ZONAL METHOD IMPLEMENTATION TO DETERMINE THE THERMAL RADIATION HEAT TRANSFER IN BIDIMENESIONAL FURNACES

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Abstract. *The radiation heat transfer is relevant in the furnace and the combustion chambers, so that knowing the temperature distribution and the gas and soot concentration is possible to estimate the radiation heat transfer, thereby to evaluate the performance of the equipment. The zonal method with Weighted Sum Gray Gas – WSGG is a powerful methodology recommended by the accuracy and low computational cost, this in turn, consists in a division of the furnace volume in small isothermal volumes to estimate the direct exchange areas and the radiative heat transfer. In this paper was implemented three ways to determine the direct exchange areas using three different weighted sum of gray gas coefficients to an bi-dimensional black wall furnace with temperature and gases concentration profile known. The results show the discrete sum integration methods for direct exchange area with three coefficients of weighted sum of gray gases are the implementation most accurate and fast.*

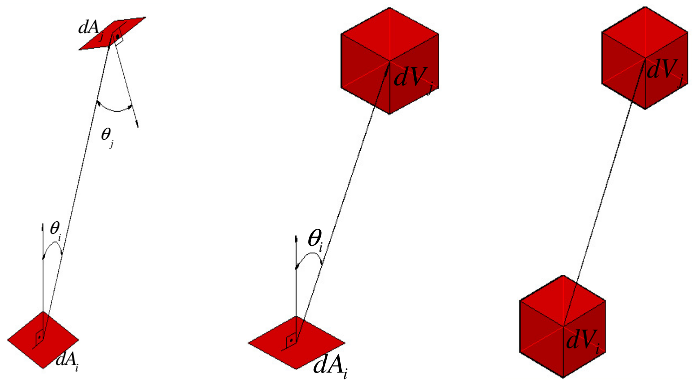
***Keywords**: Gas Concentration, Temperature Profile, Combustion Chambers*

1. NOMENCLATURE

$\mathbf{A}$	*Matrix area*	$\mathbf{I}$	Matrix identity
A_i	*Area of a surface zones*	$\mathbf{R}$	*Inverse matrix for direct exchange area*
a_s	*Weighting factor for Gray Gas surface*	r	*Distance between center of two zones*
a_s	*Weighting factor for Gray Gas volume*	$s_i s_j$	*Surface-to-surface direct exchange areas*
b	*Temperature weighting factor for Gray Gas*	$\mathbf{S}_i\mathbf{S}_j$	*Surface-to-surface total exchange areas*
ctr	Control variable	$\overrightarrow{s_i s_j}$	*Surface-to-surface directed flux areas*
$d_i d_j$	*Unified Direct exchange*	$\dot{Q}_{i-j}$	*Surface radiative heat transfer*
dd	*Unified zonal elements*	$\dot{Q}_g$	*Volume radiative heat transfer*
E_i	*Total emissive power of a black surface*	T_i	*Enclosure temperature surface*
E_g	*Total emissive power of a black volume*	T_g	*Gases temperature volume*
K	*Gray gas extinction coefficient*	V	*Volume of a gases zones*
$g_i g_j$	*volume-to-volume direct exchange areas*	*Greek symbols*	
$\mathbf{G}_i\mathbf{G}_j$	*volume-to-volume total exchange areas*	ε	*Emissivity*
$\overrightarrow{g_i g_j}$	*volume-to-volume directed flux areas*	ρ	*Reflectivity*
$g_i s_j$	*volume-to-surface direct exchange areas*	θ	*Polar angle*
$\mathbf{G}_i\mathbf{S}_j$	*volume-to-surface total exchange areas*	ϕ	*Infinity polar angle*
$\overrightarrow{g_i s_j}$	*volume-to-surface directed flux areas*		

2. INTRODUCTION

The weighted sum of gray gas method was initially established by Hottel and collaborators, apud in Hottel and Sarofim (1976). This method is based on the division of volume gas and of heat exchange surface in isothermal elements in order to account radiation heat transfer considering the interchanges between all surface to surface, surface to volume and volume to volume, see Fig(1).



(a) (b) (c)

Figure-1 Direct exchange area between surfaces -surfaces (a) between surfaces -volumes (b) and between volumes -volumes (c)

The direct exchange areas, Figure 1, are determined by:

$$s_i s_j = \int_{A_j}\int_{A_i} \frac{\cos(\theta_i)\cdot\cos(\theta_j)\cdot e^{(-K\cdot r)}}{\pi\cdot r^2} dA_i dA_j \quad (1)$$

$$g_i g_j = \int_{V_j}\int_{V_i} \frac{K\cdot K\cdot e^{(-K\cdot r)}}{\pi\cdot r^2} dV_i dV_j \quad (2)$$

$$g_i s_j = \int_{A_j}\int_{V_i} \frac{K\cdot\cos(\theta_j)\cdot e^{(-K\cdot r)}}{\pi\cdot r^2} dV_i dA_j \quad (3)$$

To take into account for the effects of multiple reflections and the emission of gases between the surfaces is used the total exchange area through the matrix representation, as realized to Rhine and Tucker (1991)

$$\overline{\mathbf{SS}} = \varepsilon\mathbf{AI}\cdot\mathbf{R}\cdot\overline{\mathbf{ss}}\cdot\varepsilon\cdot\mathbf{I} \quad (4)$$

$$\overline{\mathbf{SG}} = \varepsilon\mathbf{AI}\cdot\mathbf{R}\cdot\overline{\mathbf{sg}} \quad (5)$$

$$\overline{\mathbf{GG}} = \overline{\mathbf{gs}}\cdot\rho\mathbf{I}\cdot\mathbf{R}\cdot\overline{\mathbf{sg}}+\overline{\mathbf{gg}} \quad (6)$$

Where **R** is the following inverse matrix

$$\mathbf{R} = \left[\mathbf{AI}-\overline{\mathbf{ss}}\cdot\rho\cdot\mathbf{I}\right]^{-1} \quad (7)$$

The spectral bands effect of gas for radiation absorption, especially CO_2 and H_2O, is considered using a weighted sum of gray gases. These coefficients with direct exchange area are used to determine the direct flux area, as follow:

$$\overrightarrow{S_i S_j} = \sum_{n=1}^{N_g} a_{s,n}(T_i)\left(\overline{S_i S_j}\right)_{K=K_n} \quad (8)$$

$$\overrightarrow{S_i G_j} = \sum_{n=1}^{N_g} a_{s,n}(T_i)\left(\overline{S_i G_j}\right)_{K=K_n} \quad (9)$$

$$\overrightarrow{G_i S_j} = \sum_{n=1}^{N_g} a_{g,n}(T_i)\left(\overline{G_i S_j}\right)_{K=K_n} \quad (10)$$

$$\overrightarrow{G_i G_j} = \sum_{n=1}^{N_g} a_{g,n}(T_i)\left(\overline{G_i G_j}\right)_{K=K_n} \quad (11)$$

Where the coefficients are given by:

$$a_{s,n}\left(T_i\right)=\sum_{i=0} b_{i,n}\cdot T_w^i \tag{12}$$

$$a_{g,n}\left(T_i\right)=\sum_{i=0} b_{i,n}\cdot T_g^i \tag{13}$$

The heat exchange between the surfaces is given by:

$$\dot{Q}_{i-j}=\varepsilon_i\cdot A_i\cdot E_i-\sum_{j=1}^{m}\overrightarrow{S_iS_j}\cdot E_i-\sum_{j=1}^{l}\overrightarrow{S_iG_j}\cdot E_{g,i} \tag{14}$$

The heat exchange between the volumes is given as follow:

$$\dot{Q}_{i-j}=\sum_{j=1}^{l}\overrightarrow{G_iG_j}\cdot E_{g,i}-\sum_{j=1}^{m}\overrightarrow{G_iS_j}\cdot E_i-4\cdot K_t\cdot V_i\cdot E_{g,i} \tag{15}$$

3. METHODOLOGY

The integral solution of the direct exchange areas, Eq. (1), represent the largest computational effort of the method. The literature usually applies the Gaussian quadrature method. In this paper, it was developed the comparison between the Gauss-Legendre quadrature method with 5 and 10 terms and the integration method by discrete sum developed by Olsommer (1997) , given by:

$$d_id_j=\sum_{ddj}\sum_{ddi}\frac{\cos\left(\theta_i\right)^{ctr_i}\cdot\cos\left(\theta_j\right)^{ctr_j}\cdot K_i^{1-ctr_i}\cdot K_j^{1-ctr_j}\cdot e^{-K\cdot r_{ij}}}{\pi\cdot r_{ij}^2}\delta dd_i\,\delta dd_j \tag{16}$$

Where d_i e d_j are unified direct exchange area, s_is_j, g_is_j e g_ig_j; the terms dd_i e dd_j are the zonal exchange elements, A_i or V_i;. The control variable, *ctr,* determine if exchange factors is between areas, *crt=1,* or volumes*, crt=0.*

Another implementation realized was reduced integration proposed by Einstein (1963) to determine direct exchange areas, that take into account bi-dimensional effect by infinity walls, as follow:

$$d_id_j=\sum_{ddj}\sum_{di}\frac{2\cdot\cos\left(\theta_i\right)^{ctr_i}\cdot\cos\left(\theta_j\right)^{ctr_j}\cdot\delta d_i\cdot\delta dd_j\cdot K_i^{1-ctr_i}\cdot K_j^{1-ctr_j}}{\pi\cdot r^{1-ctr_j}}\cdot\int_0^{\pi/2} e^{-K\cdot r_{ij}/\cos\phi}\cdot\cos^{ctr_i+crt_j}\phi\,d\phi \tag{17}$$

Where ϕ represent the angle between the bi-dimensional plane and the infinity. The integration was realized using a Gaussian quadrature with 5 terms.

The direct exchange areas are three-dimensional quantities. To account the bi-dimensional energy balance applied the smoothing method for direct exchange area, as established by Lawson (1995), based on isothermic balance of the Eq.(14) and Eq. (15), as follow:

$$s_is_j^{'}=s_is_j\frac{A_i}{\sum_k s_is_k+\sum_k s_ig_k} \tag{18}$$

$$s_ig_j^{'}=s_ig_j\frac{A_i}{\sum_k s_is_k+\sum_k s_ig_k} \tag{19}$$

$$g_ig_j^{'}=g_ig_j\frac{4K_tV_i}{\sum_k g_is_k+\sum_k g_ig_k} \tag{20}$$

Theses equations are used in iterative process until that maximum discrepancy between the current e previous modified direct exchange factor is less than 10^{-10}.

To calculate the radiative heat transfer in the furnace were applied three different coefficients weighted sum of gray gas, Eq. (12) e Eq.(13), as proposed by Smith et al (1982), with three gray gases and four polynomials coefficients for temperature fit; by Truelove (1976) with four gray gases and a linear temperature fit; and by Galarça et al (2008) that used the same configuration that Smith et al (1982) with data from absorption-line blackbody distribution function.

The zonal method with weighted sums of gray gases was implemented for a cavity with black isothermal walls with dimensions shown in Fig. 2

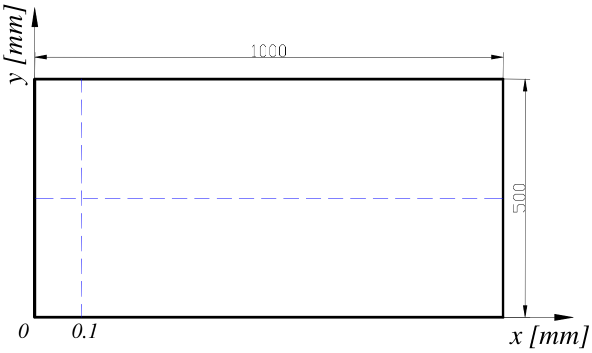


Figure 2. Isothermal Enclosure

The isothermal enclosures are determined in accordance Goutiere et al (2000), in which the gas concentrations of participating gases are assumed to be uniform and equal to 10%CO_2 and 20% H_2O; and the gases temperature profile are determined divided the isothermal enclosure in two regions, as showed by Eq. (20)

$$\begin{cases} x \le 0.1 \; T(x,y) = (14000x - 400)\cdot\left(1 - 3\cdot y_o^2 + 2\cdot y_o^3\right) + 800 \\ x > 0.1 \; T(x,y) = -\dfrac{10000}{9}(x-1)\cdot\left(1 - 3\cdot y_o^2 + 2\cdot y_o^3\right) + 800 \end{cases} \tag{20}$$

Where

$$y_o = \frac{|0.25 - y|}{0.25} \tag{21}$$

To implement the method was developed a FORTRAN code subdivided in sub-routines as show in the Fig. 3

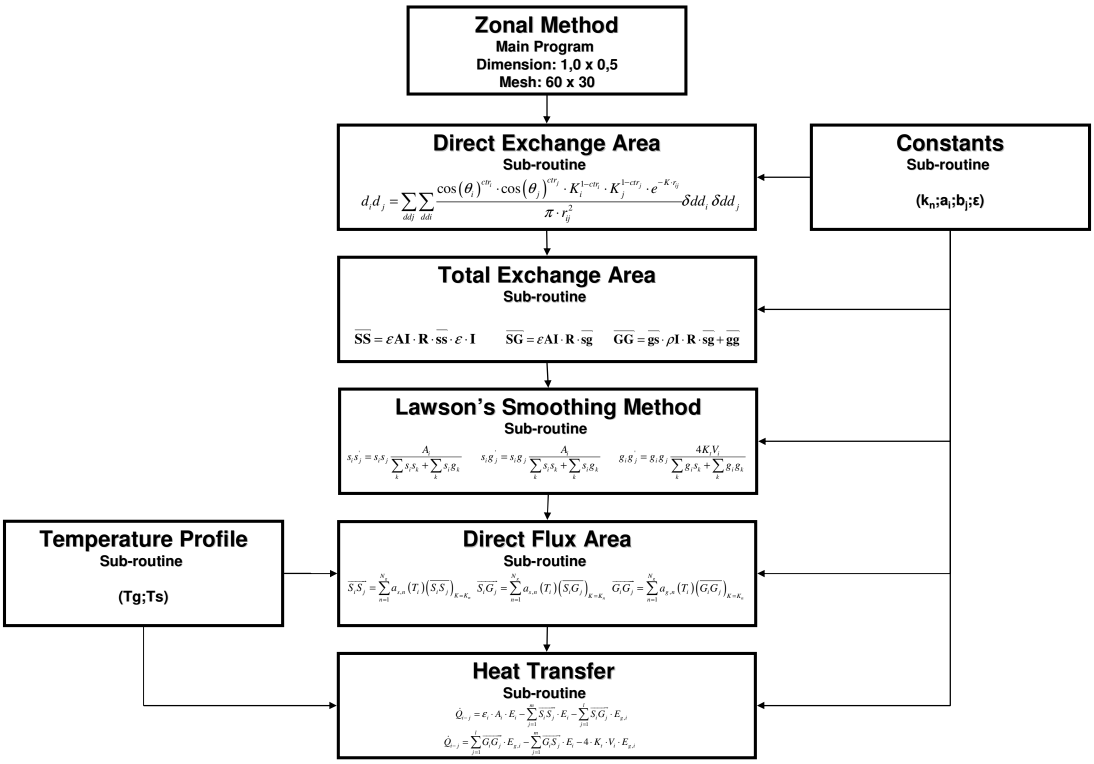


Figure 3. Implemented Code flowchart

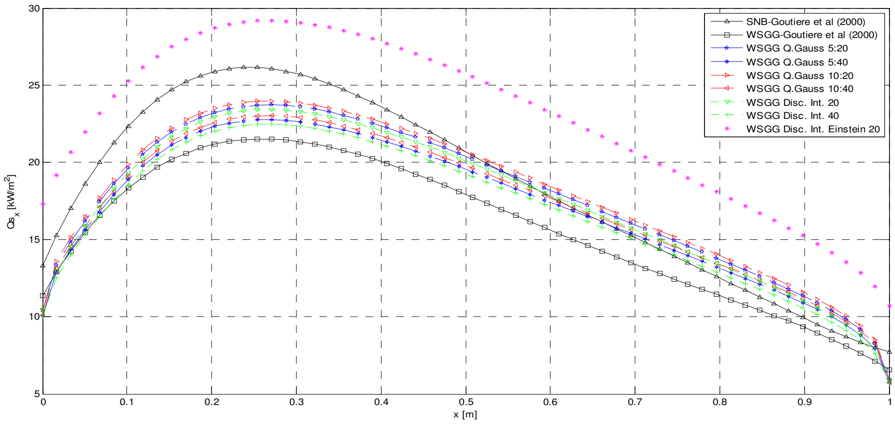
4. RESULTS

To analyze the zonal method with weighted sum of gray gases ,was initially used method suggested by Smith et al 1982, for direct exchange areas with coefficients weighted sum of gray gases . Two subdivision conditions for zonal elements are used, 20 and 40 to each direction. The percentage of average deviations were determined by the data collected by Goutiere et al (2000) using the method of weighted sum of gray gases, shown in Table 1:

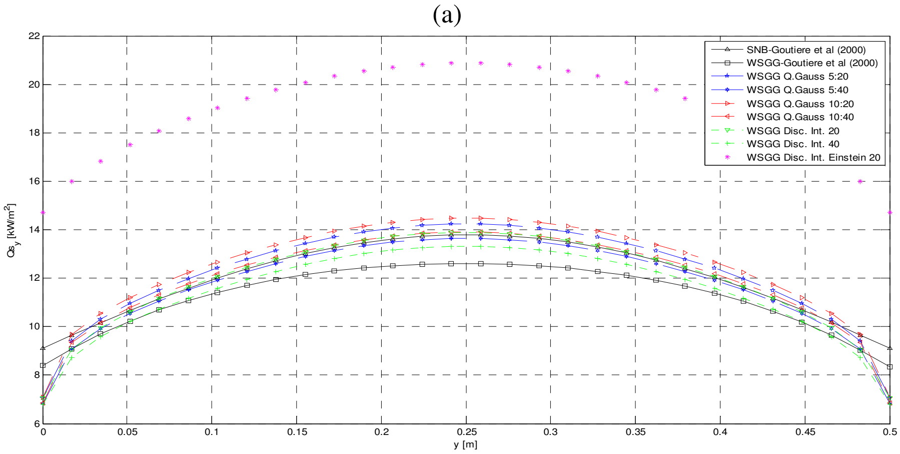
Table-1 Applied implementation for direct exchange areas

Implementation	***Average Deviation [%]***		***Time [s]***	
	20	40	20	40
Gaussian Quadrature with 5 terms	12,04	7,96	1024	6972
Gaussian Quadrature with 5 terms	13,87	9,80	1015	6977
Discrete integration	9,53	5,97	909	5399
Reduction Integration proposed by Einstein (1963)	40,98	- -	3491	- -

The results for radiative heat flux implemented in Table 1, for the bottom and the right walls of isotherm cavity are shown in Figure 4.



(a)



(b)

Figure 4. Radiative heat flux for the bottom (a) and the right (b) walls using Gaussian quadrature with 5 and 10 terms; discrete sum integration and reduced integration proposed by Einstein (1963), with 20 and 40 subdivision

The figure 4 shows that the discrete sum integration method is closest to the reference results obtained by the statistical narrow band-SNB method implemented by Goutiere et al (2000) and also has, in accordance with table 1, the smaller average deviation and processing time.

The two methods of Gaussian quadrature with 5 and 10 terms present satisfactory results with average deviations and processing times slightly higher, this results is directly related to the implemented form, which divided the subdivisions of the zones elements to adapt the quadrature points, and the expected discrepancy associated with small rays of the Eq. (1), Eq.(2) and Eq. (3), leading to function discontinuity of the direct exchange area.

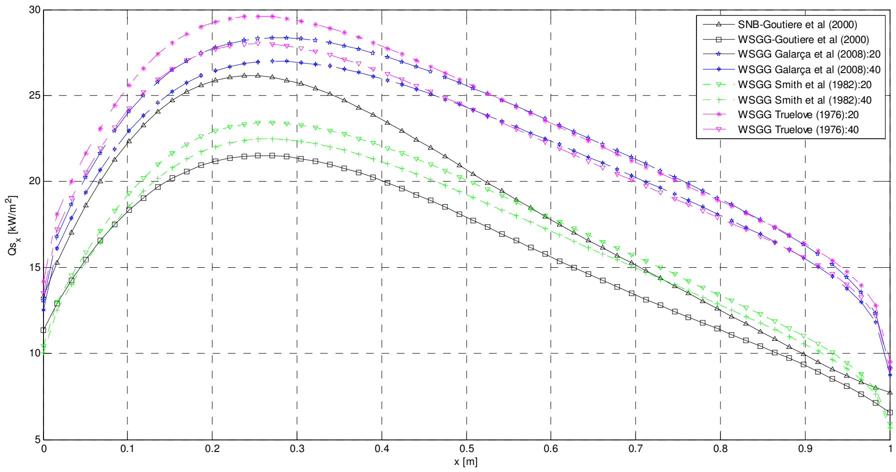
The implementation with significantly unfavorable results was established by Einstein (1963), which can be directly related to the design of bi-dimensional model proposed by Goutiere et al (2000), although the runtimes were unsatisfactory for the proposed model.

Regarding the analysis of the weighted gray gases coefficients, it was used the discrete sum integration implementation for the direct exchange areas with two zonal elements subdivisions, 20 and 40 in each direction. The percentage of average deviations were determined by the data established by Goutiere et al (2000) using the weighted sum of gray gases method, shown in Table 2:

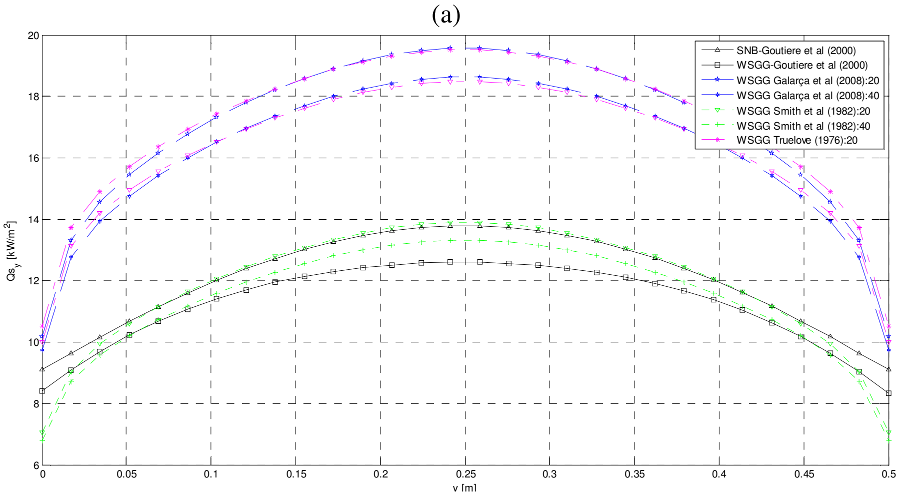
Table 2 – Implementation for different weighted gray gas coefficients

Implementation	***Average deviation [%]***		***Time [s]***	
	20	40	20	40
WSGG Galarça et al (2008)	48,82	41,83	1316	5137
WSGG Smith et al (1982)	9,53	5,97	909	5399
WSGG Truelove (1976)	38,07	32,17	1131	5841

The results for radiative heat flux implemented in Table 2, for the bottom and the right walls of isotherm cavity are shown in Figure5.



(a)



(b)

Figure 5. Radiative heat flux for the bottom (a) and the right (b) walls using the weighted sum gray gases established by Galarça et al (2008), Smith et al (1982) e Truelove (1976), with 20 and 40 subdivision

The table 2 and the figure 5 show that the best model for the weighted sum gray gas coefficients was the proposed by Smith et al (1982), since it was based on modeling spectral exponential wide bands. The weighted sum gray gas coefficients established by Galarça et al (2008) and Truelove et al (1976) in general did not represent adequately the proposed modeling.

Thus, among the evaluated implementations for calculating direct exchange areas and spectral modeling of absorption coefficients using the weighted sum of gray gases methods, the model for discrete sum integration associated with the weighted gray gases coefficients established by Smith et al (1982) shows a relevant accuracy and low processing time.

5. CONCLUSION

The implementation of the zonal method with weighted sum of gray gas showed as a method with low computational cost and with easy development when compared with other methods of radiative heat transfer solution, estimating properly the heat transfer in the furnace surfaces. Were evaluated different implementation for direct exchange area with different weighted sum gray gases coefficients, and the method of discrete sum integration with the coefficients determined by Smith et al (1982) represented the best tools to determine de radiative heat transfer in bi-dimensional furnace. However, as the present work has only been validated for a black isothermal cavity, it is important extend this validation to an actual operation condition of industrial furnaces, as well as improve the modeling of the absorption gases coefficient using spectral line-base weighted sum of gray gases method , since in the estimative the gases absorption coefficient for weighted sum method is considered as a black box model.

6. ACKNOWLEDGEMENTS

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