# MATHEMATICAL MODELING OF THE CTS - CAPTIVE TRAJECTORY SYSTEM 

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Abstract. The TTP (Pilot Transonic Wind Tunnel of IAE - Aeronautics and Space Institute) is a modern tunnel capable of aerodynamic tests in Mach number range from 0.2 to 1.3 with automatic controls of pressure (from 0.5 bar to 1.2 bar), temperature and humidity. In a transonic wind tunnel design the test section walls are semi-open to allow mass flow extraction, which is important to prevent aerodynamic chocking phenomenon, minimize shocklexpansion wave reflections from walls, and also to help on wall effects correction. To set the mass flow ratio through the walls, the test section is enveloped by a plenum chamber, in which the pressure is controlled by means of a dedicated compressor. What determines how much of the flow will pass through the walls are mainly the plenum chamber pressure and the reentry flaps positioning, located at the end of the test section. On high speed tunnels, simple aerodynamic models are connected through a device named sting wich supports the model by its back, in order to reduce as much as possible the interference effects with the clamping devices. However, there are more complex models composed of more than one body, such as aerospace vehicles with boosters coupled to a central body or artifacts suspended in aircraft wings, which cannot be correctly positioned in the test section without an auxiliary positioning device. A positioner system of three degrees of freedom called CTS (Captive Trajectory System) was designed in order to simulate a missile being jettisoned from a generic fuselage, expanding the ability of tests to be developed in the TTP. This positioner controls the position in the $X$ and $Y$ axis of the projectile and its angle of attack $\alpha$ through the linear displacement of three independent movers, controlled by step motors. The correct positioning of these three movers is required for the aerodynamic model fixed to the device to be placed in the desired position and attitude. This work demonstrates step by step the mathematical modeling performed for the device, and the equations that relate the position and attitude of the aerodynamic model with the position of the movers of the CTS positioning system designed for the TTP.

Keywords: Captive Trajectory System, Mathematical Modeling, Positioning, Attitude

## 1. INTRODUCTION

The Pilot Transonic Wind Tunnel (TTP) is one of the three wind tunnels that belongs to the Aerodynamics Division (ALA) of the Aeronautics and Space Institute (IAE), located in the Department of Aerospace Science and Technology (DCTA) in Sao Jose dos Campos - SP. Created in 1988 by a Brazilian team in conjunction with the American company Sverdrup Technology Inc., the TTP has already conducted several experiments to verify the reliability of its results in subsonic and transonic regimes (Sverdrup, 1989). Tests were conducted to examine the uniformity of the flow in the test section of the tunnel, initially with open circuit and intermittently driven by the injection system (Escosteguy, 1998 Zanin et al., 2008), then with closed circuit and directed by the injection system (Silva et al., 2009), and finally with closed circuit and driven by a two-stage axial compressor with 830 kW (Falcão et al., 2011).

TTP's test section has a cross section of 0.25 mx 0.30 m and 0.85 m in length. Aerodynamic models are fixed in the tunnel by its back through a device called sting (Figure 1), aiming to reduce the effects of the clamping devices on the results of the experiments. This clamping device has a single degree of freedom, being able to control the angle of


Figure 1. Aerodynamic models clamping device
attack of a simple model in the range of $-15^{\circ}$ to $+15^{\circ}$, while maintaining fixed the vertical and horizontal position of the center of rotation of the model. To continue the improvements, the TTP is expanding its application through the creation of a three degrees of freedom positioner, called Captive Trajectory System (CTS). Through this, the TTP is able to precisely determine the position in the X and Y axis, and angle of attack $\alpha$ of any aerodynamic model fixed to the device, and perform tests on complex models composed of more than one body, such as aerospace vehicles with boosters attached to a central body or artifacts attached to the fuselage of an aircraft.

Captive Trajectory System is the name given to the device used in high-speed tunnels for positioning bodies adjacent to the main aerodynamic model, in order to perform tests to check the interference caused by the presence of this body. Some modern devices have up to six degrees of freedom, and are able to traverse a predefined path in the middle of an experiment. They also rely on various safety sensors to prevent collisions and accurate position sensors that optimize the application of CTS and improve the quality of the results.

There are two ways of operating a Captive Trajectory System: open-loop control and closed-loop control. In the open-loop control, the CTS is operated step-by-step by the controller, who defines the desired position for the aerodynamic model, initiates the experiment, analyses the results, and then defines the second position for the model. These steps are repeated until the final position for the model is achieved. In the closed-loop control, the aerodynamic model attached to the device is instrumented with an inner balance that sends aerodynamic informations to the computer. The software uses these informations to calculate the next step for the model, and automatically sends it to that position without the controller's intervention. The loop is then reinitiated and the proceedings steps are calculated in the same way. Closed-loop control is faster and more efficient than open-loop control, but has the disadvantage of needing an inner balance installed on the aerodynamic model. On wind tunnels with large test sections this is not a problem since it is possible to create big aerodynamic models capable of supporting the inner balance. On small wind tunnels, such as the TTP, the aerodynamic models are relatively small, and inserting the inner balance into them is not always possible. For this reason, TTP's CTS is being created to support only the open-loop control.

A specific design of CTS should be created in each wind tunnel so that it can be adapted to their size and needs. The positioner designed for TTP (Figure 2), which is fixed to the plenum chamber structure, was designed according to the various constraints encountered during its development (Ubertini et al., 2012)., and has a peculiar architecture, mainly due to the reduced space for access to the test section, shown in Figure 2 by the orifice inlet.


Figure 2. Virtual model of the TTP's CTS with aerodynamic model
The position and attitude of the model fixed to the device is controlled by three linear movers, which are driven by step motors. These motors are operated remotely by the controller, using a software developed in LabView 2009 platform. Inserting in the software the desired position and attitude of the model, the computer calculates the required position on each mover, and the steps needed in each step motor so that the model is positioned at the desired location. The results are then sent to a microcontroller which, in turn, sends the data to the drivers that control the step motors. Potentiometers are used as position sensors of the movers, and continuously send information to the controller in order to monitor in real time the displacement of the device and the model fixed to it. Presence sensors prevent the impact of the sting with the walls of the hole used to access the test section (Figure 2), but possible impacts of the model with the
walls of the section are calculated only through software, given the impossibility of fixing sensors in aerodynamic models or in the tunnel walls.

The project is currently in the programming phase, when the software is being tested to adequately control the step motors through the control room of the TTP. At this phase, it is necessary to know the relationship between the position and attitude of the aerodynamic model and the linear position of the three movers. With this in mind, this article will describe step by step the mathematical modeling of the TTP's CTS, describing the equations that relate the position of the aerodynamic model with the position of each linear mover.

## 2. TTP'S CTS DESCRIPTION

Three independent movers (fixed Y mover, mobile Y mover and X mover) are connected by a sliding platform, as shown in Figure 3 below:


Figure 3. Virtual model of TTP's CTS
The aerodynamic model attached to the sting is fixed to the support, which in turn is fixed to the torpedo of the mobile and fixed Y movers by two pins. The X mover is responsible for the correct positioning of the aerodynamic model in the X axis, while the Y movers control the position in the Y axis and the angle of attack $\alpha$ of the model.

The difference between the positions of the torpedoes of the fixed and mobile $Y$ movers determines the angle of attack $\alpha$ of the aerodynamic model. When this difference is zero (the torpedoes are in the same position), the axis of symmetry of the support is parallel to the X axis and no angle is created between them. When there is any difference between the positions of the fixed Y torpedo and mobile Y torpedo, there is an angle $\beta$ between the axis of symmetry of the support and the X axis, and this angle reflects on the angle of attack $\alpha$ of the aerodynamic model fixed to the device.

When an angle is created between the axis of symmetry of the support and the $X$ axis (Figure 4), the total length of the support $(\boldsymbol{L})$ is no longer the distance between the Y movers and becomes the hypotenuse of the triangle formed. When this happens, the distance between the fixed Y mover and mobile Y mover is reduced in accordance with the Theorem of Pythagoras (Eq. (1)). This reduction is accomplished by sliding the mobile Y mover on a rail existing on the platform. For this reason, the position of the attachment point of the sting also undergoes a shift in the X axis, which must be taken into account during the mathematical modeling of the device.


Figure 4. Representation of the mobile Y mover displacement and the angle $\beta$ created between the axis of symmetry of the support and the X axis.
$L^{2}=C_{X}{ }^{2}+C_{Y}{ }^{2}$
Correctly positioning the torpedoes of the fixed Y mover, mobile Y mover and X mover, it is possible to position the aerodynamic model attached to the device in the desired position and attitude within the test section of the TTP.

## 3. MATHEMATICAL MODELING

The first step in the mathematical modeling of the TTP's CTS is to define the position ( $X_{P_{M}} ; Y_{P_{M}}$ ) and angle of attack $\left(\alpha_{P_{M}}\right)$ of the attachment point of the model on the sting $\left(P_{M}\right)$ in accordance to the displacement of the fixed Y torpedo $\left(D_{Y_{F T}}\right)$, mobile Y torpedo $\left(D_{Y_{M T}}\right)$, and X torpedo $\left(D_{X_{T}}\right)$, and the sting's tip angle $(\theta)$. In other words:
$X_{P_{M}}=f\left(D_{Y_{F T}} ; D_{Y_{M T}} ; D_{X_{T}} ; \theta\right)$
$Y_{P_{M}}=f\left(D_{Y_{F T}} ; D_{Y_{M T}} ; \theta\right)$
$\alpha_{P_{M}}=f\left(D_{Y_{F T}} ; D_{Y_{M T}} ; \theta\right)$
The second step in the mathematical modeling of the TTP's CTS is to isolate the four user defined variables $\left(D_{X_{T}}\right.$; $D_{Y_{F T}} ; D_{Y_{M T}}$ and $\theta$ ) in the equations created in the first step. Since the known information of most of the experiments that uses the CTS is the desired position and attitude of the aerodynamic model, it is important to calculate the necessary displacements on the torpedoes, in accordance to the desired position and attitude of $P_{M}$.

It's easier to visualize the nodes that need calculation if we split the apparatus into 3 dependent planes: The X mover's plane (Fig. 5), The Y movers' plane (Fig. 6) and the sting's plane (Fig. 7). The first and absolute plane is the X mover's plane, since the displacement of the X mover will slide both the Y movers' and the sting's plane. The second is the Y movers' plane, which is dependent on the position of the X torpedo, and controls the sting's plane. The third and last plane is the sting's plane, which determines the position of the model attached to the sting, in accordance to the position of the X torpedo and Y torpedoes.

Each plane has its own bi-dimensional coordinate system and nodes. The nodes of the X mover's plane define the origin of the coordinate system of the Y movers' plane, and its nodes define the origin of the coordinate system of the sting's plane.

The X mover's plane (Fig. 5) is fixed in space, which is why it is considered the absolute plan. The origin $\left(P_{0.1}\right)$ is defined as the initial position of the X torpedo ( $D_{X_{T}}=0 \mathrm{~mm} ; X_{P_{0.1}}=0 ; Y_{P_{0.1}}=0$ ), and its node $\left(X_{T}\right)$ is defined as the actual position of the X torpedo $\left(X_{X_{T}} ; Y_{X_{T}}\right)$.

The $Y$ movers' plane (Fig. 6), has its origin $\left(P_{0.2}\right)$ defined as the attachment point of the sting $\left(P_{1}\right)$, when both the fixed Y torpedo and the mobile Y torpedo are fixed to the middle of its curse ( $D_{Y_{F T}}=D_{Y_{M T}}=0 \mathrm{~mm}$ ). Instead of being
fixed in space, this plane slides along with any displacement in the X torpedo, making its nodes dependent on the origin of the coordinate system. The plane origin, on the other hand, is dependent on the X torpedo's position, making it the interface between both planes. The nodes of the Y movers' plane are the actual positions of the Y torpedoes $\left(Y_{Y_{F T}} ; X_{Y_{F T}}\right.$ and $\left.Y_{Y_{M T}} ; X_{Y_{M T}}\right)$ and the attachment point of the sting $\left(X_{P_{1}} ; Y_{P_{1}}\right) . \beta$


Figure 5. X mover's plane, origin and nodes.
The sting's plane (Fig. 7), has its origin $\left(P_{0.3}\right)$ defined as the actual position of the attachment point of the sting $\left(P_{1}\right)$, and its nodes are the interface between the first and second part of the sting ( $X_{P_{2}} ; Y_{P_{2}}$ ), and the attachment point of the model on the sting $\left(X_{P_{M}} ; Y_{P_{M}}\right)$. Just like the Y movers' plane, the sting's plane is not fixed in space, being able to move or rotate along with the attachment point of the sting. The nodes are dependent on the origin of the plane, and the origin is dependent on the position and angle of the attachment point of the sting.

Since the origin of the X mover's plane coordinate system $\left(P_{0.1}\right)$ is the position $(0 ; 0)$, the actual position of the X torpedo is simply the X position of $P_{0.1}$ plus the user defined displacement of the torpedo in the X axis $\left(D_{X_{T}}\right)$, in other words:
$X_{X_{T}}=X_{P_{0.1}}+D_{X_{T}}=D_{X_{T}}$
There is no Y displacement on the X torpedo, which means the Y position of the torpedo is equal to the Y position of the origin.
$Y_{X_{T}}=Y_{P_{0.1}}=0$
As stated before, the origin of the Y movers' plane $\left(P_{0.2}\right)$ is calculated through the position of the X torpedo. With this in mind, the equations that define the position of this point are:
$X_{P_{0.2}}=X_{X_{T}}-3,25$
$Y_{P_{0.2}}=Y_{X_{T}}-128$
The constants introduced in Eq. (7) and Eq. (8) are the measured distances between the attachment point of the sting (when the Y torpedoes are fixed to the middle of its curse) and the actual position of the X torpedo, and remain constant independent of the X torpedo's position.

Three nodes need calculation in the Y movers' plane: fixed Y torpedo's position $\left(Y_{F T}\right)$, mobile Y torpedo's position $\left(Y_{M T}\right)$ and attachment point of the sting's position $\left(P_{1}\right)$.

Since the fixed Y torpedo has no displacement in the X axis, it's X position is constant, in accordance to the origin of the coordinate system:
$X_{Y_{F T}}=X_{P_{0.2}}-150$


Figure 6. Y movers' plane, origin and nodes.
The constant introduced in Eq. (9) is half the length of the support (the distance between the support's attachment point on the fixed Y torpedo and the attachment point of the sting).

Since the initial position of the Y torpedoes is the center of its total curse, the Y position of the fixed Y torpedo is determined as the sum of the Y position of the origin of the coordinate system and the user defined Y displacement of the torpedo $\left(D_{Y_{F T}}\right)$, being it a positive or negative displacement.
$Y_{Y_{F T}}=Y_{P_{0.2}}+D_{Y_{F T}}$
The Y position of the mobile Y torpedo can be calculated in the same manner:
$Y_{Y_{M T}}=Y_{P_{0.2}}+D_{Y_{M T}}$

The X position of the mobile Y torpedo, on the other hand, is not simply calculated as the nodes calculated until now. As stated before, the difference between the Y positions of the Y movers generates an X displacement in the mobile Y torpedo's position (Fig. 4). Since the difference between the Y positions of the movers is known, as well as the total length of the support, the distance in the X axis between the fixed Y torpedo and the mobile Y torpedo can be calculated through Eq. (1):
$300^{2}=C_{X}{ }^{2}+\left(Y_{Y_{M T}}-Y_{Y_{F T}}\right)^{2}$
$C_{X}=\sqrt{300^{2}-\left(Y_{Y_{M T}}-Y_{Y_{F T}}\right)^{2}}$

Knowing the X position of the fixed Y torpedo $\left(X_{Y_{F T}}\right)$ and the distance in the X axis between the fixed and mobile Y torpedo $\left(C_{X}\right)$, the X position of the mobile Y torpedo can then be calculated through Eq. (14):
$X_{Y_{M T}}=X_{Y_{F T}}+C_{X}$

The attachment point of the sting $\left(P_{1}\right)$ is located exactly in the middle of the support that connects the fixed Y torpedo and the mobile Y torpedo. With this in mind, it is easy to realize that the position of this node is half the distance between the Y torpedoes, both in X (Eq. 15) and Y axis (Eq. 16).


Figure 7. Sting's plane, origin and nodes.
$X_{P_{1}}=X_{Y_{F T}}+\frac{X_{Y_{M T}}-X_{Y_{F T}}}{2}$
$Y_{P_{1}}=Y_{Y_{F T}}+\frac{Y_{Y_{M T}}-Y_{Y_{F T}}}{2}$
Another important thing to calculate is the $\beta$ angle (Fig. 4), which is used to determine the rotation of the sting's plane. This angle can be calculated through several ways, one of them is Eq. (17), where 300 is the total length of the support:
$\beta=\sin ^{-1}\left(\frac{Y_{Y_{M T}}-Y_{Y_{F T}}}{300}\right)$
The third and last plane is the sting's plane, which, in contrast with the first two planes, has no mover. The nodes calculated at this point are dependent only on the position and rotation of the sting's attachment point, the stings geometry and the sting's tip angle ( $\theta$ ). The sting's geometry is constant, changing only if, for some reason, the whole sting is substituted. The sting's tip angle $(\theta)$ is the user defined variable of this plane, and the only user defined variable that can only be changed manually, during the fixation of the aerodynamic model, in order to achieve higher angles of attack.

The origin of the coordinate system $\left(P_{0.3}\right)$ is the attachment point of the sting:
$X_{P_{0.3}}=X_{P_{1}}$
$Y_{P_{0.3}}=Y_{P_{1}}$
The first node of the third plane $\left(P_{2}\right)$ is the interface between the first and second part of the sting, and the second node $\left(P_{M}\right)$ is the attachment point of the model, on the sting. Since the sting's geometry is constant, the position of $P_{2}$ in accordance to $P_{0.3}$ is constant as well:
$X_{P_{2}}=X_{P_{0.3}}-\mathrm{S}_{\mathrm{X}}$
$Y_{P_{2}}=Y_{P_{0.3}}-\mathrm{S}_{\mathrm{Y}}$

Where $S_{X}$ and $S_{Y}$ are the distances between the attachment point of the sting and $P_{2}$, in the $X$ and $Y$ axis, respectively. These "constants" are expressed as variables, in order to simplify the modifications in the mathematical modeling, if, for some reason, the sting is substituted. The actual values of $S_{X}$ and $S_{Y}$ are $299,141 \mathrm{~mm}$ and 332 mm , respectively.

In order to introduce the rotation of the attachment point of the sting on the equation, the position of $P_{2}$ may also be expressed in polar coordinates:
$P_{2}=\left(446,89 ; 228^{\circ}\right)$
Whenever there is a difference between the positions of the Y torpedoes, a $\beta$ angle is created, changing the actual position of $P_{2}$. In order to achieve the new polar coordinate of $P_{2}$, this $\beta$ angle is added to the angle of Eq. (22), creating a more accurate expression for the position of $P_{2}$.
$P_{2}=\left(446,89 ; 228^{\circ}+\beta\right)$
Returning Eq. (23) to Cartesian coordinates, and making some trigonometrical transformations, the position of $P_{2}$ can be expressed as:
$X_{P_{2}}=X_{P_{0.3}}-\mathrm{S}_{\mathrm{X}} \times \cos \beta+\mathrm{S}_{\mathrm{Y}} \times \sin \beta$
$Y_{P_{2}}=Y_{P_{0.3}}-\mathrm{S}_{\mathrm{Y}} \times \cos \beta-\mathrm{S}_{\mathrm{X}} \times \sin \beta$
The attachment point of the aerodynamic model on the sting $\left(P_{M}\right)$ can be calculated in the same way, using the calculated position of $P_{2}$ as reference:
$X_{P_{M}}=X_{P_{2}}-\mathrm{S}_{\mathrm{T}} \times \cos \beta$
$Y_{P_{M}}=Y_{P_{2}}-\mathrm{S}_{\mathrm{T}} \times \sin \beta$
Where $\mathrm{S}_{\mathrm{T}}$ is the total length of the sting's tip, $182,109 \mathrm{~mm}$.
Equation (26) and Eq. (27) does not take into account the sting's tip angle ( $\theta$ ), defined during the fixation of the aerodynamic model on the sting. Considering this user defined constant, Eq. (26) and Eq. (27) can be expressed as:
$X_{P_{M}}=X_{P_{2}}-\mathrm{S}_{\mathrm{T}} \times \cos (\beta-\theta)$
$Y_{P_{M}}=Y_{P_{2}}-\mathrm{S}_{\mathrm{T}} \times \sin (\beta-\theta)$
The angle of attack $\alpha$ of $P_{M}$ is the sting's tip angle ( $\theta$ ) minus the $\beta$ angle:
$\alpha_{P_{M}}=\theta-\beta$
Equation (28), Eq. (29) and Eq. (30) defines the position and attitude of the aerodynamic model attached to the CTS, using as reference a sequence of nodes, all the way back to the position of the $X$ torpedo. As expressed in Eq. (2), Eq. (3) and Eq. (4), there are only four variables that defines the position and attitude of the aerodynamic model: $D_{X_{T}}, D_{Y_{F T}}, D_{Y_{M T}}$ and $\theta$. Joining and simplifying all the equations developed until now, it is possible to express the position and attitude of the aerodynamic model, as function of these four variables, as follows:

$$
\begin{align*}
X_{P_{M}}= & D_{X_{T}}-153,25+\frac{\sqrt{300^{2}-\left(D_{Y_{M T}}-D_{Y_{F T}}\right)^{2}}}{2}-\mathrm{S}_{\mathrm{X}} \times \cos \left(\sin ^{-1}\left(\frac{D_{Y_{M T}}-D_{Y_{F T}}}{300}\right)\right)  \tag{31}\\
& +\mathrm{S}_{\mathrm{Y}} \times \frac{D_{Y_{M T}}-D_{Y_{F T}}}{300}-\mathrm{S}_{\mathrm{T}} \times \cos \left(\sin ^{-1}\left(\frac{D_{Y_{M T}}-D_{Y_{F T}}}{300}\right)-\theta\right)
\end{align*}
$$

$$
\begin{align*}
Y_{P_{M}}= & D_{Y_{F T}}-128+\frac{D_{Y_{M T}}-D_{Y_{F T}}}{2}-\mathrm{S}_{\mathrm{Y}} \times \cos \left(\sin ^{-1}\left(\frac{D_{Y_{M T}}-D_{Y_{F T}}}{300}\right)\right)-\mathrm{S}_{\mathrm{X}} \times \frac{D_{Y_{M T}}-D_{Y_{F T}}}{300}  \tag{32}\\
& -\mathrm{S}_{\mathrm{T}} \times \sin \left(\sin ^{-1}\left(\frac{D_{Y_{M T}}-D_{Y_{F T}}}{300}\right)-\theta\right) \\
\alpha_{P_{M}}= & \theta-\sin ^{-1}\left(\frac{D_{Y_{M T}}-D_{Y_{F T}}}{300}\right) \tag{33}
\end{align*}
$$

Equation (31) and Eq. (32) calculates the position of the aerodynamic model, having as origin of the coordinate system the origin of the X mover's plane $\left(P_{0.1}\right)$. For the case of no displacement in all of the torpedoes, the result of Eq. (31) and Eq. (32) would be $-484,5 \mathrm{~mm}$ and -460 mm respectively. Since this would be the initial position of the aerodynamic model, it's better to define this point as the origin of the coordinate system, turning Eq. (31) and Eq. (32) into:

$$
\begin{align*}
X_{P_{M}}= & D_{X_{T}}+331,25+\frac{\sqrt{300^{2}-\left(D_{Y_{M T}}-D_{Y_{F T}}\right)^{2}}}{2}-\mathrm{S}_{\mathrm{X}} \times \cos \left(\sin ^{-1}\left(\frac{D_{Y_{M T}}-D_{Y_{F T}}}{300}\right)\right)  \tag{34}\\
& +\mathrm{S}_{\mathrm{Y}} \times \frac{D_{Y_{M T}}-D_{Y_{F T}}}{300}-\mathrm{S}_{\mathrm{T}} \times \cos \left(\sin ^{-1}\left(\frac{D_{Y_{M T}}-D_{Y_{F T}}}{300}\right)-\theta\right) \\
Y_{P_{M}}= & D_{Y_{F T}}+332+\frac{D_{Y_{M T}}-D_{Y_{F T}}}{2}-\mathrm{S}_{\mathrm{Y}} \times \cos \left(\sin ^{-1}\left(\frac{D_{Y_{M T}}-D_{Y_{F T}}}{300}\right)\right)-\mathrm{S}_{\mathrm{X}} \times \frac{D_{Y_{M T}}-D_{Y_{F T}}}{300}  \tag{35}\\
& -\mathrm{S}_{\mathrm{T}} \times \sin \left(\sin ^{-1}\left(\frac{D_{Y_{M T}}-D_{Y_{F T}}}{300}\right)-\theta\right)
\end{align*}
$$

In practice, the known informations of the experiment are the desired position and attitude of the aerodynamic model and the sting's tip angle, defined during the fixation of the model on the sting. With this in mind, the displacement of the torpedoes ( $D_{X_{T}}, D_{Y_{F T}}$ and $D_{Y_{M T}}$ ) must be calculated in order to achieve de desired conditions of the aerodynamic model. After some mathematical treatment on Eq. (34), Eq. (35) and Eq. (33), these three variables may be isolated, resulting in the equations below:
$D_{Y_{F T}}=Y_{P_{M}}+\left(\mathrm{S}_{\mathrm{X}}-150\right) \times \sin \left(\theta-\alpha_{P_{M}}\right)+\mathrm{S}_{\mathrm{Y}} \times \cos \left(\theta-\alpha_{P_{M}}\right)+\mathrm{S}_{\mathrm{T}} \times \sin \left(-\alpha_{P_{M}}\right)-332$
$D_{Y_{M T}}=Y_{P_{M}}+\left(\mathrm{S}_{\mathrm{X}}+150\right) \times \sin \left(\theta-\alpha_{P_{M}}\right)+\mathrm{S}_{\mathrm{Y}} \times \cos \left(\theta-\alpha_{P_{M}}\right)+\mathrm{S}_{\mathrm{T}} \times \sin \left(-\alpha_{P_{M}}\right)-332$
$D_{X_{T}}=X_{P_{M}}+\mathrm{S}_{\mathrm{X}} \times \cos \left(\theta-\alpha_{P_{M}}\right)-\mathrm{S}_{\mathrm{Y}} \times \sin \left(\theta-\alpha_{P_{M}}\right)+\mathrm{S}_{\mathrm{T}} \times \cos \left(-\alpha_{P_{M}}\right)-331,25$

## 4. CONCLUSIONS

The mathematical modeling of the Pilot Transonic Wind Tunnel's Captive Trajectory System showed that the device is capable of positioning any aerodynamic models attached to it in the desired position and attitude with a precision of 0.005 mm and $0,001^{\circ}$. This precision is the "ideal" precision, considering that all mechanical pieces are absolutely rigid and no deflection occurs. The equations that were obtained will be used in the software created in the LabView 2009 platform in order to control the step motors that controls the independent movers.

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