MecSol2013
International Symposium on Solid Mechanics
April 18-19, 2013
Porto Alegre, Rio Grande do Sul, Brazil

Frequency Response Optimization Using the Genetic Algorithm in Vibroacoustic Systems

Walter J. Paucar Casas, Régis E. Antich, Emanuel M. Cesconeto, Ricardo F. Leuck Filho
Departmento de Engenharia Mecânica
Universidade Federal do Rio Grande do Sul
walter.paucar.casas@ufrgs.br, Regis.Antich@gkndriveline.com, emanuelemc@hotmail.com, leuck.r@gmail.com

ABSTRACT

Although there are many researches carried out in the area of analysis and optimization of vibroacoustic systems, few publications show results obtained by Genetic Algorithms – GA and check its performance for frequency response optimization. This work intends to contribute in vibroacoustic analysis and optimization, through the development and implementation of one kind of algorithm to optimize the sound pressure of coupled and simplified vibroacoustic systems modeled in three dimensions. The response optimization is implemented in our own academic program of finite elements MEFLAB developed into the commercial software MATLAB®. The optimization of the response is also implemented through an own code developed for genetic algorithms. For the coupled case in study the sound pressure response is reduced adequately in the frequency interval of interest and allowed us to displace one of the two resonance responses to the boundary of the frequency interval showing its adequacy to optimize the response of coupled systems.

Keywords: Global optimization, coupled system, fluid-structure interaction

1 INTRODUCTION

A review of structural and acoustical analysis techniques using numerical methods like finite element FEM and boundary element BEM methods, followed by a survey of techniques for structural-acoustic coupling and a wide discussion over objective functions for passive noise control in structural-acoustic optimization is realized by [1]. Also, [2] demonstrates the potential of the application of normal modes in external acoustics for optimization of radiating structures.

Then, although there are many researches carried out in the area of vibroacoustic analysis and optimization, few publications present results obtained using global optimization algorithms. The genetic algorithm method is used for minimization of sound pressure in vibroacoustic systems in [3], where the system is modeled using BEM.

Some works propose a method for noise reduction using a topology optimization [4]. The sound field is modeled with the Helmholtz equation and the topology optimization is based on the interpolation functions of continuous material, density and modulus of compressibility.

The response optimization in coupled systems was explored by [5] and [6] with the classic method called Solid Isotropic Material with Penalization SIMP.

It was constructed a physical model of a box composed of thin plates of different thicknesses and bars of rectangular section [7]. Inside the fluid is air. The box was instrumented to allow data collection from both the acoustic domain as the domain structure. After constructing the physical model, a mathematical model of the box was designed for simulation; box's bars were modeled with Bernoulli beam finite elements, plates were modeled with thin flat plate finite element and fluid was modeled with boundary elements. This work demonstrates the applicability and efficiency of the developed mathematical models, when used to analyze and optimize real vibroacoustic systems.

This work is developed in order to implement algorithms for optimizing the sound pressure of coupled vibroacoustic systems in three dimensions, using the technique of Genetic Algorithms (GA) to optimize the response.

2 VIBROACOUSTIC COUPLED SYSTEMS

In order to analyze the coupling of a fluid-structure system, we consider the effect of fluid pressure on the interface Γ_I of the structure surface as being $\boldsymbol{f}_{s\Gamma_I}$, that added with the body forces $\boldsymbol{f}_{sB}$ constitute the terms of excitement in the dynamic equation:

$$\boldsymbol{k}_{ss}\boldsymbol{u} + \boldsymbol{m}_{ss}\,\ddot{\boldsymbol{u}} = \boldsymbol{f}_{s\Gamma_I} + \boldsymbol{f}_{sB} \;, \tag{1}$$

where,

$$\boldsymbol{f}_{s\Gamma_I} = \int_A \boldsymbol{N}_s^{\;T}\, q\, dA\,. \tag{2}$$

The pressure that the fluid exerts on the interface with the structure generates distributed forces q normal to the structural surface. After substituting q by $\tilde{p}$, it is obtained the equilibrium condition at the interface as:

$$\boldsymbol{f}_{s\Gamma_I} = \int_A \boldsymbol{N}_s^{\;T}\;\boldsymbol{N}_f\; dA\,\boldsymbol{p}\,. \tag{3}$$

where $\boldsymbol{N}_s$ are structural shape functions and $\boldsymbol{N}_f$ are fluid shape functions. Adding Eq. (3) in Eq. (1):

$$\boldsymbol{k}_{ss}\;\boldsymbol{u} + \boldsymbol{m}_{ss}\;\ddot{\boldsymbol{u}} + \boldsymbol{k}_{fs}\;\;\boldsymbol{p} = \boldsymbol{f}_s \tag{4}$$

where

$$\boldsymbol{k}_{fs} = -\int_A \boldsymbol{N}_s^{\;T}\;\boldsymbol{N}_f\; dA\,. \tag{5}$$

Coupling of the structural domain with the fluid domain is forced in the normal direction $\hat{n}$ of the interface surface, through an identity which ensures compatibility kinematics. This can be represented by a slipping condition of the fluid in the tangential direction of the interface:

$$\dot{\boldsymbol{v}}_{\hat{\boldsymbol{n}}} = \ddot{\boldsymbol{u}}_{\hat{\boldsymbol{n}}} \tag{6}$$

The fluid-structure coupling is described in terms of the pressure change for the fluid domain near the structural region, through the following boundary condition:

$$\frac{\partial p}{\partial \hat{\boldsymbol{n}}} = -\rho_f \, \ddot{\boldsymbol{u}}_{\hat{\boldsymbol{n}}}, \quad \text{em } \Gamma_I \tag{7}$$

Substituting Eq. (6) in Eq. (7), which means to replace the normal component $\dot{\boldsymbol{v}}_{\hat{\boldsymbol{n}}}$ by $\ddot{\boldsymbol{u}}_{\hat{\boldsymbol{n}}}$, and adopting $\tilde{\ddot{\boldsymbol{u}}} = \boldsymbol{N}_s \, \ddot{\boldsymbol{u}}$ to approximate the value of $\ddot{\boldsymbol{u}}_{\hat{\boldsymbol{n}}}$ by $\tilde{\ddot{\boldsymbol{u}}}_{\hat{\boldsymbol{n}}}$, or in discretized form by $\boldsymbol{N}_s \, \ddot{\boldsymbol{u}}$:

$$\boldsymbol{m}_{ff} \, \ddot{\boldsymbol{p}} + \boldsymbol{k}_{ff} \, \boldsymbol{p} + \boldsymbol{m}_{fs} \, \ddot{\boldsymbol{u}} = \boldsymbol{f}_f \tag{8}$$

where $\boldsymbol{m}_{fs}$ is the matrix with the terms of interface, which can be rewritten in a semi-discretized form:

$$\boldsymbol{m}_{fs} = \int_{\Gamma_I} \boldsymbol{N}_f^{\,T} \, \boldsymbol{N}_s \, d\Gamma_I \tag{9}$$

From Eqs. (4) e (8) we can organize them in a compact matrix, showing the formulation ***u-p***:

$$\begin{bmatrix} \boldsymbol{m}_{ss} & 0 \\ \boldsymbol{m}_{fs} & \boldsymbol{m}_{ff} \end{bmatrix} \begin{Bmatrix} \ddot{\boldsymbol{u}} \\ \ddot{\boldsymbol{p}} \end{Bmatrix} + \begin{bmatrix} \boldsymbol{k}_{ss} & \boldsymbol{k}_{fs} \\ 0 & \boldsymbol{k}_{ff} \end{bmatrix} \begin{Bmatrix} \boldsymbol{u} \\ \boldsymbol{p} \end{Bmatrix} = \begin{Bmatrix} \boldsymbol{f}_s \\ \boldsymbol{f}_f \end{Bmatrix} \tag{10}$$

As can be seen from Eq. (10) this matrix formulation is not symmetric, which is one of its main disadvantages, as preclude the use of several efficient algorithms developed for symmetric matrices. However, the main benefit of this formulation is the small number of degrees of freedom used in the modeling of the fluid domain. In the case of free vibrations, the force vector of Eq. (10) is zero.

The stiffness matrix of fluid-structure interaction is given by integration of Eq. (5). The mass matrix of fluid-structure interaction is given by integration of Eq. (9) and corresponds to:

$$\boldsymbol{k}_{fs} = -\boldsymbol{m}_{fs}^{T} \tag{11}$$

3 MODAL FREQUENCY RESPONSE

The modal frequency response $\boldsymbol{U}$ is expanded in terms of right eigenvectors $\boldsymbol{\Phi}$ of the coupled system according to:

$$\boldsymbol{U} = \boldsymbol{\Phi Q} \tag{12}$$

where the matrix $\boldsymbol{Q}$ considers the left eigenvectors $\overline{\boldsymbol{\Phi}}$ and the term $\boldsymbol{\Omega}^{-1}$ relates the eigenvalues λ_i with the excitation frequency ω through:

$$\boldsymbol{\Omega}^{-1} = \boldsymbol{diag}\left\{ \frac{1}{\lambda_i - \omega^2} \right\} \tag{13}$$

$$\boldsymbol{Q} = \boldsymbol{\Omega}^{-1} \overline{\boldsymbol{\Phi}}^{T} \boldsymbol{F} \tag{14}$$

The solution $\boldsymbol{u}$ is named as the frequency response:

$$\boldsymbol{u} = \sum_{i=1}^{n} \boldsymbol{\phi}_i q_i \tag{15}$$

and substituting this expression in Eq. (10), pre-multiplying it by $\bar{\boldsymbol{\phi}}_i^T$ and utilizing the orthogonality condition of the coupled system is obtained:

$$m_i \ddot{q}_i + k_i q_i = f_i \qquad (i = 1,2...,n) \tag{16}$$

where,

$$m_i = \bar{\boldsymbol{\phi}}_i^T \boldsymbol{M} \boldsymbol{\phi}_i \tag{17}$$

$$k_i = \bar{\boldsymbol{\phi}}_i^T \boldsymbol{K} \boldsymbol{\phi}_i \tag{18}$$

$$f_i = \bar{\boldsymbol{\phi}}_i^T \boldsymbol{F} \tag{19}$$

If the excitation force acting in the structure is harmonic type, then the response $\boldsymbol{u}$ will also be harmonic,

$$\boldsymbol{f} = \boldsymbol{F} e^{\mathrm{j}\omega t} \tag{20}$$

$$\boldsymbol{u} = \boldsymbol{U} e^{\mathrm{j}\omega t} \tag{21}$$

$$q_i = Q_i e^{\mathrm{j}\omega t} \tag{22}$$

which after substitution in Eqs. (15) and (16) generates the expression of the frequency response as being a superposition of coupled modes:

$$\boldsymbol{U} = \sum_{i=1}^{n} \boldsymbol{\phi}_i Q_i = \sum_{i=1}^{n} \boldsymbol{\phi}_i \left(\frac{1}{\lambda_i - \omega^2} \right) \bar{\boldsymbol{\phi}}_i^T \boldsymbol{F} \tag{23}$$

where

$$Q_i = \left(\frac{1}{\lambda_i - \omega^2} \right) \bar{\boldsymbol{\phi}}_i^T \boldsymbol{F} \tag{24}$$

The great advantage of the modal superposition involves including a limited number of modes in the calculation, usually lower than the number of degrees of freedom of the system, this way the computational cost is drastically reduced. However, a number lower of modes causes errors in the values of natural frequencies, while a number higher of modes can increase the processing time and not compensating the replacement of the direct method

4 GENETIC ALGORITHM PROGRAMMING

The problem aims to find the minimum value of a constrained nonlinear multivariable function, $f(\boldsymbol{x})$, stated as:

$$\text{Min } f(\boldsymbol{x}) \text{ under } \begin{cases} \boldsymbol{c}(\boldsymbol{x}) \leq \boldsymbol{0} \\ \boldsymbol{ceq}(\boldsymbol{x}) = \boldsymbol{0} \\ \boldsymbol{A}\ \boldsymbol{x} \leq \boldsymbol{b} \\ \boldsymbol{Aeq}\ \boldsymbol{x} = \boldsymbol{beq} \\ \boldsymbol{lb} \leq \boldsymbol{x} \leq \boldsymbol{ub} \end{cases} \tag{25}$$

where $\boldsymbol{x}$, $\boldsymbol{b}$, $\boldsymbol{beq}$, $\boldsymbol{lb}$ e $\boldsymbol{ub}$ are vector, $\boldsymbol{A}$ and $\boldsymbol{Aeq}$ are matrices, $\boldsymbol{c}(\boldsymbol{x})$ and $\boldsymbol{ceq}(\boldsymbol{x})$ are functions that return vectors, and $f(\boldsymbol{x})$ is a function that return an scalar, $f(\boldsymbol{x})$, $\boldsymbol{c}(\boldsymbol{x})$ and $\boldsymbol{ceq}(\boldsymbol{x})$ may involve nonlinear

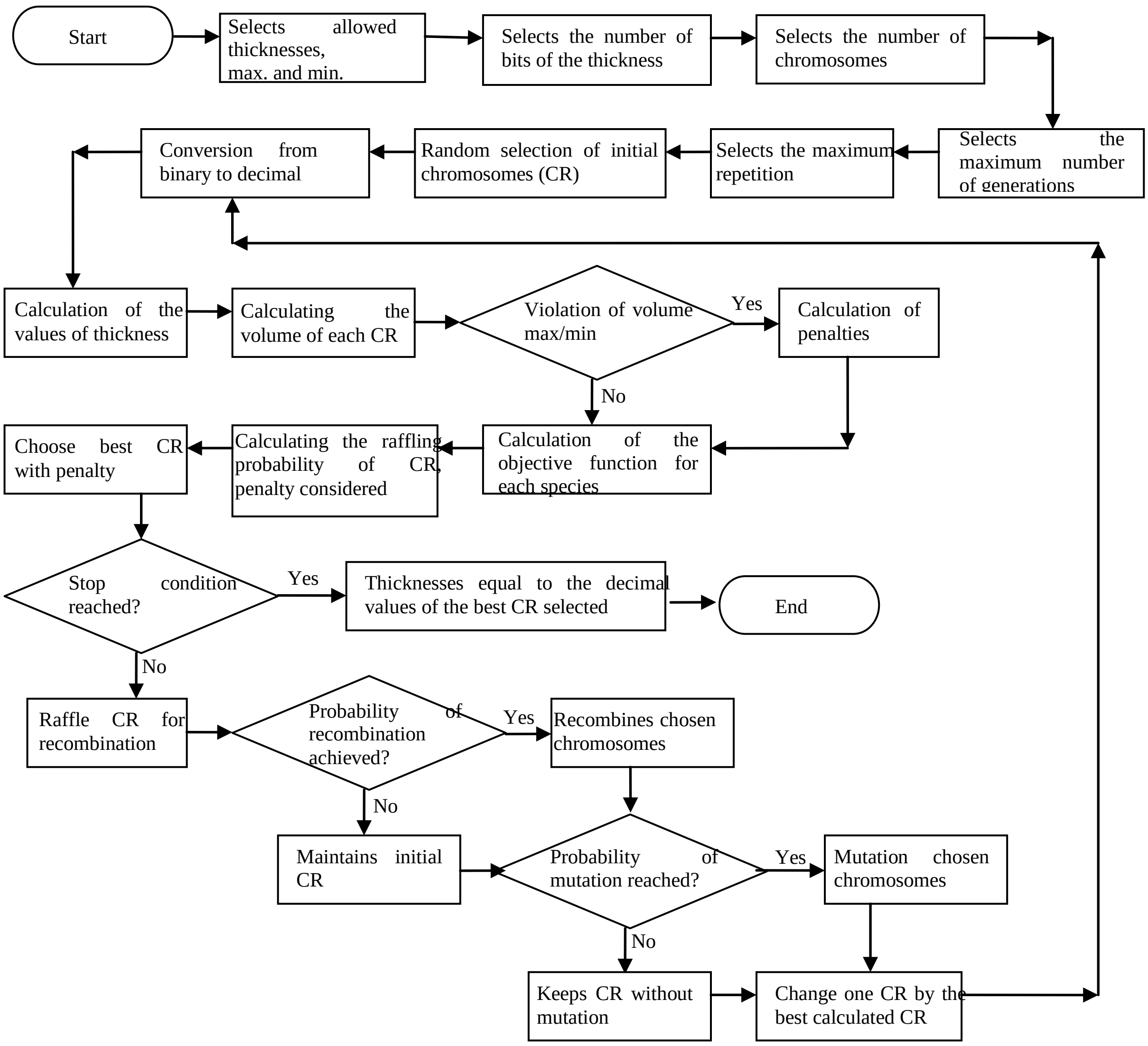

Figure 1: Flowchart of the optimization process with Genetic Algorithms

functions of the type used in this work. The function to be minimized is continuous.

The method of Genetic Algorithms was implemented into MEFLAB through a developed program using Matlab® commands to have more freedom, and not just being used the Genetic Algorithms toolbox available in Matlab ®. Optimization by Genetic Algorithm was chosen in this work because it is a global search algorithm, in this case over the lowest frequency response in structural systems and fluid-structure systems. Figure 1 shows the programmed optimization flowchart of Genetic Algorithms.

5 OPTIMIZATION OF FREQUENCY RESPONSE

There are many ways to reduce the response to vibration. We could add active or passive damping devices, or modify the physical properties of materials such as stiffness. Another way is by changing the geometrical characteristics of systems through the shape or dimensions. In this work, we chose the dimensional change of the structural domain of the coupled system by varying the thickness of plates. One of the adopted boundary conditions is to maintain the original structural mass within a small variation.

The frequency response U_{ij}, that in this work is the sound pressure, in a point *i* of measurement, resulting from an excitation force applied on one point of the structure, called the degree of freedom of input (*GDLin*), having an excitation frequency ω_j, can be obtained through the method of modal superposition for *n* coupled modes. With this method it is possible to calculate the system response using Equation (26):

$$U_{ij} = \sum_{k=1}^{GDLin} \sum_{l=1}^{n} \phi_{il} \left(\frac{1}{\lambda_l - \omega_j^2} \right) \bar{\phi}_{kl} F_k \qquad (26)$$

where ϕ_{il} is the component *i* of the right eigenvector *l*, λ_l is the eigenvalue *l*, $\bar{\phi}_{kl}$ is the component *k* of the left eigenvalue *l*. All these components are functions of the design variable, in this case the thickness variable.

The minimization of response is made through a number of discrete excitation frequencies called *freq*. In this work, the excitation is applied in the structural domain and the response measurement is realized in the fluid domain; this configuration was chosen because it represents the effects found in a cabin of a vehicle, where the excitation is transmitted from the structure to the cabin in the form of sound pressure (noise), so the point desired to reduce the sound pressure would be the driver's ear, for example. The reading point of the response is called the degree of freedom of output (*GDLout*).

A general description of the optimization problem to reduce the sound pressure, considering the structural thickness *e* as design variable, is exposed as follows::

$$\begin{aligned} &Minimize \sum_{j=1}^{freq} \sum_{i=1}^{GDLout} \left| U_{ij}(e, \omega_j) \right| \\ &e \in \Re^r \end{aligned} \qquad (27)$$

subject to constraints:

$$\begin{aligned} &\sum_{i=1}^{r} (e_i A_i) - \vartheta = 0 \\ &e_{\min} \le e \le e_{\max} \end{aligned} \qquad (28)$$

where e_i and A_i are the thickness and area of the finite element i, respectively, ϑ is the constant volume of the structure, $e_{mín}$ and $e_{máx}$ are the maximum and minimum thicknesses of the finite element, respectively.

6 RESPONSE OPTIMIZATION IN AN ACOUSTIC CAVITY OVER A SUPPORTED SQUARE PLATE –AN EXCITATION FORCE IN A POINT, MULTIPLE EXCITATION FREQUENCIES AND A MEASURING POINT

For the study is considered a square plate of side equal to 0.508 m (20 in) and thickness equal to 0.00508 m (0.2 in), simply supported at its four edges. The plate material is aluminum having the following properties:

Modulus of elasticity E = 68.948x10^9 N/m^2 (1.0x10^7 psi)

Poisson's ratio ν = 0,3

Density ρ = 2700 kg/m^3 (2.54 x10^{-4} lb-s^2/pol^4)

For the analysis, the plate is divided in 64 elements with 4 nodes per element, which means a total of 81 nodes as can be seen in

Figure 2. The non-conform plate element is chosen in MEFLAB.

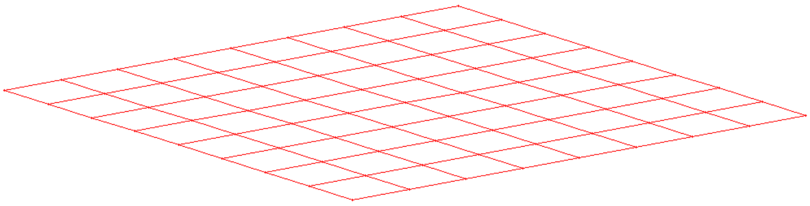

Figure 2: Plate 8x8 – 64 elements

Over the aluminum plate there is an air cavity of hexaedrical shape with 0.508 m (20 in) on each side of the base and height of 2.54 m (100 in). The cavity is filled with air having the following properties:

Density ρ = 1,29 kg/m^3 (1,21x10^{-7} lb-s^2/pol^4).

Speed of sound in air c = 330,2 m/s (13000 pol/s).

For the analysis the hexahedron of Figure 3 is divided in 512 elements with 8 nodes each one, totalizing 729 nodes. At MEFLAB natural frequencies are calculated using formulations of hexaedrical solid elements. The sidewalls of the cavity are considered rigid.

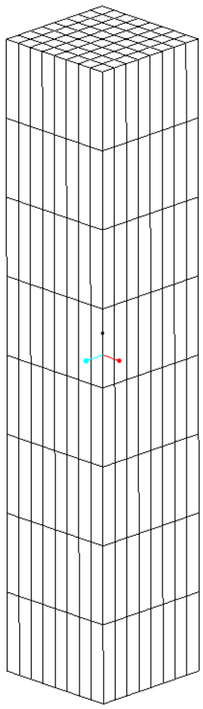

Figure 3: Cavity of air 8x8x8 – 512 elements

We consider the system of an acoustic cavity on a square plate and the corresponding frequency response analysis; proposing in this section the optimization of the sound pressure at a point of the fluid under multiple structural excitation frequencies through genetic method. For this optimization, the plate thickness is considered as the design variable and volume of the plate as constraint function.

The frequencies 60, 62, 64, 66, 68, 70, 72, 74, 76, 78 and 80 Hz are used as excitation frequencies for the applied force, each frequency is applied separately and the total response is the sum of the sound pressure of each excitation frequency at the measuring point. This way we intend to obtain a plate with a distribution of different thicknesses for each finite element, i.e. the plate has a variable thickness, but its mass should not be changed significantly, outside the specified range given as constraint.

The following data are considered during the optimization:

Force application node: 30, DOF 86 at structural domain.

Response node for optimization: 405, DOF 563 at fluid.

Excitation frequencies: 60, 62, 64, 66, 68, 70, 72, 74, 76, 78 and 80 Hz, these frequencies are chosen because there are two different resonant frequencies of the coupled system in this interval, one influenced by the fluid domain (65.4 Hz) and another (75.7 Hz and repeated because of the symmetry) influenced by the structural domain.

Number of modes for modal analysis: 10 (previous simulations showed that this amount was sufficient for the analysis of evaluated frequencies).

Minimum thickness allowed: 0,1 pol. (0,00254 m).

Maximum thickness allowed: 0,4 pol. (0,01016 m).

Minimum volume allowed: 90% of the original plate volume.

Maximum volume allowed: 110% of the original plate volume.

Number of chromosomes: 20.

Number of bits per element: 5.

Maximum number of generations: 100.

Maximum number of consecutive repetitions of the objective function value of the best individual: 15.

Results of 10 simulations for the Genetic Algorithm can be seen in Table 1.

Table 1 Simulations of optimization for the acoustic cavity over one plate.

Simulation	Generations	Decrease of original response (%)	Runtime reference (s)
1	100	42.8	15625
2	100	35.7	15858
3	89	54.2	15391
4	83	42.0	14280
5	41	45.2	7025
6	37	43.0	5939
7	33	31.7	5187
8	100	35.7	15562
9	100	35.7	14935
10	100	35.7	14883
Média	78,3	40.2	12468
Standar deviation	29,1	6.6	4473
Min. value	33	31.7	5187
Máx. value	100	54.2	15858

Figure 4 shows the obtained responses for the genetic process, shifting the curve of sound pressure to lower values than the initial response and repositioning one point of resonance closer to 79 Hz.

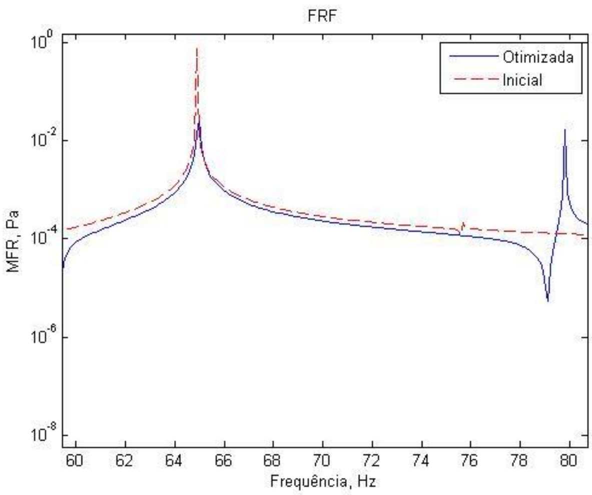

Figure 4: Sound pressure initial and optimized for 60-80 Hz interval using genetic algorithm

The configuration of thicknesses for the plate finite elements found for one of the genetic simulations can be seen in Table 2.

Table 2 Distribution of thicknesses after optimization of the acoustic cavity over one plate.

Element number	Thickness (pol)	Element number	Thickness (pol)
1	0,28	33	0,32
2	0,11	34	0,35
3	0,22	35	0,15
4	0,18	36	0,27
5	0,22	37	0,23
6	0,10	38	0,22
7	0,35	39	0,10
8	0,11	40	0,10
9	0,10	41	0,31
10	0,10	42	0,12
11	0,10	43	0,10
12	0,23	44	0,31
13	0,19	45	0,11
14	0,16	46	0,24
15	0,40	47	0,16
16	0,25	48	0,12
17	0,14	49	0,38
18	0,17	50	0,25
19	0,35	51	0,36
20	0,18	52	0,20
21	0,22	53	0,30
22	0,17	54	0,27
23	0,25	55	0,23
24	0,19	56	0,22
25	0,30	57	0,38
26	0,28	58	0,21
27	0,19	59	0,36
28	0,27	60	0,22
29	0,29	61	0,40
30	0,28	62	0,39
31	0,21	63	0,23
32	0,11	64	0,19

7 CONCLUSIONS

Concerning the response optimization of the coupled fluid-structure system; and observing the distribution of thicknesses, it was detected a configuration quite different from the original. For the studied case, the genetic method reduces adequately the initial sound pressure but with an important processing time.

8 ACKNOWLEDGEMENTS

To programs of scholarships for undergraduate research; PROBIC-FAPERGS-UFRGS (E.M.C.) and PROBITI-FAPERGS-UFRGS (R.F.L.F.).

REFERENCES

[1] Marburg, S., 2002, "Developments in Structural-Acoustic Optimization for Passive Noise Control", Archives of Computational Methods in Engineering, Vol. 9, 4, pp. 291-370.

[2] Marburg, S., Dienerowitz, F., Fritze, D. and Hardtke, H.J., 2006, "Case Studies on Structural-Acoustic Optimization of a Finite Beam", Acta Acustica United with Acustica, Vol. 92, pp. 427-439.

[3] Lee, J., Wang, S. and Dikec, A., 2004, "Topology Optimization for the Radiation and Scattering of Sound from Thin-Body Using Genetic Algorithms", Journal of Sound and Vibration, Vol. 276, n 3-5, pp. 899-918.

[4] Duhring, M.B., Jensen, J.S. and Sigmund, O., 2008, "Acoustic Design by Topology Optimization", Journal of Sound and Vibration, Vol. 317, pp. 557-575.

[5] Yoon, G.H., "Structural Topology Optimization for Frequency Response Problem Using Model Reduction Schemes", Computer Methods in Applied Mechanics and Engineering, Vol. 199, 1744-1763, 2010.

[6] Akl, W., El-Sabbagh, A., Al-Mitani, K. And Baz, A., "Topology Optimzation of a Plate Coupled with Acoustic Cavity", International Journal of Solids and Structures, Vol. 46, n. 10, 2060-2074.

[7] Marburg, S., Beer H.-J., Gier, J. and Hardtke, H.-J., 2002, "Experimental Verification of Structural-Acoustic Modeling and Design Optimization", Journal of Sound and Vibration, Vol. 252, pp. 591-615.