HYPERBOLIC HEAT TRANSFER IN GENERALIZED NEWTONIAN FLUIDS

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Abstract. *The present paper is concerned with the modelling of hyperbolic heat transfer in generalized Newtonian fluids. A general procedure, developed within the framework of thermodynamics of irreversible processes, is proposed to obtain constitutive relations that verify automatically the second law of thermodynamics and the principle of material objectivity. Such a thermodynamic approach allows a rational identification of the terms responsible for the thermomechanical coupling in the heat equation, which is a first step to better understand their influence on the fluid behaviour.*

Keywords*: hyperbolic heat conduction; finite thermal wave speed; generalized Newtonian fluids; material objectivity; second law of thermodynamics*

1. INTRODUCTION

The classical linear equation for heat conduction based on Fourier's law is a parabolic equation in terms of the temperature field. Although this parabolic equation leads to an adequate description of heat conduction in most engineering applications, it predicts an infinite wave speed of heat conduction what is physically unrealistic. It is now accepted that in situations involving very short times, extreme thermal gradients or temperatures near absolute zero may lead to a finite thermal wave speed. Consequently, any thermal disturbance exerted on a body is instantaneously felt through the whole body. Various alternative models have been proposed to lead to a finite thermal wave speed. Generally they try to replace the classical Fourier heat conduction assumption. Almost all these constitutive equations are conceived for rigid bodies at rest. Some of them clearly violate a notion of the second law of thermodynamics since the heat may flow from cold to hot regions during finite time periods. For some of them it is very difficult to assure the resulting governing equations are thermodynamically admissible or the principle of objectivity.

Vernotte, 1958, and Cattaneo, 1958, based on the concept of heat transmission by waves, independently introduced an alternative equation aiming at describing problems involving high rates of temperature change, heat flow in an extremely short period of time or very low temperatures near absolute zero. After the pioneer works of Vernotte and Cattaneo, a number of research contributions have been dedicated to the study of problems involving hyperbolic heat conduction (Jackson and Walker, 1971; Narayanamurti and Dynes, 1972; Joseph and Preziosi, 1990a,b; Kaminski, 1990; Rubin, 1992; Özisik et al, 1994; Guillemet et al, 1997; Barletta and Zanchini, 1997; Kronberg et al, 1998; Sieniutycz and Stephen Berry 2002; Christov and Jordan, 2005; Christov, 2009, Tibullo and Zampoli, 2011, for instance). The present paper is concerned with the study of sufficient conditions for an objective and thermodynamically consistent modelling of hyperbolic heat transfer in generalized Newtonian fluids using Fourier's law and an objective version of the Vernotte-Cattaneo´s law. A general procedure, developed within the framework of thermodynamics of irreversible processes presents sufficient conditions to satisfy a local version of the second law of thermodynamics. It is important to emphasize that the idea here is to suggest a physically realistic and thermodynamically consistent model for describing heat waves of finite speed. The main objective is to show an alternative approach that cannot be neglected in future experimental studies

2 - PRELIMINARY DEFINITIONS

Under suitable regularity assumptions it is possible to consider the following expressions as local versions of the first law (FLT) and second law of thermodynamics (SLT) Truesdell and Toupin, 1960, Billington and Tate, 1981:

$$FLT : \rho \dot{e} = -div(\boldsymbol{q}) + \boldsymbol{T} : \boldsymbol{D} \tag{1}$$

$$SLT : d = (d_1 + d_2) \geq 0 \ \text{ with } \ d_1 = \boldsymbol{T} : \boldsymbol{D} - \rho(\dot{\psi} + s\dot{\theta}) \ \text{ and } \ d_2 = -(1/\theta)\boldsymbol{q} \cdot \boldsymbol{g} \tag{2}$$

where $(\dot{\ })$ denotes the material time derivative of ( ); ρ is the mass density; $\boldsymbol{T}$ the Cauchy stress tensor; $\boldsymbol{D} = 1/2\ [\boldsymbol{grad}(\boldsymbol{v}) \cdot \boldsymbol{grad}(\boldsymbol{v})^{\mathrm{T}}]$ the deformation rate tensor; e the internal energy per unity mass, θ the absolute temperature; s the total entropy per unit mass; $\psi = (e - \theta s)$ the Helmholtz free energy per unit mass; $\boldsymbol{q}$ the heat flux vector and $\boldsymbol{g} = \boldsymbol{grad}(\theta)$. d is the rate of energy dissipation per unit volume. The quantity d_1, defined in (2), is usually called the intrinsic dissipation and the quantity d_2 the thermal dissipation. The second law of thermodynamics makes a distinction between possible processes $(d \geq 0)$ and impossible processes $(d < 0)$. The possible processes may be reversible (the rate of energy dissipation d is always equal to zero) or not. This local version of the SLT does not exclude the possibility of unusual behaviours such as a decreasing temperature if heat is added to the medium. To exclude the possibility of such kind of unusual behaviour, the present study is restricted to fluids that always satisfy a further restrictive constraint:

$$d \geq 0 \text{ AND } d_2 \geq 0 \tag{3}$$

It is simple to verify that the second constraint in (3) leads to the classical heat conduction inequality $-\boldsymbol{q} \cdot \boldsymbol{g} \geq 0$ since the absolute temperature θ is a positive quantity. This relation implies that heat flows in the direction of decreasing temperature when $\boldsymbol{q}$ is parallel to the temperature gradient.

3. CONSTITUTIVE FRAMEWORK

3.1. Helmholtz free energy and intrinsic dissipation

In this section, it is presented an abstract framework to define a particular family of incompressible fluids that encompasses the so-called *generalized Newtonian fluids*. The study is restricted to this class of fluids due to the limited space, but all the theory can be extended to encompass the *viscoelastic* and *elasto-viscoplastic* constitutive equations proposed in da Costa Mattos, 1998, 2012. By definition, for this class of fluids, the free energy ψ is only a differentiable function of the absolute temperature θ

$$\psi = \psi(\theta) \tag{4}$$

To set up a general constitutive theory it is then necessary to consider aspects of the second law of thermodynamics since dissipative behaviour must be taken into account. The relation with dissipative mechanisms is introduced through a dissipation potential ϕ which is a differentiable, convex and isotropic function of $\boldsymbol{D}$ and θ . Restricting the study to incompressible fluids, the potential ϕ is such that the intrinsic dissipation d_1 is supposed to have the following form

$$d_1 = (\partial \phi / \partial \boldsymbol{D}) : \boldsymbol{D} \tag{5}$$

with ϕ such that $\phi(\boldsymbol{D},\theta) \geq 0 \ \ \forall \ (\boldsymbol{D},\theta)$ and $\phi(\boldsymbol{D} = \boldsymbol{0},\theta) = 0$. Definition (5) implies that $d_1 = 0$ when $\boldsymbol{D} = \boldsymbol{0}$. The parcel d_1 represents the rate of energy dissipation due to viscous phenomena. For the sake of simplicity, the present study is restricted to incompressible fluids. Considering the incompressibility constraint $\dot{\rho} = 0 \Rightarrow tr(\boldsymbol{D}) = 0$ and noting P the hydrostatic pressure which is a multiplier associated to the constraint $tr(\boldsymbol{D}) = 0$, it is useful to use the following notation:

$$\boldsymbol{T} = P\boldsymbol{1} + \boldsymbol{\sigma} \tag{6}$$

Where $\boldsymbol{1}$ is the unit tensor and $\boldsymbol{\sigma}$ is the part of the Cauchy stress tensor to be defined by the constitutive relations and Assuming incompressibility constraint and using the balance of mass equation, from the definition of the intrinsic dissipation in (2) and eq. (5), the following expression can be obtained:

$$d_1 = \boldsymbol{\sigma} : \boldsymbol{D} - \rho((\partial \psi / \partial \theta)\dot{\theta} + s\dot{\theta}) \tag{7}$$

From (5) and (7) it is possible to verify that

$$0 = (\boldsymbol{\sigma} - \partial \phi / \partial \boldsymbol{D}) : \boldsymbol{D} - \rho(\partial \psi / \partial \theta + s)\dot{\theta} \tag{8}$$

Since the following relations must always hold in any admissible process, it comes that:

$$s = -(\partial \psi / \partial \theta) \tag{9}$$

$$\boldsymbol{T} = -P\boldsymbol{1} + \boldsymbol{\sigma} \quad \text{ with } \quad \boldsymbol{\sigma} = \partial \phi / \partial \boldsymbol{D} \tag{10}$$

3.2. Generalized Cattaneo law and thermal dissipation

The classic Cattaneo law $\boldsymbol{q}+\tau\dot{\boldsymbol{q}} = -k\ \boldsymbol{g}$ is not objective and, therefore, it is only adequate for the analysis of heat transfer in rigid bodies at rest. k is a positive function of θ usually called the thermal conductivity and τ a nonnegative constant. A possible alternative to circumvent this limitation is to adopt the following generalized Cattaneo law:

$$\boldsymbol{q}+\tau\breve{\boldsymbol{q}} = -k\ \boldsymbol{g} \tag{11}$$

Where $\breve{\boldsymbol{q}}$ is an objective time derivative of $\boldsymbol{q}$ which will be adopted in the place of $\dot{\boldsymbol{q}}$ in order to assure objectivity. The material time derivative of an objective vector quantity is not necessarily objective. Therefore, in order to assure objectivity, it is necessary to use some special kind of time derivative in rate type constitutive equations. A large number of definitions of objective time derivatives can be found in the literature (Jaumann, Truesdell, Cotter-Rivling, Gordon-Schowalter, etc. (Truesdell and Noll, 1965, Billington and Tate, 1981, for instance). The choice of a particular derivative is important and can be interpreted as a constitutive assumption. The present study is restricted to co-rotational derivatives:

For an arbitrary vector $\boldsymbol{q}$: $\breve{\boldsymbol{q}} = \dot{\boldsymbol{q}} - \boldsymbol{\Omega}\boldsymbol{q}$ (12)

where $\breve{\boldsymbol{q}}$ an objective time derivative of an arbitrary objective vector $\boldsymbol{q}$. $\boldsymbol{\Omega}$ is a time dependent skew-symmetric second order tensor which can be associated to microscopic motions of the material structure or substructure. The objective derivative $\breve{\boldsymbol{q}}$ is equal to the material derivatives of $\boldsymbol{q}$ as it would appear to an observer in a frame of reference attached to the particle and rotating with it at an angular velocity equal to the instantaneous value of the spin $\boldsymbol{\Omega}$. The choice of a particular derivative is important and must be interpreted as a constitutive assumption. Different possible choices of $\boldsymbol{\Omega}$ can be found in litterature, mainly connected with objective derivatives proposed for second order tensor quantities. For instance: $\boldsymbol{\Omega} = \boldsymbol{W} = 1/2\ [\boldsymbol{grad}(\boldsymbol{v}) - \boldsymbol{grad}(\boldsymbol{v})^{\mathrm{T}}]$, $\boldsymbol{W}$ being the vorticity tensor (Jaumann derivative); $\boldsymbol{\Omega}=\dot{\boldsymbol{R}}\boldsymbol{R}^{\mathbf{T}}$, $\boldsymbol{R}$ being the rotation tensor (Green/Naghdi derivative). Eq. (12) implies that the thermal dissipation must have the following form

$$d_2 = \frac{\boldsymbol{q}}{\theta k} \cdot (\boldsymbol{q}+\tau\breve{\boldsymbol{q}}) \tag{13}$$

4. THE SECOND LAW OF THERMODYNAMICS - SUFFICIENT CONDITIONS

Eqs. (9), (10), (11) form a set of objective constitutive equations. In this section sufficient conditions are proposed in order to assure that these equations are also thermodynamically admissible, i.e. for any particular set of constitutive equations obtained within the context proposed on this paper, the inequalities in (3) are satisfied in any process.
Since ϕ is a convex, differentiable function of $\boldsymbol{D}$ such that $\phi(\boldsymbol{D}) \geq 0\ \ \forall\ \boldsymbol{D}$ and $\phi(\boldsymbol{D} = \boldsymbol{0}) = 0$,
a classical result of convex analysis (Rockafellar, 1970) is that

$$\frac{\partial\phi}{\partial\boldsymbol{D}} : \boldsymbol{D} \geq 0, \quad \forall\ \boldsymbol{D} \tag{14}$$

Therefore, from eq. (5), it is possible to conclude that, in this case, $d_1 \geq 0$ in all processes. Although eq. (11), the generalized version of Cattaneo equation, is indeed objective, the limitations regarding the second law of thermodynamics are the same of the case of the Vernotte-Cattaneo equation for rigid bodies undergoing small transformations. From eq. (13) and the fact that the thermal conductivity is a positive quantity, it is simple to verify that the thermal dissipation d_2 is always positive provided

$$\boldsymbol{q} \cdot (\boldsymbol{q}+\tau\breve{\boldsymbol{q}}) \geq 0 \tag{15}$$

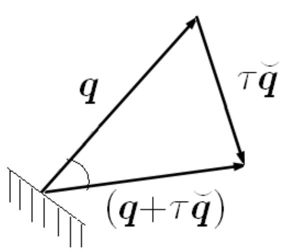

Figure 1: Representation of the condition $\boldsymbol{q} \cdot (\boldsymbol{q}+\tau\breve{\boldsymbol{q}}) \geq 0$

This means the vectors $\boldsymbol{q}$ and $(\boldsymbol{q}+\tau\breve{\boldsymbol{q}})$ must be in the same half space in order to assure the constraint $d_2 \geq 0$ (see fig. 1). Therefore eq. (15) is a necessary condition to assure that heat flows in the direction of decreasing temperature when $\boldsymbol{q}$ is parallel to the temperature gradient and that eqs. (9)-(11) form a set of objective thermodynamically admissible constitutive equations.

5. SOME REMARKS ABOUT THE HEAT EQUATION

In the case of the particular family of fluids defined in section 3, the thermomechanical coupling can be very important. A few recent studies show how the choice of the thermodynamic potentials ψ and ϕ is important and how it can affect the heat equation in rate type constitutive equations (da Costa Mattos and Lopes Pacheco, 2009, for instance). Nevertheless, different from the present paper, most of these works are concerned with inelastic solids (not fluids) undergoing small transformations and the influence of the objective time derivatives is not considered.

From the definition of the Helmholtz free energy per unit mass ψ and of the intrinsic dissipation d_1 it is possible to obtain the following alternative local form for the first law of thermodynamics

$$div(\boldsymbol{q}) = d_1 - \rho\mathbb{C}\dot{\theta} \text{ with } \mathbb{C} = -\theta\,\frac{\partial^2\psi}{\partial\theta^2} \tag{16}$$

$\mathbb{C}$ is usually called the specific heat. The term d_1 can be very important in the case of pseudoplastic fluids. It plays a role in eq. (16) that is similar to a heat source. It is important to remark that $d_1 \geq 0$ for the family of fluids studied here. Taking the material derivative of eq. (16) it is possible to obtain

$$div(\dot{\boldsymbol{q}}) = \dot{d}_1 - \rho\dot{\mathbb{C}}\dot{\theta} - \rho\mathbb{C}\ddot{\theta} \tag{17}$$

Finally, combining eqs (11), (12) and (17), the following local version of the FLT, which will be called the heat equation, can be obtained

$$\rho\mathbb{C}(\dot{\theta} + \tau\ddot{\theta}) + \rho\dot{\mathbb{C}}\dot{\theta} = div(k\ \boldsymbol{grad}(\theta)) + d_1 + \tau\dot{d}_1 - \tau div(\boldsymbol{\Omega q}) \tag{18}$$

The terms d_1 and $\dot{d}_1$ are responsible for the thermomechanical couplings in the heat equation and the term $\tau div(\boldsymbol{\Omega q})$ may be important in turbulent flows. If $\phi \equiv 0$ (hence $d_1 = 0$ and $\dot{d}_1 = 0$), $\mathbb{C}$ and k are constants and $\boldsymbol{\Omega q} = \boldsymbol{0}$ (for instance, $\boldsymbol{\Omega} = \boldsymbol{0}$ in irrotational flows, for some material derivatives and $\boldsymbol{\Omega q} \approx \boldsymbol{0}$ in the case of small transformations) equation (18) reduces to the classical hyperbolic heat equation

$$\rho\mathbb{C}(\dot{\theta} + \tau\ddot{\theta}) = k\nabla^2\theta \tag{19}$$

where the symbol ∇^2 denotes the Laplacian operator with respect to the present position.

5.1. Generalized Newtonian fluids. An example

The dissipation function ϕ for a generalized Newtonian fluid has following particular form

$$\phi(\boldsymbol{D}) = \varphi\ (\boldsymbol{D_I});\quad \boldsymbol{D_I} = \boldsymbol{D} : \boldsymbol{D}\quad \text{with}\quad \varphi \not\geq 0 \Rightarrow \boldsymbol{T} = -P\boldsymbol{1} + \boldsymbol{\sigma}\quad \text{with}\quad \boldsymbol{\sigma} = 2\frac{\partial\varphi}{\partial\boldsymbol{D_I}}\boldsymbol{D} \tag{20}$$

The term ($\partial\varphi / \partial\boldsymbol{D_I}$) is usually called the dynamic viscosity. It allows modelling the dependence of viscosity on the rate of deformation and temperature. In order to make easier a comparison with the classic hyperbolic equation obtained from the Cattaneo equation, it will be considered in the following that $\mathbb{C}$ is a constant. This is obtained if the free energy has the following particular form:

$$\psi(\theta) = \psi_o - \int_{\theta_0}^{\theta} c\log(\xi)d\xi \tag{21}$$

With ψ_o and c being positive material constants and θ_o a reference temperature. An example of a generalized Newonian fluid is the so-called Ostwald´s fluid, which is defined as follows

$$\phi(\boldsymbol{D}) = \nu\ (\boldsymbol{D} : \boldsymbol{D})^{2n} \Rightarrow \boldsymbol{\sigma} = 2n\nu(\boldsymbol{D_I})^{2n-2}\boldsymbol{D} \tag{22}$$

with ν >0 and n >0. If $n = 1$, the fluid is Newtonian. This model (with $n > 1$) is often used to describe flows of molten polymers in ducts. It gives a reasonable description if there are no abrupt changes in the geometry of the duct and, hence, elastic effects are negligible. A thermo-dependent theory may be obtained by considering ν a positive function of the absolute temperature θ. Using (22) and expression (5) proposed for d_1, the following equations are obtained

$$d_1 = 2n\nu(\boldsymbol{D} : \boldsymbol{D})^n \quad \dot{d}_1 = 4n^2\nu(\boldsymbol{D} : \boldsymbol{D})^{n-1}(\boldsymbol{D} : \dot{\boldsymbol{D}}) \tag{23}$$

Finally, assuming a constant thermal conductibility k it is possible to obtain the following heat equation for the Ostwald´s fluid:

$$\rho c(\dot{\theta} + \tau\ddot{\theta}) = k\nabla^2\theta + 2n\nu(\boldsymbol{D} : \boldsymbol{D})^n + \tau 4n^2\nu(\boldsymbol{D} : \boldsymbol{D})^{n-1}(\boldsymbol{D} : \dot{\boldsymbol{D}}) - \boldsymbol{div}(\boldsymbol{\Omega q}) \tag{24}$$

6. FOURIER LAW AND HYPERBOLIC HEAT TRANSFER – AN ALTERNATIVE APPROACH

In this section it is proved that an adequate alternative thermomechanical formulation can lead to a hyperbolic heat equation even if Fourier´s law is considered. The main idea is that the Helmholtz postulate is not necessarily valid outside the states of equilibrium. Similarly as in section 3, the free energy ψ is a differentiable function of the absolute temperature $\theta : \psi = \psi(\theta)$. In this case, the intrinsic dissipation has an additional term and classical Fourier´s law is assumed to hold

$$d_1 = \grave{d}_1 - \frac{\rho\tau}{\theta}\dot{\theta} \quad \text{and} \quad \boldsymbol{q} = -k\ \boldsymbol{g} \quad \text{with } \grave{d}_1 = \left(\frac{\partial\phi}{\partial\boldsymbol{D}}\right) : \boldsymbol{D} \ \text{ and } \ \tau \geq 0 \tag{25}$$

Similarly as in section 3.1, it is possible to conclude that the following constitutive equations must hold

$$s = -\left(\frac{\partial\psi}{\partial\theta} - \frac{\tau}{\theta}\dot{\theta}\right) \text{ and } \boldsymbol{T} = -P\boldsymbol{1} + \boldsymbol{\sigma}, \quad \text{with} \quad \boldsymbol{\sigma} = \frac{\partial\phi}{\partial\boldsymbol{D}} \tag{26}$$

If ϕ is a convex function of $\boldsymbol{D}$ such that $\phi(\boldsymbol{D},\theta) \geq 0 \ \ \forall\ (\boldsymbol{D},\theta)$ and $\phi(\boldsymbol{D} = \boldsymbol{0},\theta) = 0\ \forall\ \theta$, then $\grave{d}_1 \geq 0$. Using Fourier´s law it is simple to verify that

$$d_2 = -\frac{1}{\theta}\boldsymbol{q}\cdot\boldsymbol{g} = \frac{k}{\theta}\boldsymbol{g}\cdot\boldsymbol{g} \geq 0 \tag{27}$$

In this case, it is possible to assure that heat flows in the direction of decreasing temperature when $\boldsymbol{q}$ is parallel to the temperature gradient. The Clausius-Duhen inequality $d \geq 0$ will only be satisfied if

$$d = \underbrace{\left(\frac{\partial\phi}{\partial\boldsymbol{D}}\right) : \boldsymbol{D}}_{\geq 0} + \underbrace{\frac{k}{\theta}\boldsymbol{g}\cdot\boldsymbol{g}}_{\geq 0} - \frac{\rho\tau}{\theta}\dot{\theta} \geq 0 \Leftrightarrow \left(\frac{\partial\phi}{\partial\boldsymbol{D}}\right) : \boldsymbol{D} + \frac{k}{\theta}\boldsymbol{g}\cdot\boldsymbol{g} \geq \frac{\rho\tau}{\theta}\dot{\theta} \tag{28}$$

Combining eqs. (1), (4), $(25)_1$ and (26) it is possible to obtain the following version of the first law of thermodynamics:

$$div(\boldsymbol{q}) = \grave{d}_1 - \rho(\mathbb{C}\dot{\theta} + \tau\ddot{\theta}) \text{ with } \mathbb{C} = -\theta\ \frac{\partial^2\psi}{\partial\theta^2} \tag{29}$$

Assuming that the Fourier Law holds, the following heat equation is obtained

$$\rho\mathbb{C}(\dot{\theta} + \tau\ddot{\theta}) = div(k\ \boldsymbol{grad}(\theta)) + \grave{d}_1 \tag{30}$$

Therefore, hyperbolic heat transfer is possible provided constraint (28) holds. If $\phi \equiv 0$ (hence $\grave{d}_1 = 0$) and $\mathbb{C}$, k are constants, eq.(30) is reduced to the classical hyperbolic heat equation (19). If the particular example of the Ostwald´s fluid in which ϕ and ψ are defined, respectively, by eqs. (20) and (21), the following heat equation is obtained:

$$\rho c(\dot{\theta} + \tau\ddot{\theta}) = k\nabla^2\theta + 2n\nu(\boldsymbol{D} : \boldsymbol{D})^n \tag{31}$$

Green and Naghdi, 1977, have presented reservations regarding the Clausius-Duhen inequality. They claim that there are some processes that can be thermodynamically admissible even if they do not verify the Clausius-Duhen inequality.

The most adequate constraint would be $d_1' \geq 0$ and $d_2 \geq 0$ instead of $d \geq 0$ and $d_2 \geq 0$. It is not the goal of the present paper to perform a discussion about the most adequate local version of the SLT. But, if the alternative constraints are considered as an adequate version of the SLT, then the set of equations defined by (4), (25) and (26) are thermodynamically admissible in any process, and condition (28) is no longer necessary.

7. CONCLUDING REMARKS

This study presents adequate thermomechanical formulations that can lead to the modelling of heat waves of finite speed in generalised Newtonian fluids. The main objective is to show alternative approaches that cannot be neglected in future experimental studies. The generalization of the demonstrations to more complex material behaviours can be performed by considering a more sophisticated constitutive theory under the framework of thermodynamics of irreversible processes. All constitutive equations obtained in da Costa Mattos and Pacheco (2009) in the case of inelastic damageable solids undergoing small transformations, and in da Costa Mattos (1998, 2012) in the case of non Newtonian fluids, can be extended to account for hyperbolic heat transfer within this context.

8. REFERENCES

Barletta, A., Zanchini, E., 1997. “Hyperbolic heat conduction and local equilibrium: a second law analysis”. Int, J. Heat Mass Transfer. Vol. 40, pp. 1007–1016.
Billington, E.W. and Tate, A, 1981. “The Physics of Deformation and Flow”. McGraw-Hill, N.Y.
Cattaneo, C., 1958. “A form of heat conduction equation which eliminates the paradox of instantaneous propagation”. C. R. Hebdomadaires des Séances Acad. Sci. Vol. 247, pp- 431–433.
Christov, C. I., Jordan, P. M., 2005. “Heat conduction paradox involving second-sound propagation in moving media”. Phys. Rev. Lett. 94, 154301.
Christov, C.I., 2009. “On frame indifferent formulation of the Maxwell–Cattaneo model of finite-speed heat conduction”. Mech. Res. Comm. Vol. 36, pp. 481 - 486.
da Costa Mattos, H. S., 1998. “A thermodynamically consistent constitutive theory for fluids”. Int. J. of Non-linear Mech. Vol. 33, pp. 97 – 110.
da Costa Mattos, H.S. and Lopes Pacheco P.M.C., 2009. “Non-isothermal low-cycle fatigue analysis of elasto-viscoplastic materials”. Mech. Res. Comm. Vol. 36, pp. 428-436.
da Costa Mattos, H. S., 2012. “On thermodynamics, objectivity and the modeling of non-isothermal viscoelastic fluid behavior”. Nonlinear Analysis: Real World Applications, Vol. 13, no 5, pp. 2134–2143.
Green, A.E.. and Naghdi, P.M., 1977. “On thermodynamics and the nature of the second law”. Proc. R. Soc. Lond. A. Vol. 357, pp. 253-270.
Guillemet. P., Bardon, J.P., Rauch C., 1997. “Experimental route to heat conduction beyond the Fourier equation”. Int. J. Heat Mass Transfer. Vol. 40, pp. 4043-4053.
Jackson, H.E., Walker, C.T., 1971. “Thermal conductivity, second sound and phonon–phonon interactions in NaF”. Physical Review B. Vol. 3, pp. 1428–1439.
Joseph, D.D., Preziosi, L.,1990. “Heat waves”. Rev. Mod. Phys. Vol. 61, pp. 41–73.
Joseph, D.D., Preziosi, L.,1990. Addendum to the paper “Heat waves”. Rev. Mod. Phys. Vol. 62, pp. 375–391.
Kaminski, W., 1990. “Hyperbolic heat conduction equation for materials with a nonhomogeneous inner structure”. J. Heat Transfer. Vol. 112, pp. 555–560.
Kronberg, A.E., Benneker, A.H., Westerterp, K.R., 1998. Notes on wave theory in heat conduction: a new boundary condition. Int. J. Heat and Mass Transfer. 41, 127–137.
Narayanamurti, V., Dynes, R.C., 1972. “Observation of second sound in bismuth”. Phys. Rev. Lett. Vol. 28, pp. 1461–1465.
Özisik, M.N., Tzou, D.Y., 1994. “On the wave theory in heat conduction”. J. Heat Transfer. Vol. 116, pp. 526–535.
Rockafellar, R. T., 1970. Convex Analysis, Princeton University Press.
Rubin M. B., 1992. “Hyperbolic heat conduction and the second Law”. Int. J. Engng. Sci. Vol. 30, pp. 1665-1676.
Sieniutycz, S., Stephen Berry, R., 2002. “Variational theory for thermodynamics of thermal waves”. Physical Review E. Vol. 65, 046132.
Tibullo, V., Zampoli. V., 2011. “A uniqueness result for the Cattaneo–Christov heat conduction model applied to incompressible fluids”. Mech. Res. Comm. Vol. 38, 77–79.
Truesdell, C. and Noll W., 1965. “The Nonlinear Fields Theories of Mechanics”. Enc. Phys, Vol. III/3. Springer, Berlin.
Truesdel, C. and Toupin, R. A., 1960. “The classical fields theory”, Handbuch der Physik III/I, Springer Verlag, Berlin.
Vernotte, P., 1958, “Les paradoxes de la théorie continue de l'équation de la chaleur”. C. R. Hebdomadaires des Séances Acad. Sci. Vol. 246, pp. 3154–3155.