DIMENSIONLESS DYNAMIC MODEL FOR THE SIMULATION OF MICROALGAE BIOMASS PRODUCTION IN COMPACT PHOTOBIOREACTORS

Emerson Dilay, emerson197212@hotmail.com
André Bellin Mariano, andrebmariano@gmail.com
Núcleo de Pesquisa e Desenvolvimento em Energia Auto-Sustentável, Universidade Federal do Paraná, CP 19011
Curitiba, PR, Brazil, CEP: 81531-980

Juan Carlos Ordonez, ordonez@caps.fsu.edu
Departament of Mechanical Engineering and Center for Advanced Power Systems, Florida State University, Tallahassee, Florida, 32310, USA

José Viriato Coelho Vargas, vargasjvcv@gmail.com
Núcleo de Pesquisa e Desenvolvimento em Energia Auto-Sustentável, Universidade Federal do Paraná, CP 19011
Curitiba, PR, Brazil, CEP: 81531-980

Abstract. *Photobioreactors simulation is currently the subject of scientific research in many countries due to interest in biofuel production. One challenge in the assessment of photobioreactors behavior is the lack of information about algae growth properties. In this paper, a dimensionless mathematical model is introduced for assessing the transient microalgae growth, O_2, CO_2 concentrations and temperature as function of light intensity. An equation to calculate the growth rate independently of algae specie is introduced. Photobioreactor geometry is considered as well. The equipment was discretized in space by the volume element method, VEM. Balances of energy and species together with thermodynamics, heat transfer and chemistry empirical and theoretical correlations are applied to each volume element. The resulting system of ordinary differential equations with respect to time is capable of delivering temperatures and concentrations as functions of space and time, even with a coarse mesh. The numerical results are capable of predicting the transient and steady state photobioreactor biomass production with low computational time. The results are dimensionless and they represents the behavior of any equipment build with functional and physical characteristics similar to the system analyzed by the model. As a result, the model is expected to be a useful tool for simulation, design, and optimization of photobioreactors.*

Keywords*: Numerical Simulation, Microalgae, Volume Element Method, Photobioreactor, Bioprocess*

1. INTRODUCTION

The present study attempts to mathematically and computationally model a physical system capable of cultivating microalgae. The model is based on a synthesis of mathematics, biology, thermodynamics, heat and mass transfer and chemical kinetics. In this form, biological and chemical information conceptually feed the mathematical model whose purpose is to describe the temporal evolution of the cultivation system of microalgae (a compact photobioreactor) from known initial data. The phenomena were described using the volume element method (Vargas *et al.*, 2001), which discretizes the domain (in this case the photobioreactor) into small control volumes. The result of this work was a computational tool to evaluate in a normalized way the influence of design and operating conditions in the productivity of algal biomass. The results are dimensionless and they represent the behavior of equipment build with functional and physical characteristics similar to the system analyzed by the model. A mathematical equation is introduced to represent the kinetics of growth independent of the algae specie.

2. MATHEMATICAL MODEL

A schematic representation of a compact photobioreactor is illustrated in Fig. 1. In this model, the Law of the Conservation of Energy is used to calculate the temperature of the walls of the photobioreactor tubes and the temperature of the culture medium. The Law of the Conservation of Species (Continuity) is used to calculate the concentration of microalgae, CO_2 and O_2 as a function of time in the photobioreactor. The photobioreactor was discretized in space using the Volume Element Method, VEM, proposed by Vargas *et al.* (2001). This procedure resulted in a system of ordinary differential equations which was solved numerically using the classical Runge-Kutta method with fourth order accuracy and adaptive stepping.

In the context of systems engineering, the overall system is divided in three integrated subsystems: (I) solar collector; (II) degasser column and (III) pump illustrated in Fig. 1. The subsystem pump is responsible only for generating a pressure difference sufficient to create flow in the system, and it will not be modeled in this work. Therefore, the mathematical model is developed according to that division with emphasis on the solar collector.

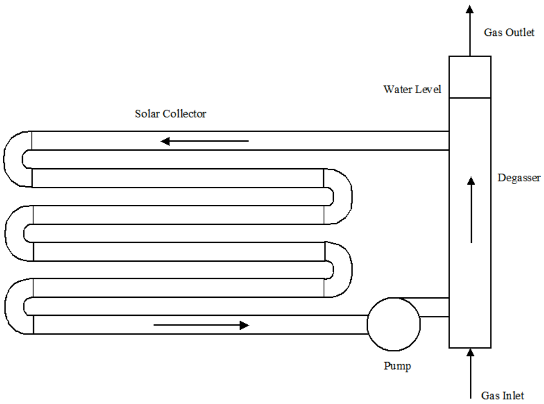


Figure 1. Schematic representation of a compact photobioreactor

2.1 Dimensionless variables

In order to make the results general, for the system configuration proposed, dimensionless variables are needed. An important constraint in biomass production is the total system volume defined as $V_T = L \cdot W \cdot H$ where L represents the length, the W width and H the height of the photobioreactor. Then, it makes sense to put all the system dimensions in function of the total system volume. This way, dimensionless tube length, diameter, thickness, volume and area are defined by:

$$L_{ref} = V_T^{1/3}, \tilde{d} = \frac{d}{V_T^{1/3}}, \tilde{e} = \frac{e}{V_T^{1/3}}, \tilde{V} = \frac{V_i}{V_T}, \tilde{A} = \frac{A}{V_T^{2/3}} \tag{1}$$

Dimensionless reference mass flow rate, mass, specific heats, density, time, mass flow rate and temperatures are defined by:

$$\dot{m}_{ref} = \frac{\dot{W}}{\Delta p}, \ \tilde{m}_i = \frac{m_i}{\rho_w V_T}, \ \tilde{c}_i = \frac{c_i}{c_w}, \tilde{\rho} = \frac{\rho}{\rho_w} \ \tilde{t} = \frac{t}{\dot{m}_{ref}/(\rho_w V_T)}, \ \dot{M} = \frac{\dot{m}_i}{\dot{m}_{ref}}, \ \tau_i = \frac{T_i}{T_\infty} \tag{2}$$

where $\dot{W}$ is the pump work rate, Δp is the pump pressure variation, ρ_w is the water specific mass, c_w is the water specific heat and T_∞ is the atmospheric temperature.

Dimensionless heat capacity rates, transparent tube absorbed radiation, emitted radiation and microalgae growth rate are defined by:

$$H_i = \frac{h_i A_i}{\dot{m}_{ref} c_w}, \ \tilde{I}_i = \frac{\alpha I_i A_i}{\dot{m}_{ref} c_w}, \ \tilde{\varepsilon}_i = \frac{\varepsilon_i \sigma A_i}{\dot{m}_{ref} c_w}, \ \tilde{\psi}_i = \frac{\psi_i}{\dot{m}_{ref}/(\rho_w V_T)} \tag{3}$$

where h_i is the convection coefficient, A_i is the tube surface area, I_i is the light irradiation, α is the absorptivity, ε is the emissivity and ψ is the microalgae growth rate.

2.2 Solar collector

The subsystem type I, the solar collector, consists of a bundle of transparent tubes. The growth of microalgae occurs in this bundle of tubes. Light powers the photosynthesis process and consequently increases biomass. Apart from photosynthetically active radiation (PAR), the tubes receive heat via solar radiation and also thermal interaction with the atmospheric air. The photosynthetically active radiation (PAR) is electromagnetic radiation contained in the range 400-700 µm in wavelength, which is directly responsible for the process of photosynthesis.

The bundle of tubes was divided into smaller elements by creating a volume element model, VEM (Vargas *et al.,* 2001). Each element is formed by the tube wall and the contents of the tube. The principles of thermodynamics, heat and mass transfer and empirical relationships were applied to each part of the generic volume element. The objective was to obtain a system of ordinary differential equations (ODE) that capture the system behavior (Dilay *et al.,* 2010).

The fluid inside the tubes consists of water, O_2, CO_2 and microalgae. Applying the law of the conservation of species in the volume element gives Eq. (4). This equation corresponds to the increase in mass of species $i = 1$, microalgae. Where: Y_1 is the concentration of the specie $i = 1$, microalgae,

$$\tilde{V}^{(j)}\frac{dY_i^{(j)}}{d\tilde{t}} = \frac{\dot{M}}{\tilde{\rho}}\left(Y_i^{(j-1)} - Y_i^{(j)}\right) + \tilde{V}^{(j)}Y_i^{(j)}\tilde{\psi} \quad (i=1) \tag{4}$$

The CO_2 (i=2) and O_2 (i=3) are calculated using the law of the conservation of species in form of Eq. (5). The variable R_i represents the rate of generation of specie i in relation to specie 1 (microalgae).

$$\tilde{V}^{(j)}\frac{dY_i^{(j)}}{d\tilde{t}} = \frac{\dot{M}}{\tilde{\rho}}\left(Y_i^{(j-1)} - Y_i^{(j)}\right) + R_i\tilde{V}^{(j)}Y_1^{(j)}\tilde{\psi} \quad (i=2,\, 3) \tag{5}$$

Microalgal growth rate is influenced by temperature and solar radiation. Each species has different rates of irradiation and appropriate temperatures. Inadequate lighting and temperatures inhibit growth and may even lead to cell death. Molina Grima *et al.* (1994), Richmod (1992), Sanchez *et al.* (2008), Seronotti *et al.* (2004) and Subramanian and Thajuddin (2005) proposed equations to attempt to correlate the specific growth rate μ, as a function of temperature T and the average light intensity inside the tubes, I_{avg}. However, these correlations need experimental parameters not always available. In this work, the goal is to assess the photobioreactor behavior independently of the algae type. Therefore makes sense the use of equations in a general format. Correlations with the same trends of those available in the literature but independent of algae specie are proposed here.

Equation (6) represents the trend of algae growth, where $\tilde{\mu}_{\max}$ is the maximum growth rate. This rate is evaluated like a quadratric equation. The best growth rate is reached when medium temperature is τ_{opt}. The instantaneous temperature in each element is represented by τ. The average light intensity inside the tubes $\tilde{I}_{avg}^{j}$ is a function of light intensity, that reaches the tube surface, $\tilde{I}_j$, tube diameter, D, algae concentration and a coefficient k that are implicit in $\tilde{\chi}$. The light intensity, $\tilde{I}_j$, is computed using a ray tracing algorithm (Dilay *et al*, 2010).

$$\tilde{\psi} = \tilde{\mu}_{\max}\left(1 - e^{\tilde{I}_{avg}^{j}}\right),\ \tilde{\mu}_{\max} = 1 - \left(\tau - \tau_{opt}\right)^2,\ \tilde{I}_{avg}^{j} = \frac{\tilde{I}_j}{\tilde{\chi}}\left(1 - e^{-\tilde{\chi}}\right),\ \ \tilde{\chi}_j = D \cdot Y_j \cdot k$$
(6)

Applying the law of the conservation of energy in each of the volume elements gives Eq. (7). The energy transfer interactions that occur are the heat flow from the solid wall of the tube, and the energy transported along with the flow of fluid between elements.

$$\frac{d\tau}{d\tilde{t}} = \frac{1}{\tilde{m}^{(j)}}\left\{\frac{H}{\tilde{c}}\left(\tau^{(j)} - \tau_p^{(j)}\right) + \dot{M}\left(\tau^{(j-1)} - \tau^{(j)}\right)\right\} \tag{7}$$

2.2.1. *Side wall of the volume element (tube wall)*

At the solid wall there is no mass transport. The tube wall experiences three heat transfer interactions: radiation from the incident light, heat transfer by convection with the outside air and heat transfer by convection with the liquid medium contained in the tube. The application of energy balance on the wall of the transparent tubes leads to Eq. (8). The effect of solar radiation is represented by $\dot{Q}_{rad}^{(j)}$, Eq. (9).

$$\frac{d\tau}{d\tilde{t}} = \frac{1}{\tilde{m}^{(j)}}\left\{\frac{H}{\tilde{c}}\left(\tau^{(j)} - \tau_p^{(j)}\right) + \frac{H}{\tilde{c}}\left(1 - \tau_p^{\ (j)}\right) + \frac{1}{\tilde{c}}\tilde{Q}_{rad}^{(j)}\right\} \tag{8}$$

$$\tilde{Q}_{rad}^{(j)} = \tilde{I}^{(j)} - \tilde{\varepsilon}^{(j)}\left(\tau_p^{(j)} - 1\right) \tag{9}$$

2.3 Degasser column

Equations (4), (5), (6), (7), (8) and (9) are used to describe the phenomena in the degasser column. The O_2 generated by photosynthesis is eliminated up to the limit of water saturation and new CO_2 are injected until reach saturation level.

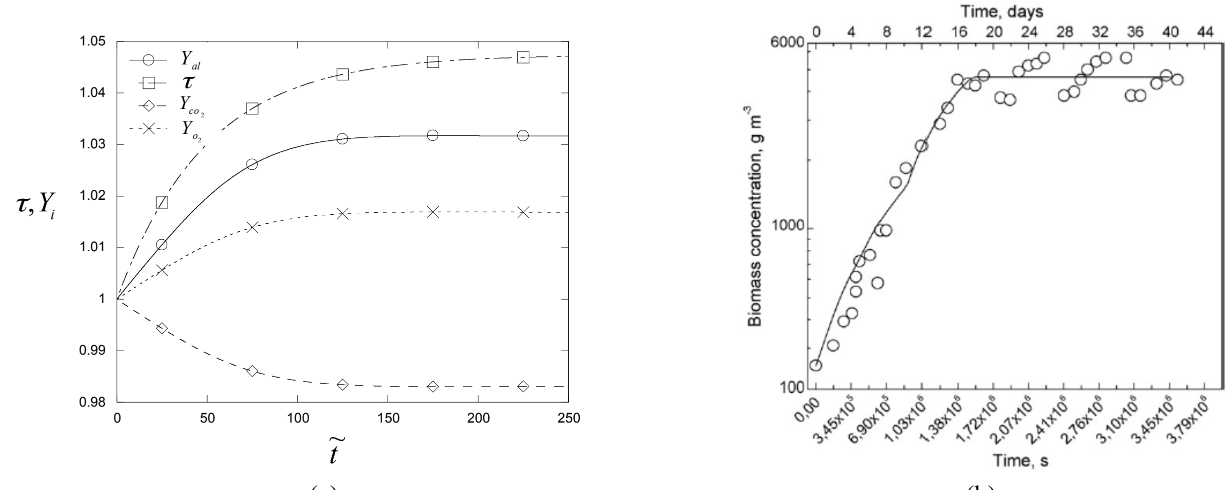
4. NUMERICAL METHOD

The system of ordinary differential equations (ODE) represented by Equations (4), (5), (6), (7), (8) and (9) resulting from the mathematical model was solved using the Runge-Kutta method with fourth order accuracy and adaptive stepping. This method was chosen to enable the evaluation of the transient nature of the equipment. The method was programmed using FORTRAN.

5. RESULTS AND DISCUSSION

The variation of algal concentration versus time can be seen in Fig. 2(a). The algae concentration curve follows a trend compatible with experimental data available in the literature as showed in Fig. 2(b) (Concas *et al.* 2010). In the beginning of the process, the algal concentration is low and this permit a high light flux inside the cultivation medium. Then as the photosynthesis rate is proportional to light flux, the algal growth rate is high. As the concentration increases, the light flux decreases and, consequently, the growth rate decreases. At this point the transient is finished and the PBR reaches the steady state. The curves of O_2 concentration growths and the curve of CO_2 decrease as a result of photosynthesis process.

In the initial moment the PBR temperature is in equilibrium with the ambient. The thermal radiation process heats up the tubes and production medium. As the temperature heats up, the convection process between the PBR and the external air increase. At same point the radiation process is balanced with the convection process and, then, the temperature is stabilized. The algal growth model is, also, function of temperature. Then the temperature feeds back the algal growth rate.



(a) (b)

Figure 2. (a) Dimensionless temperature, O_2, CO_2, algae biomass concentration in function of dimensionless time and (b) Algal biomass concentration in function of time in a coil photobioreactor (Concas *et al.* 2010)

6. CONCLUSIONS

The evaluation of the biomass of microalgae in photobioreactors according to the geometry through dimensionless mathematical modeling as demonstrated in this article presents a fundamental tool in developing such equipment.

Dimensionless groups were identified. The numerical results presented are normalized. They are valid to any geometric configuration with functional and physical characteristics similar to the system analyzed by the model. As the time needed to evaluate a configuration takes several minutes, the dimensionless model is an away to save computational time when is necessary to compare different projects. This way, one computational solution represents not one, but several similar projects.

Future studies should focus on optimizing the placement of tubes, minimizing the system size, evaluation of concentrations of N_2, P, and others species present in the production medium, and inclusion of photo-inhibition effects.

7. ACKNOWLEDGEMENTS

The authors gratefully acknowledge support from CNPq, NILKO and UFPR.

8. REFERENCES

Dilay, E., Ribeiro, R.L.L., Pulliam, R., Ordonez, J.C., Vargas, J.V.C., 2010, "The transient response of a compact photobioreactor for microalgae cultivation", 13rd Brazilian Congress of Thermal Sciences and Engineering, Uberlandia, MG, Brazil.

Concas, A., Pisu, M., Cao, G., 2010, "Novel simulation model of the solar collector of BIOCOIL photobioreactors for CO_2 sequestration with microalgae", Chemical Engineering Journal, vol. 157, p. 297-303.

Molina Grima, E., García Camacho, F., Sánchez Pérez, J.A., Fernández Sevilla, J., Acién Fernández, F.G. and Contreras Gómez, A., 1994, "A mathematical model of microalgal growth in light limited chemostat cultures", Journal Chem. Technol. Biotechnol., vol.61, p.167-173.

Richmond, A., 1992, "Mass culture of cyanobacterium", MAN, N.H. & CARR, N.G. (eds). Photosynthetic Prokaryotes. Plenum Press, New York.

Sánchez, J.F., Servilla, J.M.F., Acién, F.G., Ceron, M.C., Parra J.P., Grima E.M., 2008, "Biomass and lutein productivity of Scenedesmus almeriensis: influence of irradiance, dilution rate and temperature", Applied Microbiol. Biotechnol., v. 79, p. 719–729.

Serenotti, F., Crespi, B.A. and Torres, L.G., 2004, "Contribuição à modelagem da produção de Spirulina maxima em fotobioreatores" Revista Univ. Rural, vol.23, p.8–17.

Subramanian, G. and Thajuddin, N, 2005, "Cyanobacterial biodiversity and potential applications in biotechnology", Current Science, vol.89, no.1, p.47-57.

Vargas, J.V.C., Stanescu, G., Florea, R. and Campos, M.C., 2001, "A numerical model to predict the thermal and psychrometric response of electronic packages", ASME Journal of Electronic Packaging, vol.123, no.3, p.200-210.

Vargas, J.V.C., Ordonez, J.C., Dilay, E., Parise, J.A.R., 2009, "Modeling, simulation and optmization of a solar collector driven water heating and absorption cooling plant", Solar Energy, no. 83, p. 1232-1244.

9. RESPONSIBILITY NOTICE

The authors are the only responsible for the printed material included in this paper.