

HYSTERESIS LOOPS DUE TO GASEOUS CAVITATION

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Abstract. *This work presents a consistent thermodynamic model to describe the gas release phenomenon in homogeneous liquid-gas flows undergoing isothermal transformations. The liquid-gas mixture is regarded as a pseudo-fluid whose constitutive behavior is obtained from two thermodynamic potentials - the Helmholtz free energy and a pseudo-potential of dissipation. Thanks to the inclusion of the concentration of dissolved gas in the liquid in the list of state variables, along with the gas volume fraction and the mass densities of the liquid and gas constituents, a suitable and simple expression for the rate of gas release is derived. Numerical simulations carried out for an air-water mixture, subjected to cyclic expansion-contraction loadings, illustrate the influence of the gas release on the mechanical response of the fluid, which exhibits dispersion, attenuation and hysteresis.*

Keywords: *gaseous cavitation, free gas, internal variables theory, irreversible processes.*

1. INTRODUCTION

Liquids are known to be able to absorb certain amount of gas in contact with their free surfaces. Through a diffusive process, the gas dissolves in the liquid and stays there, unless its temperature is raised beyond the boiling point or its pressure is lowered below the saturation pressure of the dissolved gas. In this last process, usually referred to as gas release or, alternatively, gaseous cavitation, the gas comes out of solution as small bubbles that either are carried by the fluid flow stream as free gas or migrate towards the free surface or the upper parts of the systems where the liquid is confined, forming small and discrete cavities.

The most relevant problems in which gaseous cavitation plays an important role are those associated with transient fluid flows in piping systems. Typical examples include water distribution systems, sewage networks, nuclear reactor piping systems and aviation fuel pipelines. The relevance of these applications seems to be responsible for concentrating the great majority of the scientific works related to free gas and gaseous cavitation on this specific context (Swaffield , 1972; Kranenburg , 1974; Wiggert and Sundquist , 1979; Baasiri and Tullis , 1983; Chaudry et al. , 1990; Kessal and Bennacer , 2005).

In contrast to the problem related to the free gas influence on the wave propagation in liquid-gas mixtures for which there exists a significant amount of theoretical and experimental studies (Swaffield , 1972; Chaudry et al. , 1990; Wylie and Streeter , 1993), only a few works are devoted to the gaseous cavitation phenomenon. Theoretical works are restricted to a few number of works (Kranenburg , 1974; Wiggert and Sundquist , 1979; Wylie and Streeter , 1993; Kessal and Bennacer , 2005). The rate of gas release is commonly taken into account as a non-negative source term in the balance equation of mass for the free gas. It is equivalent to assume a one-way process, inasmuch as the reverse process (i. e., the gas absorption phenomenon) takes place in a time scale greater than the one associated with the gas release (Baasiri and Tullis , 1983; Wylie and Streeter , 1993). This assumption implies that the released gas stays as free gas or in cavities and does not re-dissolves in a subsequent rise in pressure. Since only a small portion of dissolved gas actually evolves during the life of most practical systems transients, it is generally assumed that mass of dissolved gas remains constant at the initial value during a transient event.

Aiming to advance the theoretical knowledge in the filed of gaseous cavitation, a new gas release model is proposed in

this work. It is derived in the context of an internal variables theory taking as basic assumptions homogeneous liquid-gas mixtures. Besides the mass densities of the liquid and the free gas, the gas volume fraction and the concentration of dissolved gas in the liquid are treated as state variables. Taking into account the classical assumptions to describe the gas release phenomenon, a new expression for the rate of gas release is proposed, without appealing to the extension of the Henry's law to states of non-equilibrium (Sander, 1999).

2. BALANCE EQUATIONS AND FUNDAMENTAL ASSUMPTIONS

Let $B_t \subset \mathbb{R}^3$ be an open bounded set which represents the current configuration of continuum body in the Euclidean vector space \mathbb{R}^3 and let $I \subset \mathbb{R}$ be an open subset of the real numbers, which stands for an interval of observation. For a given frame of reference, let $\mathbf{x} \in B_t$ and $t \in I$ represent the position vector and time instant, respectively. Let the mass density and the velocity fields be represented by $\varrho(\mathbf{x}, t)$ and $\mathbf{v}(\mathbf{x}, t)$. Let also the fluid pressure and the extra stress (symmetric) tensor due to motion be denoted by $p(\mathbf{x}, t)$ and $\mathbf{S}(\mathbf{x}, t)$. By representing the internal energy per unit volume as $e(\mathbf{x}, t)$, the heat flux vector as $\mathbf{q}(\mathbf{x}, t)$, the heat generation by unit volume as $r(\mathbf{x}, t)$, the entropy per unit volume as $s(\mathbf{x}, t)$ and the absolute temperature by $\theta(\mathbf{x}, t)$, then the balance equations can be expressed as follows:

$$\dot{\varrho} + \varrho \nabla \cdot \mathbf{v} = 0, \quad (1)$$

$$\varrho \dot{\mathbf{v}} = -\nabla p + \nabla \cdot \mathbf{S} + \varrho \mathbf{g}, \quad (2)$$

$$\dot{e} + e \nabla \cdot \mathbf{v} = -p \nabla \cdot \mathbf{v} + \mathbf{S} \cdot \mathbf{D} - \nabla \cdot \mathbf{q} + r, \quad (3)$$

$$d := -(p + \Psi) \nabla \cdot \mathbf{v} - \dot{\Psi} - \dot{\theta} s + \mathbf{S} \cdot \mathbf{D} - \frac{\mathbf{q} \cdot \nabla \theta}{\theta} \geq 0. \quad (4)$$

for all $(\mathbf{x}, t) \in B_t \times I$, in which \mathbf{D} stands for the symmetric parcel of the velocity gradient $\nabla \mathbf{v}$, $\dot{\chi}$ stands for the time material derivative of the material property χ and $\Psi := e - \theta s$ is the Helmholtz free energy per unit volume.

Equations (1-3) are the local forms of the balance of mass, momentum and energy, whereas Eq. (4) stands for a local version of the SLT. It defines the dissipated energy rate per unit volume of fluid d and makes a distinction between possible ($d \geq 0$) and impossible ($d < 0$) processes. These equations describe the behavior of the body, which is a homogeneous two-component two-phase mixture, whose constituents are a liquid and a gas. The gas exists freely dispersed as small bubbles in suspension as well as dissolved in the liquid. Both constituents share the same temperature θ .

To properly take the gas release into account, the concentration of dissolved gas in the liquid c (the quantity of mass of dissolved gas per unit volume of liquid) and the gas volume fraction α (the ratio between the volume of free gas in suspension and the total volume of the mixture) are considered as internal variables. Since by assumption free gas is always present in the fluid, α is subjected to the constraint $\alpha \in (0, 1)$, which is considered a constitutive restriction and is properly accounted for by the constitutive equations. Since there is no vapor along with the free gas in suspension, the mass density of the fluid can be defined as:

$$\varrho := (1 - \alpha)(\rho_l + c) + \alpha \rho_g, \quad (5)$$

in which ρ_l and ρ_g stand for the mass densities of the liquid and the free gas, respectively, both of them assumed as being compressible fluids. Taking into account the interchange of mass of gas between the liquid and gaseous phases along with Eq. (5) and that the mixture is supposed to undergo isothermal transformations ($\dot{\theta} = 0$ and $\nabla \theta = 0$), the balance equations

may be rewritten as follows:

$$(1 - \alpha)\dot{\rho}_l - \rho_l\dot{\alpha} + (1 - \alpha)\rho_l\nabla\cdot\mathbf{v} = 0, \quad (6)$$

$$(1 - \alpha)\dot{c} - c\dot{\alpha} + (1 - \alpha)c\nabla\cdot\mathbf{v} + \Gamma = 0, \quad (7)$$

$$\alpha\dot{\rho}_g + \rho_g\dot{\alpha} + \alpha\rho_g\nabla\cdot\mathbf{v} - \Gamma = 0, \quad (8)$$

$$((1 - \alpha)(\rho_l + c) + \alpha\rho_g)\dot{\mathbf{v}} = -\nabla p + \nabla\cdot\mathbf{S} + \varrho\mathbf{g}, \quad (9)$$

$$d := -(p + \Psi)\nabla\cdot\mathbf{v} - \dot{\Psi} + \mathbf{S}\cdot\mathbf{D} \geq 0, \quad (10)$$

for all $(\mathbf{x}, t) \in B_t \times I$, in which Γ stands for the rate of mass transfer per unit volume of fluid.

When $\Gamma > 0$, the gas dissolved in the liquid evolves in the fluid as free gas in suspension. On the other hand, if $\Gamma < 0$, then the free gas dissolves in the liquid. Finally, if $\Gamma = 0$, then there is no interchange of mass of gas between the liquid and gaseous phases and, consequently, the mass of dissolved gas in the liquid and the mass of free gas are conserved independent from each other.

3. CONSTITUTIVE THEORY

The constitutive relations describing the macroscopic mechanical behavior of the pseudo-fluid are derived in the framework of the thermodynamics of irreversible processes. In this theory, once the local state of the material has been characterized by means of an appropriate choice of the set of state variables, two thermodynamic potentials - the Helmholtz free energy and a pseudo-potential of dissipation - are sufficient to derive a complete set of constitutive equations.

3.1 Helmholtz free energy and state laws

Following the classic assumption of the thermodynamics of irreversible processes, the free energy per unit volume of fluid Ψ is supposed to be a function of the state variables $\rho_l, \rho_g, c, \alpha$ and θ . Since the pseudo-fluid is regarded as a mixture of two constituents, its behavior comprise a combination of the liquid and gas thermo-mechanical properties, taking α as a weighting factor. Thus, we choose

$$\Psi = \Psi(\rho_l, \rho_g, c, \alpha, \theta) := \Psi'(\rho_l, \rho_g, c, \alpha, \theta) + \bar{I}(\alpha), \quad (11)$$

in which

$$\Psi'(\rho_l, \rho_g, c, \alpha, \theta) := (1 - \alpha)\rho_l\Psi_l(\rho_l, \theta) + (1 - \alpha)c\Psi_c(\theta) + \alpha\rho_g\Psi_g(\rho_g, \theta), \quad \bar{I}(\alpha) := \begin{cases} 0 & , \text{if } \alpha \in (0, 1) \\ +\infty & , \text{otherwise.} \end{cases} \quad (12)$$

In the above expression, Ψ' is a smooth function which describes the thermo-mechanical properties of the pseudo-fluid and $\bar{I}(\alpha)$ represents the indicator function of the convex set $(0, 1)$ (Moreau et al. , 1988). The term $\bar{I}(\alpha)$ is the non-smooth parcel of the free energy and is included to take the internal constraint $\alpha \in (0, 1)$ into account as a constitutive assumption. The terms Ψ_l and Ψ_g represent the free energies per unit mass of liquid and gas, respectively. Finally, the term Ψ_c denotes the free energy per unit mass of dissolved gas which is incorporated in the free energy of the mixture to take the gaseous cavitation phenomenon into account.

The state laws for the pseudo-fluid, relating the reversible components of the thermodynamic forces to the state variables, are obtained from the free energy potential and defined as follows:

$$B^{e_l} := \frac{\partial \Psi'}{\partial \rho_l} = (1 - \alpha) \left(\Psi_l + \frac{p_l}{\rho_l} \right) = (1 - \alpha) g_l, \quad (13)$$

$$B^{e_g} := \frac{\partial \Psi'}{\partial \rho_g} = \alpha \left(\Psi_g + \frac{p_g}{\rho_g} \right) = \alpha g_g, \quad (14)$$

$$B^c := \frac{\partial \Psi'}{\partial c} = (1 - \alpha) \Psi_c, \quad (15)$$

$$B^\alpha := \frac{\partial \Psi'}{\partial \alpha} + h, \quad \text{with } h \in \partial_\alpha \bar{I}(\alpha), \quad (16)$$

in which;

$$\frac{\partial \Psi'}{\partial \alpha} = \rho_g \Psi_g - \rho_l \Psi_l - c \Psi_c, \quad p_l := \rho_l^2 \frac{\partial \Psi_l}{\partial \rho_l} \quad \text{and} \quad p_g := \rho_g^2 \frac{\partial \Psi_g}{\partial \rho_g}. \quad (17)$$

In the above equations, p_l and p_g stand for the liquid and gas pressures, respectively. The term h in Eq. (16) is an element of the subdifferential set $\partial_\alpha \bar{I}(\alpha)$ with respect to α of the convex function $\bar{I}(\alpha)$. The subdifferential of the indicator function $\bar{I}(\alpha)$ at α is given by the set (Moreau et al. , 1988);

$$\partial_\alpha \bar{I}(\alpha) := \{h \in |\bar{I}(\alpha^*) - \bar{I}(\alpha) \geq h(\alpha^* - \alpha); \quad \forall \alpha^* \in (0, 1)\}. \quad (18)$$

A straightforward calculation shows that $\partial_\alpha \bar{I}(0 < \alpha < 1) = \{0\}$ and $\partial_\alpha \bar{I}(\alpha) = \emptyset$ if $\alpha \notin (0, 1)$. It is important to remark that the restriction $\alpha \in (0, 1)$ is effectively taken into account through the constitutive law Eq. (16), since this relation implies that the subdifferential $\partial_\alpha \bar{I}(\alpha)$ is not empty. Keeping in mind that time derivatives must be left derivatives in order to cope with the principle of determinism, it comes out that:

$$\dot{\Psi} = \frac{\partial \Psi'}{\partial \rho_l} \dot{\rho}_l + \frac{\partial \Psi'}{\partial \rho_g} \dot{\rho}_g + \frac{\partial \Psi'}{\partial c} \dot{c} + \frac{\partial \Psi'}{\partial \alpha} \dot{\alpha} + \lim_{\substack{\Delta t \rightarrow 0 \\ \Delta t > 0}} \frac{\bar{I}(\alpha(t)) - \bar{I}(\alpha(t - \Delta t))}{\Delta t}. \quad (19)$$

In computing $\dot{\Psi}$ care should be taken since Ψ is not a smooth function. By using the definition of the subdifferential in Eq. (19) along with the state laws Eqs. (13-16), it can be shown that Eq. (10) can be reduced to:

$$d := -(p + \Psi) \nabla \cdot \mathbf{v} + \mathbf{S} \cdot \mathbf{D} - B^{\rho_l} \dot{\rho}_l - B^c \dot{c} - B^{\rho_g} \dot{\rho}_g - B^\alpha \dot{\alpha} \geq 0. \quad (20)$$

To obtain a complete set of constitutive equations, it suffices to specify a pseudo-potential of dissipation from which complementary laws are derived in such a way that the local version of the SLT is always verified.

3.2 Pseudo-potential of dissipation and complementary laws

To introduce the irreversible behavior of the pseudo-fluid, and also to ensure that the SLT is always satisfied, we assume that there exists a pseudo-potential of dissipation Φ , which is an objective, convex and differentiable function of \mathbf{D} , \dot{c} , $\dot{\alpha}$ and B^Γ . Moreover, the pseudo-potential Φ is assumed to have the following properties:

$$\Phi(\mathbf{D}, \dot{c}, \dot{\alpha}, B^\Gamma; \alpha, c, \theta) \geq 0 \quad \text{and} \quad \Phi(\mathbf{0}, 0, 0, 0; \alpha, c, \theta) = 0, \quad (21)$$

whatever the values of \mathbf{D} , \dot{c} , $\dot{\alpha}$, B^Γ , α , c and θ . The additional information associated with the dissipative behavior of the pseudo-fluid is obtained from Φ through the following complementary laws:

$$\mathbf{S} := \frac{\partial \Phi}{\partial \mathbf{D}}, \quad B^{\dot{c}} := \frac{\partial \Phi}{\partial \dot{c}}, \quad B^{\dot{\alpha}} := \frac{\partial \Phi}{\partial \dot{\alpha}}, \quad \Gamma := \frac{\partial \Phi}{\partial B^\Gamma}. \quad (22)$$

In addition, if the rate of energy dissipation d is supposed to have the form,

$$d := \mathbf{S} \cdot \mathbf{D} + B^{\dot{c}} \dot{c} + B^{\dot{\alpha}} \dot{\alpha} + B^{\Gamma} \Gamma - \bar{I}(\alpha) \nabla \cdot \mathbf{v}, \quad (23)$$

then we get from the convexity property of Φ that, for any actual evolution:

$$d := \mathbf{S} \cdot \mathbf{D} + B^{\dot{c}} \dot{c} + B^{\dot{\alpha}} \dot{\alpha} + B^{\Gamma} \Gamma \geq \Phi(\mathbf{D}, \dot{c}, \dot{\alpha}, B^{\Gamma}; c, \alpha, \theta) - \Phi(\mathbf{0}, 0, 0, 0; c, \alpha, \theta). \quad (24)$$

In view of Eq. (21), it is easy to see that $d \geq 0$ for any actual evolution of the pseudo-fluid and so the SLT given by the inequality Eq. (10) is always satisfied.

The constitutive assumptions made so far are sufficient to partially characterize the mechanical behavior of the pseudo-fluid. Since the mass balance equations given by Eq. (6) plus Eq. (7) along with Eq. (8) define subspaces of the linear space spanned by \mathbf{v} , $\dot{\rho}_l$, $\dot{\rho}_g$, \dot{c} and $\dot{\alpha}$, then in order that Eq. (20) be equal to Eq. (23) for any actual evolution one must have:

$$-(p + \Psi) \nabla \cdot \mathbf{v} + \mathbf{S} \cdot \mathbf{D} - B^{\rho_l} \dot{\rho}_l - B^{\rho_g} \dot{\rho}_g - B^c \dot{c} - B^{\alpha} \dot{\alpha} = \mathbf{S} \cdot \mathbf{D} + B^{\dot{\alpha}} \dot{\alpha} + B^{\dot{c}} \dot{c} + B^{\Gamma} \Gamma - \bar{I}(\alpha) \nabla \cdot \mathbf{v} + \eta \left((1 - \alpha) \dot{\rho}_l - \rho_l \dot{\alpha} + (1 - \alpha) \rho_l \nabla \cdot \mathbf{v} \right) + \xi \left((1 - \alpha) \dot{c} + \alpha \dot{\rho}_g + (\rho_g - c) \dot{\alpha} + ((1 - \alpha)c + \alpha \rho_g) \nabla \cdot \mathbf{v} \right), \quad (25)$$

in which η and ξ are Lagrange multipliers. Since $\alpha \in (0, 1)$ by hypothesis and Eq. (25) holds for any independent evolution of \mathbf{v} , $\dot{\rho}_l$, $\dot{\rho}_g$, \dot{c} and $\dot{\alpha}$, η and ξ can be eliminated from the above equations to render the following relationships among the thermodynamic forces,

$$p = (1 - \alpha)p_l + \alpha p_g + cB^{\dot{c}} \quad , \quad B^{\dot{\alpha}} = p_g - p_l - \frac{cB^{\dot{c}}}{(1 - \alpha)} \quad , \quad B^{\Gamma} = \Psi_c - g_g + \frac{B^{\dot{c}}}{(1 - \alpha)}. \quad (26)$$

Except to the terms in which $B^{\dot{c}}$ appears at the right-hand side of Eq. (26), these relationships establish that the mixture pressure is the mean pressure between the liquid and free gas pressures, the thermodynamic force associated with the rate of evolution of α is equal to the pressure difference between the liquid and the gas and, finally, the thermodynamic force associated with the rate of gas mass transfer between is equal to the difference of the free energies in these two phases.

3.3 Constitutive equations

Taking into account the results of experimental tests available in literature, specific forms are prescribed to both thermodynamic potentials in order to obtain the desired constitutive equations. The forms of Ψ_l , Ψ_g and Ψ_c are those responsible for describing the reversible behavior of the liquid, the free gas and the dissolved gas as independent and isolated constituents. By assuming that the liquid and free gas are simple compressible substances and that the free energy of the dissolved gas depends on the saturation pressure $p_s = p_s(\theta)$, these free energies can be postulated as:

$$\Psi_l = \Psi_l(\rho_l, \theta) := -C_l \theta \log \theta + a_l^2 \left(\frac{\rho_l^o}{\rho_l} + \log \rho_l \right), \quad (27)$$

$$\Psi_g = \Psi_g(\rho_g, \theta) := -C_g \theta \log \theta + a_g^2 \log \rho_g, \quad (28)$$

$$\Psi_c = -C_g \theta \log \theta + a_g^2 \log \frac{p_s}{a_g^2} + a_g^2. \quad (29)$$

In the above expressions, C_l and C_g are assumed to be constants. They represent the specific heats at constant volume of the liquid and free gas, respectively. The material parameters ρ_l^o , a_l^2 and a_g^2 are temperature dependent, being the last two the square of the isothermal wave propagation velocities in the pure liquid and pure gas, respectively. Without losing generality, if we assume that both constituents behave as Newtonian fluids a simple and appropriate choice for Φ is:

$$\Phi(\mathbf{D}, \dot{c}, \dot{\alpha}, B^{\Gamma}; \alpha, c, \theta) := \frac{\lambda}{2} (\text{tr}(\mathbf{D}))^2 + \mu \mathbf{D} \cdot \mathbf{D} + \frac{\beta}{2n} \langle B^{\Gamma} \rangle^{2n}. \quad (30)$$

in which $\beta = \beta(c, \theta)$ and $n = n(\theta)$ are positive material constants, with $n > 1/2$, associated with the gas cavitation phenomenon and $\lambda = \lambda(\alpha)$, $\mu = \mu(\alpha)$. These material parameters are such that $\mu \geq 0$ and $\lambda + 2/3\mu \geq 0$. By definition, $\langle \chi \rangle := \max\{0, \chi\}$. The first two terms in Eq. (30) are classic and expresses the behavior of a Newtonian fluid. The last one is unusual and reflects irreversible changes within the internal structure of the fluid associated with the gas release. Since Φ does not depend on \dot{c} and $\dot{\alpha}$, it comes out by Eq. (22) that $B^{\dot{c}} = 0$ and $B^{\dot{\alpha}} = 0$. As a result, the specific choices expressed by Eq. (27-30) give rise to the following constitutive equations:

$$p = p_l = a_l^2 (\rho_l - \rho_l^o) = p_g = a_g^2 \rho_g \quad , \quad \mathbf{S} = \lambda \text{tr}(\mathbf{D})\mathbf{I} + 2\mu\mathbf{D} \quad , \quad \Gamma = \beta (a_g^2)^{2n-1} \langle \log \frac{p_s}{p} \rangle^{2n-1}, \quad (31)$$

It can be seen that $\Gamma \geq 0$; the gaseous cavitation is considered an one-way process with mass being transferred from the gas dissolved in the liquid to the gas in suspension. The dissolved gas will evolve whenever $p < p_s$, which is a constitutive parameter and is related to the saturation concentration of dissolved gas by the Henry's law (Sander, 1999).

4. NUMERICAL SIMULATIONS

To better understand the influence of gas release on the mechanical behavior of the liquid-gas mixture, simple numerical simulations are carried out. For this purpose, we consider a piston-cylinder system, whose cross-sectional area is $A = \pi m^2$ and initial volume is $V_o = 2\pi m^3$. The mixture can have its original volume expanded and contracted along the time by means of external actions and remains homogeneous throughout its volume, so that the convective terms can be neglected. Thus, the time rate of the volume change per unit volume of the whole mixture within the system is described by the term $\nabla \cdot \mathbf{v} = \widehat{\nabla \cdot \mathbf{v}}(t)$, which is assumed to be a prescribed input function of the time. Since Γ does not depend on c , Eqs. (6) and (8) can be reduced to the following set of ordinary differential equations for p and α , with the aid of Eq. (31):

$$\frac{dp}{dt} = \left[\frac{a_l^2 \rho_l a_g^2 \rho_g}{\rho (a_l^2 \rho_l \alpha + a_g^2 \rho_g (1 - \alpha))} \right] \frac{\rho}{\rho_g} (\Gamma - \rho_g \widehat{\nabla \cdot \mathbf{v}}(t)), \quad \frac{d\alpha}{dt} = \frac{\alpha(1 - \alpha) \widehat{\nabla \cdot \mathbf{v}}(t) (a_l^2 \rho_l - a_g^2 \rho_g) + \Gamma(1 - \alpha) a_g^2}{\alpha a_l^2 \rho_l + (1 - \alpha) a_g^2 \rho_g}. \quad (32)$$

In what follows, we investigate the mechanical response of the mixture governed by Eq. (32) to a cyclic loading depicted in Fig. 1(L) for the first cycle only. The maximum value in absolute terms reached by $\widehat{\nabla \cdot \mathbf{v}}(t)$ can be expressed by AU/V_o , in which U stands for the maximum velocity of the piston during its stroke. One cycle of the input function is characterized by two strokes: one expansion stroke of elapsed time $2\Delta t_1$ followed by a rest period of Δt_2 and one contraction stroke, having an elapsed time $2\Delta t_1$, also followed by a rest period of Δt_2 . Initially $\alpha(t = 0) = 10^{-8}$ and $p_o = p(t = 0) = 30p_{atm}$, in which $p_{atm} = 101.325\text{kPa}$. The saturation pressure is assumed to be $p_s = 125\text{ kPa}$ and the gas release is computed taking as material parameters $\beta = 0.038$ and $n = 2.2$ (Zielke et al., 1990). The initial-value problem formed by Eq. (32) is numerically approximated by using the classic Adams-Moulton method, which is up to order twelve, with initial time-steps of the order of $0.05\mu\text{s}$ and relative errors of 10^{-6} .

The mixture is subjected to four successive cycles. The rest periods of time are constant and equal to $\Delta t_2 = 1\text{ s}$. Three different rates of volumetric deformation are imposed, by assigning different values to the maximum velocity of the piston $U = 0.8; 1.6; 3.2\text{ m/s}$. To isolate the influence of the rate of volumetric deformation on the mixture response, a constant value of $\Delta V/V_o = (AU\Delta t_1)/V_o = 0.31\%$ is assumed for all the three simulations. The product $U\Delta t_1$ is taken constant, rendering $\Delta t_1 = 7.6621; 3.8311; 1.9155\text{ ms}$ for $U = 0.8; 1.6; 3.2\text{ m/s}$, respectively. To allow an easier comprehension of the forthcoming analysis, the letters A to E have been inserted at different positions in the (first) cycle illustrated in Fig. 1(L). The obtained responses are illustrated graphically in Figs. 1(R) to 3. In all the graphs the letters A to E, previously shown in Fig. 1(L), have also been marked as a reference for the first cycle for $U = 3.2\text{ m/s}$. The time histories of the α response are presented in Fig. 1(L) for $U = 0.8; 1.6; 3.2\text{ m/s}$. Since $\Delta t_1 \ll \Delta t_2$ the expansion and contraction strokes are visualized in Fig. 1(R) as vertical straight lines. Due to the gas release, a continuously increasing gas volume fraction response is observed during the periods of rest after the expansion strokes. Since after the contraction strokes the pressure remains above p_s , no gas release takes place during the subsequent rests periods rendering flat gas volume fraction responses.

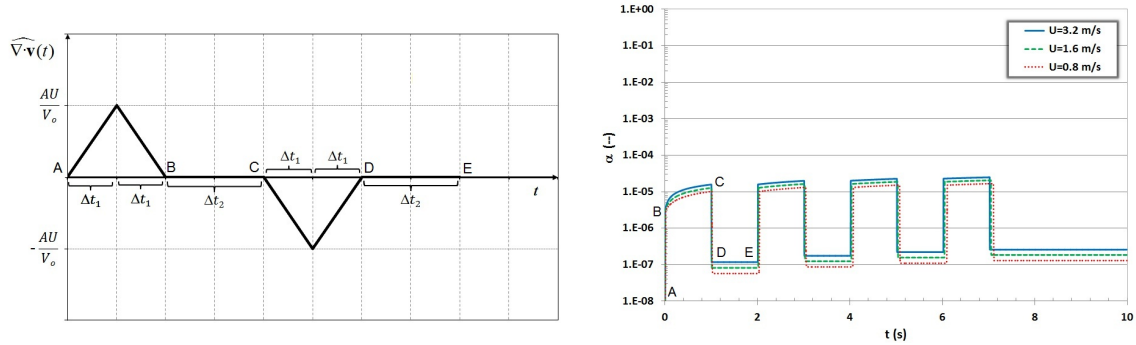


Figure 1. First cycle of $\nabla \cdot \mathbf{v} = \widehat{\nabla \cdot \mathbf{v}}(t)$ (left)(L) and history of the α response for $U = 0.8; 1.6; 3.2$ m/s (right)(R).

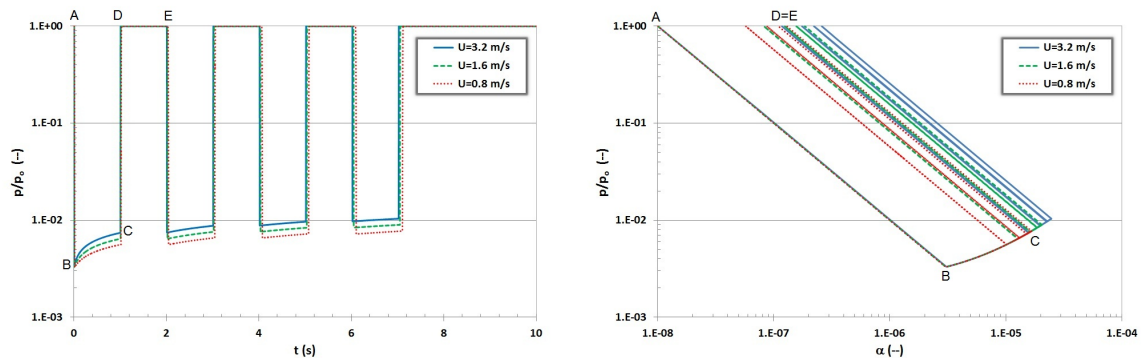


Figure 2. Histories of p/p_o as a function of t (left)(L) and as a function of α (right)(R) for $U = 0.8; 1.6; 3.2$ m/s.

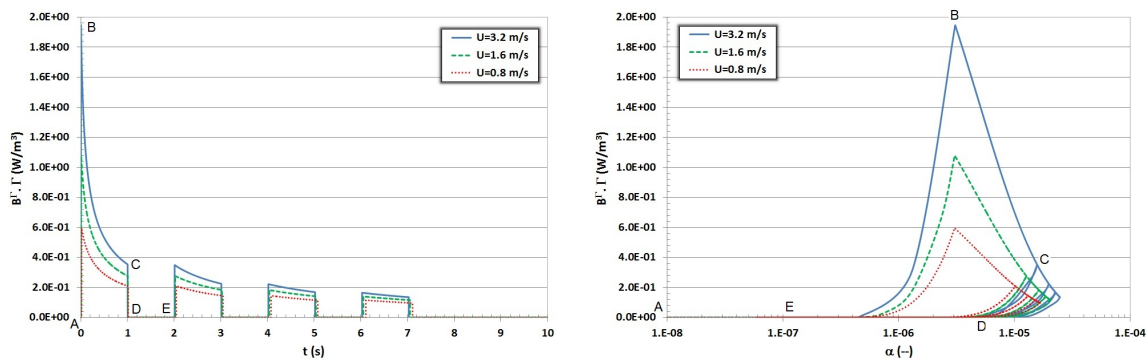


Figure 3. Histories of $B^T \Gamma$ as a function of t (left)(L) and as a function of α (right)(R) for $U = 0.8; 1.6; 3.2$ m/s.

The normalized pressure p/p_o as a function of the time t is plotted in Fig. 2(L) for $U = 0.8; 1.6; 3.2$ m/s. The pressure recovery observed after each expansion stroke increases continuously cycle after cycle, being more pronounced for the greatest rate of deformation. After completing each cycle, the pressure returns to its initial value p_o . The normalized pressure can be plotted against α , as shown in Fig. 2(R), to describe the paths of the loading-unloading cycles experienced by the mixture. Peculiarities of liquids and solids aside, Fig. 2(R) resembles the same pattern (in a upside down position) observed in cyclic tests of load-unload carried out in metallic materials which exhibits elasto-viscoplastic mechanical behaviors. More specifically, it can be seen parallel paths of loading (A-B path) and unloading (C-D path) intercalated by time-dependent viscous creep (B-C path), with the presence of residual deformation (A-E path).

To highlight one of the main advantages and distinguishable features of having a consistent thermo-mechanical theory supporting the gas evolution expression, it is presented in Figs. 3(L) and 3(R) the dissipated energy rate per unit volume $B^T \Gamma$ as a function of t (Fig. 3(L)) and as a function of α (Fig. 3(R)), for $U = 0.8; 1.6; 3.2$ m/s. The dissipated energy rate associated with the gas release is the only active dissipative process in Eq. (23) for this example. This non-negative term is non-null as long as the pressure is less than the saturation pressure of dissolved gas. The area delimited between the curve

and the x -axis in Fig. 3(L) gives the amount of dissipated energy per unit volume in each cycle. The larger the rate of volumetric deformation (characterized by the U values) is, the greater the amount of dissipated energy in a cycle becomes. The total amount of dissipated energy per unit volume at the end of the fourth cycle for $U = 3.2$ is approximately 1.27 J/m^3 . When the dissipated energy rate is plotted against the gas volume fraction, the influence of the rate of volumetric deformation on the formations of loops of hysteresis are put in evidence in Fig. 3(R).

5. CONCLUSIONS

A new model for the gas release in homogeneous liquid-gas flows has been presented in the context of the thermodynamics of irreversible processes. In spite of having no influence on the mechanical properties of the liquid-gas mixture or even not having to necessarily appear in the list of dependent variables in the governing equations, the concentration of the dissolved gas in the liquid is of crucial importance to derive a consistent thermodynamic expression for the rate of gas release, without appealing to the extension of the Henry's law to states of non-equilibrium.

6. ACKNOWLEDGEMENTS

The authors would like to thank PETROBRAS S.A., CNPq, CAPES and FAPERJ for the continuous support of all research activities of this group over the years.

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