GEOMETRIC OPTIMIZATION FOR THE MAXIMUM HEAT TRANSFER DENSITY RATE FROM CYLINDERS ROTATING IN NATURAL CONVECTION

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Abstract. *In this paper we investigates the thermal behaviour of an assembly of consecutive cylinders in a counter-rotating configuration cooled by natural convection with the objective of maximizing the heat transfer density rate (heat transfer rate per unit volume). A numerical model was used to solve the governing equations that describe the temperature and flow fields and an optimisation algorithm was used to find the optimal structure for flow configurations with two or more degrees of freedom. It was found that the optimized spacing decreases and the heat transfer density rate increases as the Rayleigh number increases, for the optimized structure. For the single scale configuration it found that the optimized spacing decreases and the maximum heat transfer density rate increases, as the cylinder rotation speed was increased at each Rayleigh number. Results further showed that there was an increase in the heat transfer density rate of the rotating cylinders over stationary cylinders. For a multi scale configuration it was found that there was almost no effect of cylinder rotation on the maximum heat transfer density rate, when compared to stationary cylinders, at each Rayleigh number; with the exception of high cylinder rotation speeds, which serve to suppress the heat transfer density rate. It was, however, found that the optimized spacing decreases as the cylinder rotation speed was increased at each Rayleigh number.*

Keywords: *Natural convection, Rotating cylinders, Heat transfer density rate, Optimal packing*

1. NOMENCLATURE

D	cylinder diameter
$\mathbf{g}$	gravitational acceleration
H_d	downstream flow length
H_u	upstream flow length
k	thermal conductivity
$\hat{k}$	unit vector
P	pressure
$\mathcal{P}$	Power
q	total heat transfer rate
q'''	heat transfer density rate
R	solution residual
S_0	spacing between large cylinders
t	time / time step
T	fluid temperature
$\mathcal{T}$	torque
u, v, w	velocity components
$\mathbf{U}$	velocity vector
x, y, z	Cartesian coordinates

Greek Symbols

α	thermal diffusivity
β	thermal expansion coefficient
μ	viscosity
ν	kinematic viscosity
ρ	fluid density
ω_0	cylinder angular velocity

Non-Dimensional Numbers

Pr	Prandtl number
$\tilde{q}$	dimensionless heat transfer density rate
Ra	Rayleigh number

Subscripts

m	maximum
opt	optimum
sn	in the direction of the surface normal
w	wall

Accents

$\tilde{\cdot}$	dimensionless variable

2. INTRODUCTION

Efficiency is a key aspect in design, which has become prevalent in the design of heat transfer devices such as heat sinks and pin fins. Research has been and is still being conducted on this subject with the aim of extracting more and more heat from a given space through the maximizing of the packing of heat-generating material per unit volume. This drive to augment heat transfer devices has become reinforced by modern electronic systems which produce high amounts of heat due to the ever increasing power-to-volume ratio employed in such systems. The strive for greater heat transfer density rates has been the driving force behind many of the miniaturization efforts, augmentations and unconventional ways of designing heat transfer devices. This has lead researchers to study the optimized configurations for various architectures such as: the optimal spacing of parallel plates in forced convection, natural convection and mixed convection (Bejan and Sciubba, 1992; Bejan and Morega, 1994; da Silva *et al.*, 2004; Bello-Ochende and Bejan, 2004b); the optimal spacing of cylinders in forced convection and natural convection (Stanescu *et al.*, 1996, 1995); and various optimized multi-scale structures (Bejan and Fautrelle, 2003; Bello-Ochende and Bejan, 2004a; da Silva and Bejan, 2005; Bello-Ochende and Bejan, 2005a; Bello-Ochende *et al.*, 2010; Bello-Ochende and Bejan, 2005b), etc.

The heat transfer and fluid flow around a single rotating cylinder has been studied previously (Badr and Dennis, 1985; Chiou and Lee, 1993; Panda and Chhabra, 2011; Gshwendtner, 2004; Mohanty *et al.*, 1995; Oesterle *et al.*, 1998; Ozerdem, 2000; Paramane and Sharma, 2009, 2010; Yan and Zu, 2008; Nobari *et al.*, 2009).

More recent studies have been conducted by Joucaviel *et al.* (2008), with a single scale structure of a row of heat-generating rotating cylinders cooled by forced convection. The authors reported that the effect of rotation was beneficial from the point of view of maximising the heat transfer density rate. The results also showed that a counter-rotation configuration increases the heat transfer density rate more efficiently when compared to a co-rotation configuration. Similarly, Bello-Ochende *et al.* (2011) built onto the work by Bello-Ochende and Bejan (2005a) by considering a multi-scale constructal design with rotating cylinders.

The study presented in this paper builds onto prior research by Bello-Ochende and Bejan (2005b), in which the authors optimized the cylinder-to-cylinder spacings in a multi-scale constructal design of heat-generating cylinders (without cylinder rotation) cooled by natural convection for one and two degrees of freedom. These classical results will be used as a reference (benchmark) for the results reported in this paper. It is the purpose of this paper to maximize the heat transfer density rate of a single and multi scale configuration of heat-generating rotating cylinders in steady laminar single-phase natural convection, through constructal theory and design (Bejan and Lorente, 2008; Bejan, 2000; Bejan *et al.*, 2004; Bejan and Lorente, 2004). According to this method, the flow configuration is free to morph (change) in the pursuit of maximising global performance subject to global constraints. The resulting optimal configuration is determined through the use of a mathematical optimization routine.

3. NUMERICAL MODEL

Consider a row of infinity long, rotating and heat-generating cylinders as shown in Fig. 1. The large diameter cylinders (D_0) are aligned along their centreline and smaller cylinders (diameter D_1) are inserted in the entrance (converging) region of the channels formed between the larger diameter cylinders to form a stacking. The objective is to select the number of cylinders (large and small) in the stacking or in other words to select the cylinder-to-cylinder spacing (S_0) and the small cylinder diameter (D_1) in such a manner that the overall thermal heat transfer between the cylinders and the ambient air is maximized. The flow is assumed steady, laminar, incompressible and two-dimensional. All thermophysical properties are assumed constant. The temperature variations are assumed sufficiently small relative to the absolute temperature so that the Boussinesq approximation is valid. Figure 1 shows the elemental volume with computational boundary conditions that characterises these assemblies.

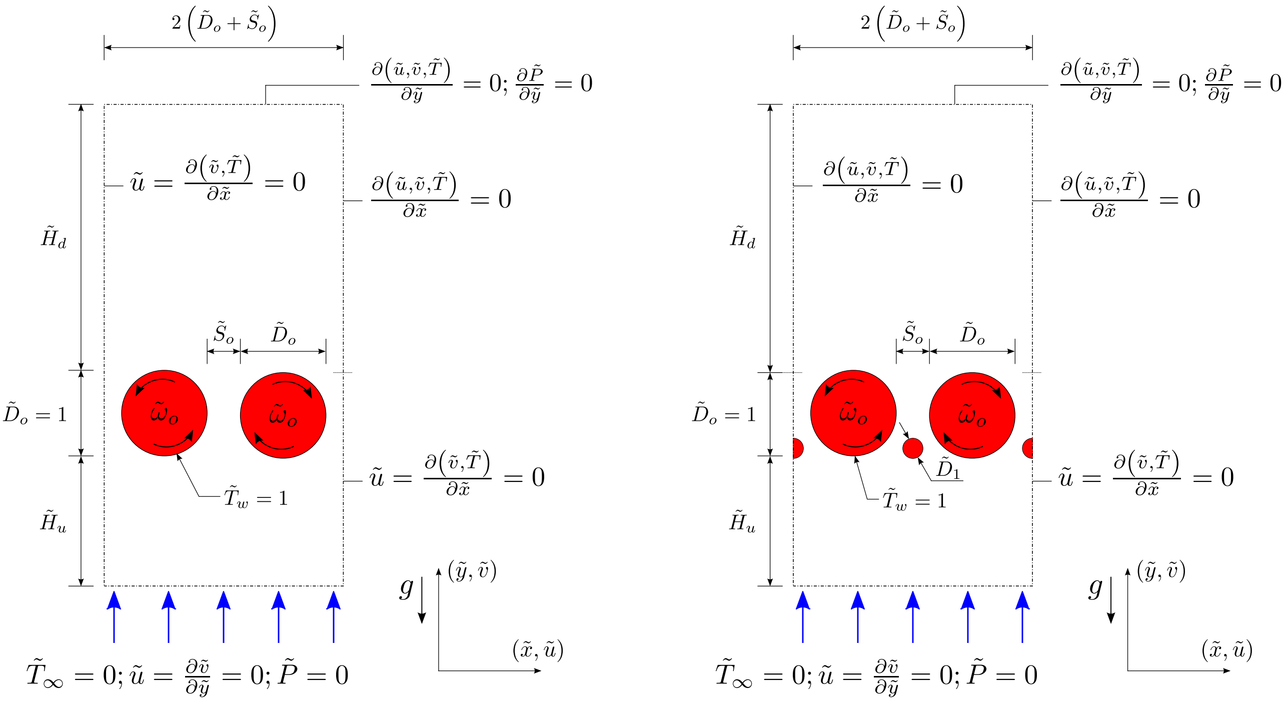

Figure 1: The computational domain and boundary conditions for a set of counter-rotating cylinders. (Left: Single scale configuration; Right: Multi scale configuration)

The numerical work of solving these computational domains is based on dimensionless formulation, where the dimensionless form of the continuity, momentum and energy equations is, respectively:

$$div\ \tilde{\mathbf{U}}=0 \tag{1}$$

$$\left(\frac{Ra}{Pr}\right)^{1/2}\frac{D\tilde{\mathbf{U}}}{D\tilde{t}}=-\tilde{\nabla}\tilde{P}+\tilde{\nabla}^2\,\tilde{\mathbf{U}}+\left(\frac{Ra}{Pr}\right)^{1/2}\tilde{T}\hat{k} \tag{2}$$

$$(RaPr)^{1/2}\frac{D\tilde{T}}{D\tilde{t}}=+\tilde{\nabla}^2\,\tilde{T} \tag{3}$$

where $\tilde{\mathbf{U}}=[\tilde{u},\tilde{v},\tilde{w}]$ is the dimensionless velocity field, $\tilde{\nabla}^2=\partial^2/\partial\tilde{x}^2+\partial^2/\partial\tilde{y}^2+\partial^2/\partial\tilde{z}^2$, $\hat{k}=[0\quad 1\quad 0]$ is a unit vector indicating the directions in which gravity acts, $Pr=\nu/\alpha$ is the Prandtl number and the Rayleigh number is defined in terms of the large cylinder diameter as:

$$Ra_{D_0}=\frac{g\beta\left(T_w-T_\infty\right){D_0}^3}{\alpha\nu} \tag{4}$$

The cylinder-to-cylinder spacing and the small cylinder diameter is varied and thus we are interested in the geometric configuration that maximizes the overall heat transfer between the cylinders and the surrounding fluid. The dimensionless quantity used to evaluate this configuration is the dimensionless heat transfer density rate. The heat transfer density rate is $q^{'''}=q^{'}/2D_0(D_0+S_0)$, where $q^{'}$ is the sum of the total heat transfer rate integrated over the surface of the cylinders:

$$q^{'}=\sum_{i=1}^{di}\frac{D_0}{2}\int_0^{2\pi}k(\nabla T)_{sn}\mathrm{d}\theta+\sum_{i=1}^{dj}\frac{D_1}{a}\int_0^{2\pi}k(\nabla T)_{sn}\mathrm{d}\theta \tag{5}$$

where di is the number of large diameter cylinders, dj is the number of small diameter cylinders, $a=2$ or $a=4$ for a complete and half cylinder respectively, and the subscript sn denotes that gradient of T is taken with respects to the normal direction to the cylinder surface. The corresponding dimensionless heat transfer density rate is:

$$\tilde{q} = \frac{q^{'}}{2D_0\left(D_0 + S_0\right)k\left(T_w + T_\infty\right)} \tag{6}$$

4. NUMERICAL METHODS

Equations 1 to 3 were solved using a finite volume package (OpenCFD Ltd, 2011), with hexahedron elements. The velocity-pressure coupled equations were solved using the Pressure-Implicit with Splitting of Operators (PISO) procedure, proposed by Issa (1986). The solution routine of solving the governing equations, for each geometric configuration, was automated through the use of a programming script (Python Software Foundation, 2011) and a few numerical tools (Jones *et al.*, 2011; Gschaider, 2011). This automated routine was then coupled to a mathematical optimisation algorithm (Jones *et al.*, 2011), based on the original algorithm implemented by Kraft (1994). The governing equations were discretised using the Finite Volume Method (FVM) and all internal variable fields were initialised as being 0 (i.e. $\mathbf{U}^0 = \mathbf{0}$, $P^0 = 0$ and $T^0 = 0$). The convergence criteria, used to solve the governing equations, is set based on the residual of each of the variable fields:

$$R_t^k\left(\mathbf{U}\right) \leq 10^{-4}, \quad R_t^k\left(P\right) \leq 10^{-4}, \quad R_t^k\left(T\right) \leq 10^{-6} \tag{7}$$

In which k is the iteration counter and t is the time step.

The mesh design received special attention and was tested extensively in the range $10^2 \leq Ra \leq 10^4$, for $\tilde{\omega}_0 = 0$ and $Pr = 0.72$, with the grid varying from one geometric configuration to the next. A structured mesh, consisted of hexahedron elements was used. The number of elements per unit length, upstream ($\tilde{H}_u$) and downstream ($\tilde{H}_u$) lengths for $Pr = 0.72$ were also selected based on the extensive mesh independence studies.

The constrained optimisation problem (derived from Eq. 6) can formally be written as:

$$\begin{aligned}
&\underset{w.r.t.\ \mathbf{x}}{\text{minimise:}} \\
&\qquad f\left(\mathbf{x}\right) = -\tilde{q}\left(\mathbf{x}\right), \qquad \mathbf{x} = \left[x_1, x_2\right]^T \in \mathbb{R}^n \\
&\text{such that:} \\
&\qquad g_2\left(\mathbf{x}\right) = -\left[\left(\frac{x_1 + D_0}{2}\right)^2 + \left(\frac{D_0 - x_2}{2}\right)^2\right]^{0.5} \leq 0 \\
&\qquad\qquad\quad = -\left[\left(x_1 + D_0\right)^2 + \left(D_0 - x_2\right)^2\right]^{0.5} \leq 0
\end{aligned} \tag{8}$$

where $x_1 = \tilde{S}_0$ and $x_2 = \tilde{D}_1$. The inequality constraint $g_2\left(\mathbf{x}\right)$ is added to Eq. 8 to ensure that the large diameter cylinders and small diameter cylinders do not overlap. The convergence criteria used to terminate the optimisation algorithm was:

$$\|\mathbf{x}^i - \mathbf{x}^{i-1}\| \leq 10^{-3} \tag{9}$$

where $\|\cdot\|$ is the Euclidean norm.

5. NUMERICAL RESULTS

5.1 Single scale

The optimal cylinder spacing and corresponding maximum heat transfer density rates, for a single scale configuration, are summarised in Fig. 2. From this figure it can be seen that the optimal cylinder-to-cylinder spacing decreases and the maximum heat transfer density rate increases as the Rayleigh number is increased (for any cylinder rotation speed). It can also be seen from Fig. 2 that the effect of cylinder rotation is beneficial for both improving the optimal cylinder packing and for enhancing the maximum heat transfer density rate. From Fig. 2 the optimal cylinder-to-cylinder spacing and heat transfer density rate were correlated by two power law functions respectively, with an error of less than 1%:

$$\tilde{S}_{0,opt} = -0.05\tilde{\omega}_0^{0.86} + 1.69Ra^{-0.25} \qquad \tilde{q}_m = 0.32\tilde{\omega}_0^{0.49} + 0.71Ra^{0.29} \tag{10}$$

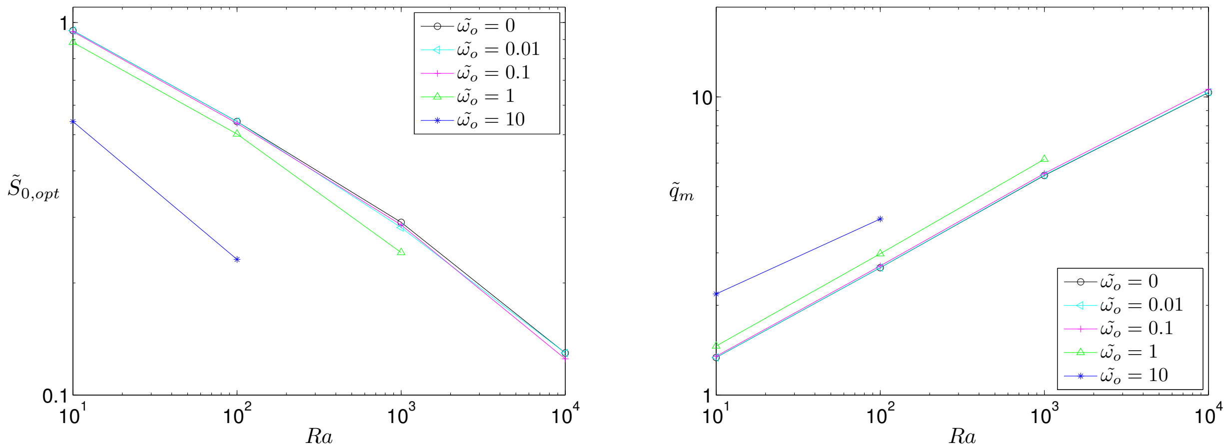

Figure 2: Single scale results for a row of rotating cylinders shown in Fig. 1 for $Pr = 0.72$. (Left: Optimal cylinder-to-cylinder spacings; Right: Maximum heat transfer density rates)

5.2 Multi scale

The optimal cylinder packing and maximum heat transfer density rates, for a multi scale configuration, are summarised in Fig. 3. From this figure it can be seen that the effect of cylinder rotation has little to no impact on the improvement on the optimal cylinder packing nor maximum heat transfer density rate. There is however the exception at a higher rotational speed of $\tilde{\omega}_0 = 10$ which serves to suppress the maximum heat transfer density rate as well as to reduce the optimal cylinder-to-cylinder spacing.

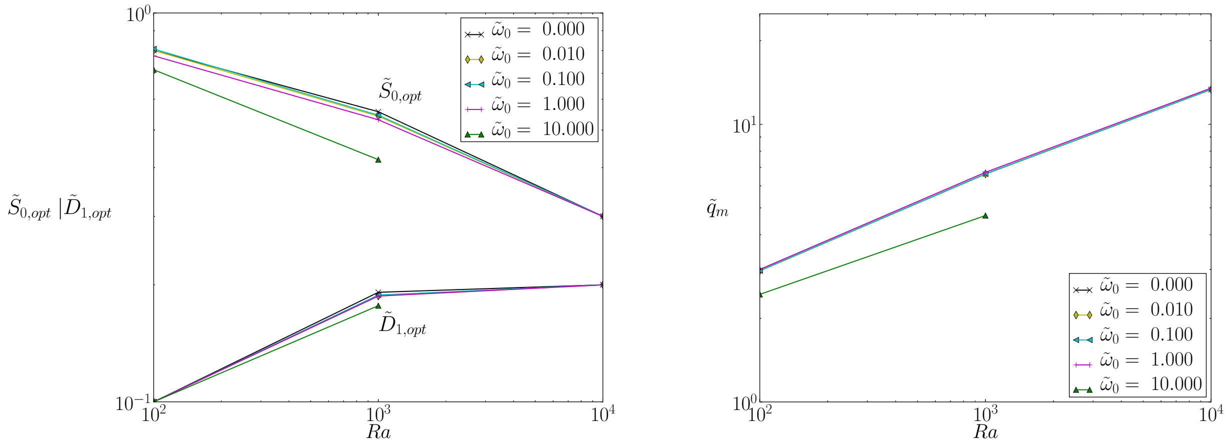

Figure 3: Multi scale results for a row of rotating cylinders shown in Fig. 1 for $Pr = 0.72$. (Left: Optimal cylinder packing; Right: Maximum heat transfer density rates)

5.3 Power input

For calculating the power input requirement only the effect of viscous shear in the fluid was considered. The effect of acceleration and friction forces of the cylinder itself was neglected. The power is given in its dimensionless form by using the following variable:

$$\tilde{\mathcal{P}} = \frac{\mathcal{P}}{(2\mu\alpha^2/D_0)\,(Ra_{D_0}Pr)} \tag{11}$$

where the power is calculated from the viscous force:

$$\mathcal{P} = \mathcal{T}\,\omega_0 = \frac{\mu\,\omega_0\,D_0}{2}\left(\frac{\partial U}{\partial x}\right)_{sn} \tag{12}$$

The non-dimensional power input required to rotate the cylinders, for both single scale and multi scale configurations, is summarised in Fig. 4. From this figure it can be seen that the power input required to rotate the cylinders increases significantly as the cylinder rotational speed is increased. It can also be seen the the power input required to rotate the cylinders increase as the Rayleigh number is increase (for any rotational speed).

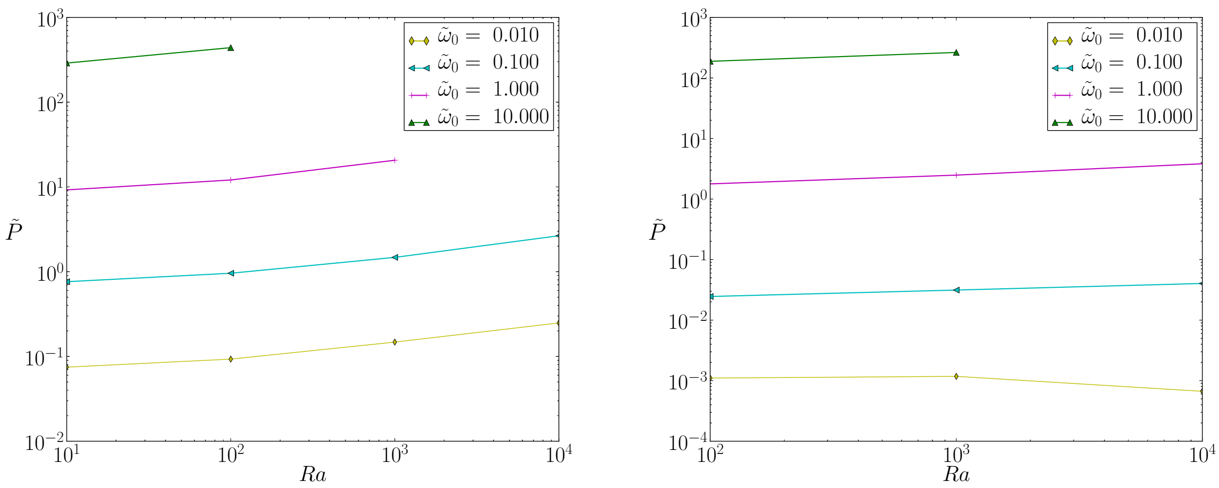

Figure 4: Non-dimensional power input required to rotate the cylinders shown in Fig. 1 for $Pr = 0.72$. (Left: Single scale configuration; Right: Multi scale configuration)

6. CONCLUSION

In this paper we showed numerically the effect of counter-rotation on a row of heat-generating cylinders which were cooled by natural convection. The cylinder-to-cylinder spacing was optimized for each flow regime and rotational speed on the cylinders. In the Rayleigh number range considered it was shown that the maximum heat transfer density rate increased and the optimal cylinder-to-cylinder spacing decreased with an increase in cylinder rotation speed for a single scale configuration at each Rayleigh number. For a multi scale configuration, the effect of increasing the rotation of the large diameter cylinders has little to no impact on the heat transfer density rate with the exception of a high rotation speed which serves to suppress the heat transfer density rate.

7. ACKNOWLEDGEMENTS

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