} - \left\{\begin{matrix} \text{Heat consumed} \\ \text{by replacing} \\ \text{fluid} \end{matrix}\right\} - \left\{\begin{matrix} \text{Heat loss} \\ \text{to} \\ \text{surrounding} \end{matrix}\right\} - \left\{\begin{matrix} \text{Heat consumed} \\ \text{or input by external} \\ \text{heating system} \end{matrix}\right\}$$

Em mathematical symbols this expression is written as follows:

$$Q_{ac} = Q_u - Q_g - Q_p - Q_G \tag{1}$$

where the heat accumulated in the storage tank is given by

$$Q_{ac} = \frac{d}{dt}\left(\rho C_p V T\right) \tag{2}$$

and the useful heat obtained by the solar collector can be formulated using the collector efficiency η which is expressed as a function of the temperature difference between the circulating working fluid in the solar collection area and the ambient temperatures and of the linear collectors coefficients a and b, average solar radiation and collector area A_c. The linear collector coefficients specifies the type of collector to be used.

$$Q_u = \eta I_r A_c, \qquad \eta = a + \frac{b}{2I_r}\left(T - T_a\right) \tag{3}$$

The heat loss by the working fluid consumption in the heating process is given by:

$$Q_g = \dot{w}_s C_p T - \dot{w}_e C_p T_e \tag{4}$$

where T_e is the replacing fluid temperature.

The heat losses from the storage tank to the ambient are grouped in a heat transfer global coefficient U_p for the exposed tank area A_p is given by:

$$Q_p = (UA)_p (T - T_a) \tag{5}$$

Substituting these quantities in the energy balance gives:

$$\frac{d}{dt}\left(\rho C_p V T\right) = \eta I_r A_c - \left(\dot{w}_s T - \dot{w}_e C_p T_e\right) - U_p A_p \left(T - T_a\right) - Q_G \tag{6}$$

The heat storage fluid volume can be considered constant and also its physical properties (density and specific heat) can be assumed constant due to the small variation in temperature. Substituting the efficiency expression in Eq.(6) gives:

$$\rho C_p V \frac{dT}{dt} = \left(a + \frac{b}{2}\frac{\left(T - T_a\right)}{I_r}\right) I_r A_c - \dot{w}_s C_p T + \dot{w}_e C_p T_e - U_p A_p \left(T - T_a\right) - Q_G \tag{7}$$

$$\frac{dT}{dt} + \frac{1}{\rho C_p V}\left(\dot{w}_s C_p + U_p A_p - \frac{b}{2} A_c\right) T = \frac{\left(a I_r A_c + \dot{w}_s C_p T_e + \frac{b}{2} A_c T_a + U_p A_p T_a - Q_G\right)}{\rho C_p V} \tag{8}$$

The solar radiation can be considered as a function of time with a maximum daily value I_R.

$$I_r(t) = I_R sin(\varpi t) \tag{9}$$

This relation gives a good representation of the natural behaviour of solar radiation distribution of clear days, Fig. 2.

Substituting all of the defined expressions Eq. (8) and rearranging gives:

$$\frac{dT}{dt} + P(t)T = Q(t) \tag{10}$$

The initial condition for this system is taken to be as $T(0)=T_o$ and the variables $P(t)$ and $Q(t)$ are defined as:

$$P(t)=\frac{\dot{w}_s C_p + U_p A_p - \frac{b}{2} A_c}{\rho C_p V} \tag{11}$$

and;

$$Q(t)=\frac{aI_r A_c + \dot{w}_e C_p T_e - \frac{b}{2} A_c T_a + U_p A_p T_a - Q_G}{\rho C_p V} \tag{12}$$

where the collector parameter efficiency b efficiency, the solar radiation I_r, the fluid inlet temperature, T_e, and the ambient temperature T_a for N days (where N is equal two times the period P) are defined for a continues variation in the form:

$$b(t)=b_o\left[\sum_{n=0}^{N}(-1)^n u(t-12n)\right] \tag{13}$$

$$I_r(t)=I_R sin(wt)\left[\sum_{n=0}^{N}(-1)^n u(t-12n)\right] \tag{14}$$

$$T_e(t)=T_{io}+\left(T_{ioM}-T_{io}\right)sin\left(w_o t\right) \tag{15}$$

$$T_a(t)=T_{am}+\Delta sin\left(w_a t\right) \tag{16}$$

where $u(t)$ is the step function of Heaviside. Note that the step function is used to equate the values $b(t)$ and $I_r(t)$ to zero during the night. In this case t_d is the sunshine period duration and Δ is the maximum ambient temperature excess relative to the average value T_{am}. For simplicity the distribution of the ambient temperature can be expressed in function of frequency function w_a.

$$T_a(t)=T_{am}+(T_M-T_{am})sin[w_a(t-t_{dev})] \tag{17}$$

for a deviation, t_{dev}, and daily maximum temperature, T_M.

For the energy consumption, the fluid mass flow rate, $\dot{w}_e$, can be given as:

$$\dot{w}_e(t)=\frac{1}{2}\dot{w}_{eo}\left(1+sin\left(w_o t\right)\right) \tag{18}$$

in this manner various forms of the hot water consumption can be handled.

Considering an optional system as heat exchanger that consume a variable energy quantity, $Q_G(t)$, with a known consumption frequency, w_{cg};

$$Q_G(t) = \frac{1}{2} Q_{G_o}(T)\left\{1 + \cos\left[\cos\left(w_{cg} + t\right)\right]\right\} \tag{19}$$

The frequencies of the consumption of the fluid w_o, the variation of solar radiation w, of ambient temperature variation w_a and of the optional system energy consumption w_{cg}, are defined as:

$$w_o = \frac{\pi}{t_{w_o}}, \; w = \frac{\pi}{t_d}, \; w_a = \frac{\pi}{t_{dt}}, \; w_{cg} = \frac{\pi}{t_{cg}}$$

where t_{wa}, t_d, t_{dt} and t_{cg} are the corresponding duration periods.

The collector efficiency is defined as a linear function of the following related coefficients (Svard et.. (1981))

$$a = F_R(\tau\alpha)_n \quad , \; b = -F_R U_L$$

The solution of Eq. (10) can be put in a closed form as

$$T = T_o \exp(-P(t)) + \exp(-P(t))\int_0^t Q(\tau)\exp(-P(\tau))d\tau \tag{20}$$

In this form the transient temperature distribution of the fluid in the storage tank, assuming constant energy consumption, $Q_G = Q_{G_o}$, is given by the expression:

$$\begin{aligned} T(t) = T_o \exp(-P_o(t)) + \frac{\exp(-P(t))}{\rho C_p V}\Big[aA_c\int_0^t I_r(\tau)\exp(P(\tau))\Big]d\tau + \\ C_p\int_0^t T_a(\tau)\dot{w}_e(t)\exp(P_o(\tau))d\tau - \mathrm{A_c}\int_0^t T_a(\tau)\mathrm{b}(\tau)\exp(\mathrm{p}(\tau))\mathrm{d}\tau \\ + U_p A_p\int_0^t T_a(\tau)\exp(P_o(\tau))d\tau - \frac{\exp(-P_o(t))}{\rho C_p V}Q_{G_o}\left(1 - \exp(P_o(t))\right) \end{aligned} \tag{21}$$

This work investigates the performance of a residential flat plate solar collector. The characteristics of a residential solar collector is the rate of water consumption which is assumed to be constant. The influence of the working fluid volume, collector area, global heat transfer coefficient, the solar radiation, and the working fluid flow rate are studied. The daily solar radiation is estimated using mathematical relation obtained from the literature. This equation is validated through comparison com experimental results. The developed equations are solved using MAPLE_V to calculate the temperature of the working fluid in the storage tank.

3. RESULTS AND DISCUSSION

Most of the residential solar collectors fabricate this type of systems without any pre-investigation of thermal performance or economic feasibility which results in high construction cost. To minimise the cost an optimum design has to be developed. This design would tie the cost, thermal performance, number of users and other parameter such as the collector weight and architectural aspects. The thermal performance would be better expressed directly in terms of the hot water temperature. The construction cost depends basically on the prices of the residential solar collector components and the collector size. Then it would be interesting to develop a model that expresses the water storage tank temperature in terms of collector design parameters.

In this section the performance of a residential solar collector in function of various design and construction parameters is investigated. Among others are the solar absorber plate area, the water storage tank volume, solar radiation, type of collector (collector efficiency), and the water consumption flow rate. The water consumption is assumed to be constant and in a steady state. The day time is considered to be in hours to express a real situation and would be better visualised. In the following graphical representations the zero hour is equal to 6:00 AM and so on. The various parameters used in the calculation are given in Tab. 1.

Table 2- Design and construction parameters used in the calculation

Specific heat (J/kg^oC)	4179.14	T_e (^oC)	20
ρ (kg/m^3)	995.7	T_o (^oC)	20
U_P ($J/m^{2o}C$)	8	w_e (kg/s)	0.056
T_a (^oC)	20	w_s (kg/s)	0.056
b ($J/sm^{2o}C$)	-8.86	Inside diameter (m)	0.0254
A_c (m^2)	2.0	Tank volume (m^2)	0.1
a (m)	0.83	I_R ($J/s.m^2$)	998.5

Before the simulation of the solar system is started the solar radiation intensity is compared to an experimental data obtained by Abugderah (1991) and measured on the date of 14 of august of 1990. As can be seen from Fig. 2 the theoretical data obtained by the mentioned relation gives satisfactory results of a clear solar day.

One of the most important factor of the residential solar collector thermal performance is the solar radiation intensity. To make use of the mentioned equation various maximum solar radiation intensities were simulated during the whole day. The comparative results are shown in Fig. 3 and as can be verified system thermal performance is increases as the solar intensity increases. The highest thermal performance is obtained during the time interval between 12:00 and 16:00 PM. After wards the water storage tank temperature drops due to the decrease of the solar intensity and ambient temperature which are directly proportional to the system thermal performance.

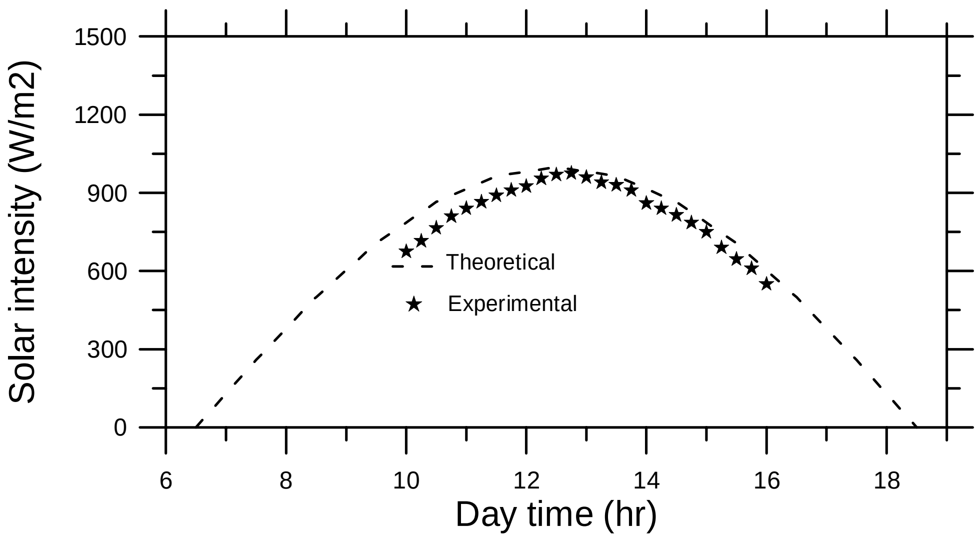

Figure 2 - Solar radiation intensity of a clean day.

One of the residential solar collector characteristics is the water consumption which is usually a function of the family members. The project design is based on the consumption flow rate. Figure 4 presents the hot water temperature versus the day time for different consumption flow rates. As can be seen from the figure the hot water temperature is very sensitive to the consumption flow rate. As expected the hot water temperature is directly proportional to the flow of consumption. Therefore, knowing the necessity of the water consumption the solar collection size can be calculated depending on the desired hot water temperature.

Figure 5 presents the temperature of the working fluid in the tank for different solar collection areas. The day time period is assumed to be *12 hr* long where the zero hour would be the sun rise time. As expected the energy stored in the water tank is a strong function of this area. Note that for $A_c = 4m^2$ the fluid temperature reaches its highest value at about *2hours* after mid day. At about *3 hours* after sun rise the temperature starts to change significantly until the maximum temperature is reached. Afterwards the fluid temperature starts to fall because of the decrease in the solar radiation intensity.

The main parameter of residential water heating systems is the quantity (volume) of the water to be heated. The volume of water tank is based on the number of the hot water temperature and consumption. Figure 6 presents the fluid temperature versus day time for different volumes of the heated fluid in the storage tank. The lower the volume of the heated water the higher the temperature reached and loses heat more rabidly. When the volume equals $0.005\ m^3$ the fluid reaches a temperature of about 63^oC at mid day then starts to lose heat due to the high temperature difference between the fluid and ambient temperatures. For volume of $0.5\ m^3$ the highest temperature is reached at time of *3 hours* after mid day and it is practically maintained until the end of the day period. Of course, a higher desired temperature can obtained by increasing the collector area.

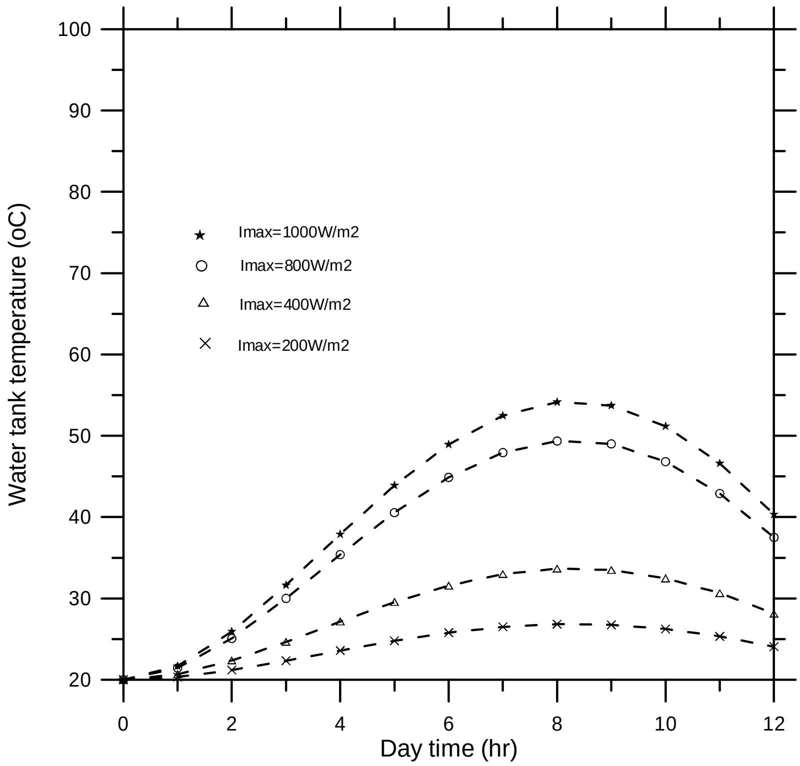

Figure3 - Fluid tank temperature versus day time for different solar radiation intensities

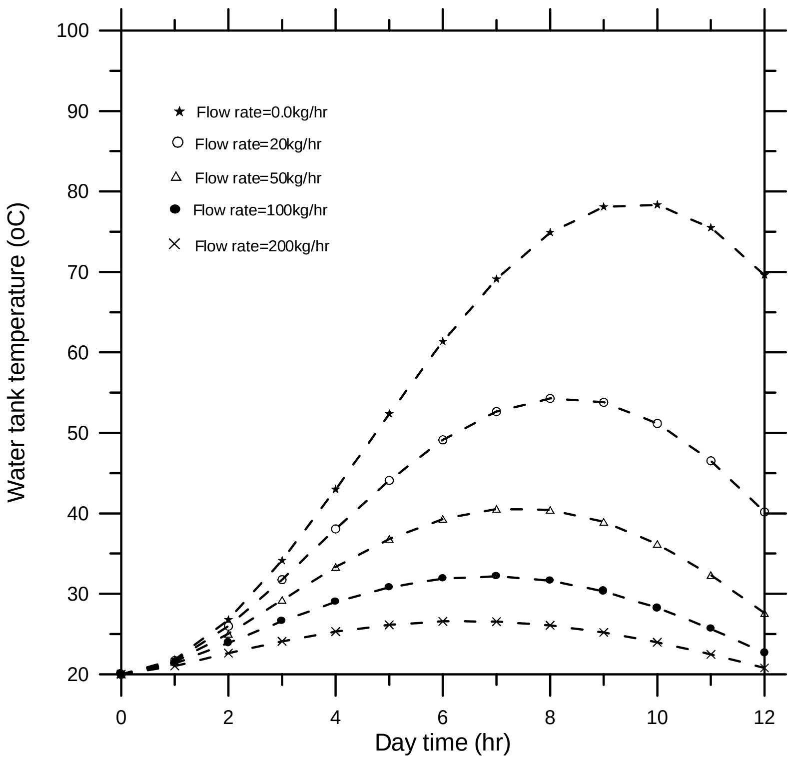

Figure 4 - Fluid tank temperature versus day time for different consumption flow rates

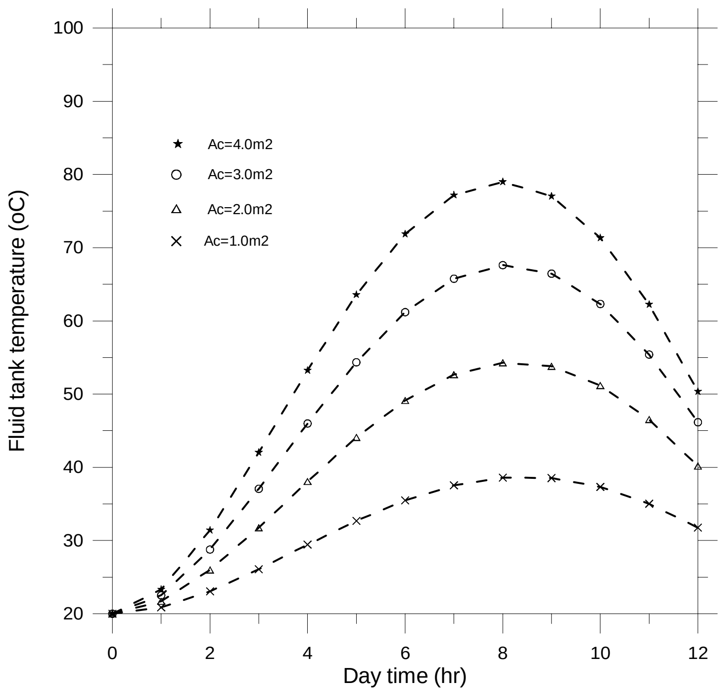

Figure 5 - Storage fluid temperature versus day time for different collector plate area

Figures 7 and 8 show the change of the heated fluid temperature versus day time for different total heat loss coefficient and type of collector, respectively. The total heat transfer coefficient consists of bottom, top and edge losses. The top losses depends on losses by convection, conduction and radiation and can be minimised by the use of one or more transparent covers. The radiation losses can be lowered using a selective collector plate and glass covers because it reflects the infrared radiation. The bottom losses can be handled by the use of an insulation materials. The edge losses are usually small compared with the collector area. The efficiency of thermal solar systems depend on the type of collector used. These values are checked and the results are shown in Fig. 8. As can be seen the thermal performance of the solar system is not very insensitive with respect to collector type.

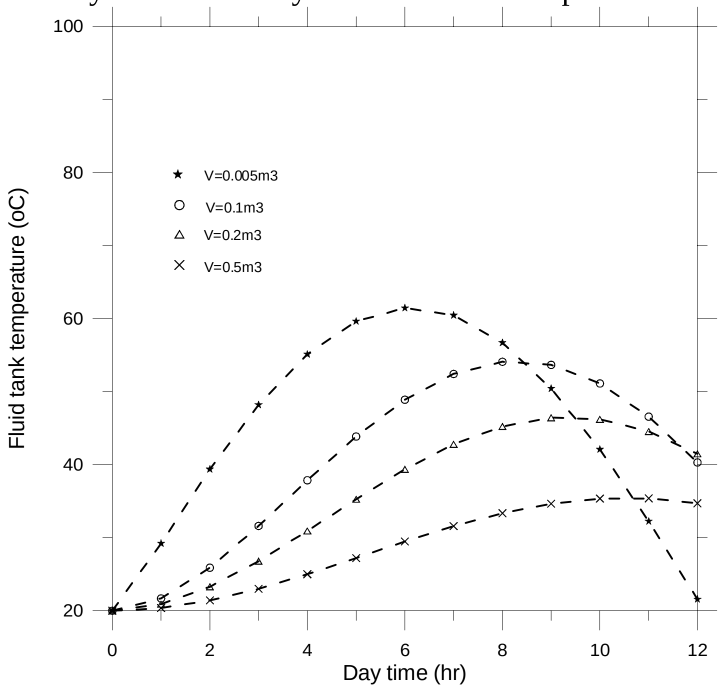

Figure 6 - Storage fluid temperature versus day time for different collector plate area

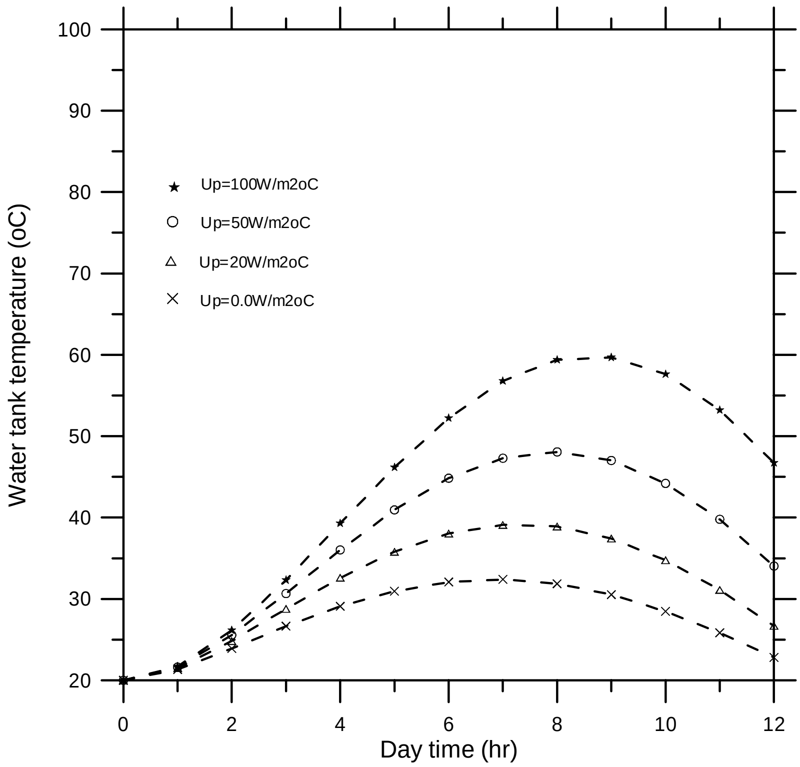

Figure 7 - Storage fluid temperature versus day time for different global heat transfer coefficient

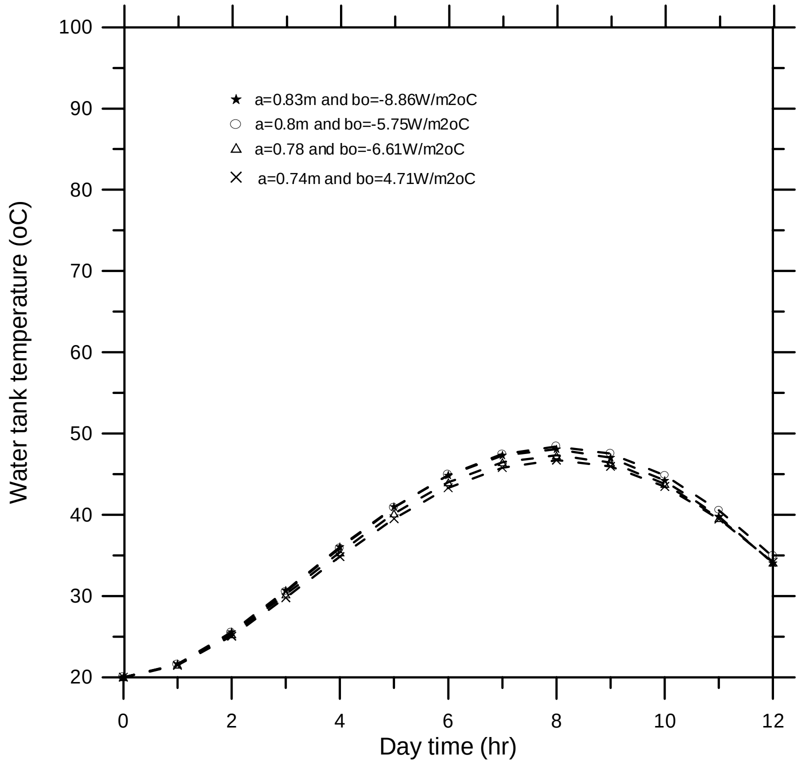

Figure 8 - Storage fluid temperature versus day time for different collector types

CONCLUSIONS

Water and ambient heating is one of the most important applications of solar energy conversion. To optimise the solar system it is necessary to have some type of model to calculate the influence of various parameters on its thermal performance. The developed procedure calculates the fluid temperature in function of working fluid proprieties and of the ambient and the system construction parameters. It also permits the design and the optimisation of the system before construction and therefore permits economic visibility study. It is worth mentioning that the choice of the collector size, besides the thermal performance, depends also on space availability and sometimes on architectural requirements.

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