CORRELATION FOR THE COOLING PROCESS OF VERTICAL STORAGE TANKS UNDER NATURAL CONVECTION FOR HIGH PRANDTL NUMBER

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Abstract. A new dimensionless correlation for the Nusseltnumber associated to cooling processes in vertical storage tanks for high Prandtl number is presented. The correlation depends on the average temperature of the tank and is a function of Rayleigh and Prandtl numbers, aspect ratio and heat transfer coefficient. It can be used to correct the temperature profiles obtained through a global method. A Finite Volume method is employed using a transient twodimensional model in cylindrical coordinates assuming laminar flow. Thermal losses to the external environment are considered for the all walls (top, bottom and sidewalls) of the tank. Several cases of cooling were simulated, varying the aspect ratio, the cooling rate and the volume of the tank. The temperature fields obtained from numerical simulation are used to obtain the internal coefficient of heat transfer in various configurations. Later, the coefficient of heat transfer and other variables are used in the application of Buckingham theorem to determine the dimensionless groups that represent the cooling process. The correlation is checked with numerical simulations results with good agreement.

Keywords: Natural Convection, Storage Tanks, Buckingham, Correlation.

1. NOMENCLATURE

- area [m²], aspect ratio [-] А
- modified aspect ratio [-] Ac
- specific heat at constant pressure $[J kg^{-1} K^{-1}]$ c_p C
- coefficient [-]
- D inner diameter of the tank [m]
- e insulation [m]
- acceleration of gravity $[m s^{-2}]$ g
- heat transfer coefficient [W m⁻² K⁻¹] h
- Η internal tank height [m]
- characteristic dimension of the tank [m] L
- number of volume elements [-] n
- Nu Nusselt number [-]
- pressure [Pa] р
- Pr Prandtl number [-]
- heat flux [W m⁻²] q
- R total radius [m]
- Ra_H Rayleigh number [-]
- modified Rayleigh number [-] Ra

t time [s] T temperature [K] *z*-direction velocity $[m s^{-1}]$ и \overline{U} overall heat transf. coefficient [W m⁻² K⁻¹] \hat{U} dimensionless overall heat transf. coef. [-] v *r*-direction velocity $[m s^{-1}]$ coordinate [m] r, z

Greek symbols

- thermal diffusivity [m² s⁻¹] α
- thermal expansion coefficient [K⁻¹] β
- k thermal conductivity [W m⁻¹ K⁻¹]
- dynamic viscosity [Pa s] μ
- kinematic viscosity $[m^2 s^{-1}]$ υ
- specific mass [kg m⁻³] ρ
- θ dimensionless temperature [-]
- volume [m⁻³] Ω

2. INTRODUCTION

Many areas of engineering use thermal storage to optimize performance, such as in the petrochemical, food industries and in liquid heating/cooling systems. This effective optimization requires an extensive knowledge of the thermal and dynamic behaviour of the liquid within the tanks. Alternately, these tanks are submitted to the heat transfer processes by natural and mixed convection. Regarding the natural convection process, it is largely covered by theoretical, experimental and numerical approaches.

Under actual operating conditions, some heat is transferred through all the walls of the thermal reservoir. However, in cooling processes, some authors (Kwak et al., 1998; Lin and Armfield, 1999; Oliveski et al. 2005) often use an adiabatic boundary condition hypothesis at the top and/or bottom of the cylindrical tanks. Regarding the sidewalls boundary conditions, studies (Kwak et al., 1998; Oliveski et al., 2005) can be found using prescribed temperature or heat flow. For cooling processes in vertical cylindrical tanks, there are few works (Reindl, Beckman and Mitchell, 1992; Cotter and Charles, 1993; Oliveski et al., 2003a-b, Rodríguez et al., 2009) using heat flux variable with the time and space on all the walls (side, top and bottom).

Another important observation that should be noted is about real-time analysis. In Reindl and Beckman (1992) the average Nusselt number was presented during a maximum time of 1000s for several Rayleigh numbers, while in Rodríguez et al. (2009), the numerical simulations were performed for 15 h, and a correlation for the Nusselt number was presented. Cotter and Charles (1992), for example, presented the time dependence of the Nusselt number for several oil viscosity values. In this work, the average temperature of the liquid, the wall temperature and the ambient temperature were considered to obtain the average Nusselt number. Cooling of oil tanks was also numerically and experimentally studied by Oliveski et al. (2005).

There are not many correlations in the literature on typical thermal reservoir cooling processes. Correlations of Oliveski et al. (2003b) and Rodríguez et al. (2009) are based on numerical simulations, applied to the cooling of vertical cylindrical tanks by natural convection. The first one is restricted to tanks of a fixed volume and was not presented in a fully dimensionless form. On the other hand, the second work is fully dimensionless but is appropriate only for certain cooling times and initial temperatures of the thermal reservoir.

The present work proposes a new correlation for Nusselt number in natural convection in a vertical cylindrical tank submitted to an unsteady cooling process. The results of the cooling process are obtained by numerical simulation through Finite Volume method. All tank walls (top, bottom and sidewalls) are considered to be subjected to heat transfer. After experimental validation of the numerical model, cooling of the tanks is simulated under forty different configurations, obtaining correlations of the Nusselt number as a function of thermal losses to the environment, aspect ratio and tank volume. Comparing to results available in the literature, we offer a temperature-dependent correlation which can be applied to correct temperature profiles found through global methods.

3. PROBLEM DEFINITION

This work is focused on the natural convection process inside vertical cylindrical tanks, as shown in Fig. 1.



Figure 1. (a) Storage tank scheme. (b) Computational domain.

The external convection heat transfer coefficient h_e is set to 10 W/m² K, in accordance with laboratory and common practical situations (Cotter and Charles, 1993, and Adihkari et al., 1995): he = 11 and 9.5 W/m2K, respectively. Despite the differences in these values, its effect is much attenuated when combined with the overall heat transfer coefficient (\overline{U} , Eq. 1), because the more significant value corresponds to the insulation resistance.

Thirty-two different configurations are used, relating the aspect ratio, cooling rate and tank volume. For each volume (0.1 and 0.2 m³), four aspect ratios are used (1.0, 1.5, 2.0 and 2.5). In addition, for each of these cases four different insulation thicknesses (*e*) are tested, resulting in the following overall heat transfer coefficient (Eq. 1): 1.0, 2.0, 3.0 and 5.0 W m⁻² K⁻¹.

$$\overline{U} = \frac{1}{\frac{e}{k} + \frac{1}{h_e}} \tag{1}$$

The characterization of the flow regime in the natural convection processes can be achieved through the Rayleigh number, based on the difference temperature or heat flow through the walls. When the Rayleigh number is based on a temperature difference, the conventional procedure would be to take the working fluid and wall temperatures, and these temperatures should be associated with the direction in which the process has the highest thermal gradient. However, in this case, the choice of the temperature difference is not an obvious task due to the thermal stratification process and its degradation inside the tank. In a numerical simulation in which the stratified temperature field is the initial condition, it must be considered the temperature difference between the top and bottom of the tank, because this is the direction of the largest thermal gradient. In this case, the characteristic length would be the distance between these two positions (top and bottom), which corresponds to the height of the tank. Moreover, in a simulation where the initial temperature

field is uniform, as is the case of this work, there is no temperature difference between the fluid and walls. There is only one temperature difference between the working fluid and the external environment. In this case the thermal process should be characterized by the modified Rayleigh number (Ra^*), which is based on heat flow (q'') between the external environment and the computational domain, as indicated by Eqs. (1) and (2), respectively.

$$Ra = \frac{g\beta q^{*}L_{c}^{4}}{\kappa\alpha\nu}$$
(2)

$$q'' = \overline{U} \left(\overline{T}_w - T_{amb} \right) \tag{3}$$

where g is the acceleration of gravity, β is the thermal expansion coefficient, α is the thermal diffusivity, v is the kinematic viscosity, T_w is the average temperature of the internal face of the tank walls, T_{amb} is the ambient temperature and L_c is the characteristic dimension of the tank, defined as:

$$Lc = \Omega / A_{Tot} \tag{4}$$

where Ω is the volume and A_{Tot} is the total area of the internal face of the tank.

4. MATHEMATICAL MODEL

For the cases studied in this work, the most critical are those corresponding to $\overline{U} = 5.0 \text{ Wm}^{-2}\text{K}^{-1}$. For this cooling condition and aspect ratios between 1.0 and 2.5, the Rayleigh number, based on the height of the tank, ranged from 5.4×10^{11} to 6.0×10^{12} , for the volume of 0.1 m³, and from 1.4×10^{12} to 1.6×10^{13} , for the volume of 0.2 m³. Some authors (Oliveski et al., 2003a; Oliveski et al.; 2003b; Rodriguez et al., 2009) have conducted their numerical simulations, with similar geometric and thermal conditions, assuming the hypothesis of laminar flow. These authors present a careful validation or numerical verification, thereby ensuring that the laminar flow approach was adequate. In addition, Papanicolaou and Belessiotis (2002) in a study with main aspect ratio (L/D) =1 and Prandtl number ranging from 2.965 to 5.388, found that laminar regime can be obtained for Rayleigh numbers (based on the tank length) up to Ra_H=10¹³ while turbulent flow should be expected for Ra_H $\leq 5 \times 10^{13}$. Thus, taking into account the cited works and the range parameters of this study, the laminar flow can be safely assumed. In addition, it is assumed that the flow in the vertical tank is axisymmetric. Thus, a two-dimensional laminar model is used. For these conditions, the transient natural convection inside a storage tank is governed by the continuity, momentum and energy transport equations (5-8), respectively.

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u)}{\partial z} + \frac{1}{r} \frac{\partial (\rho r v)}{\partial r} = 0$$
(5)

$$\frac{\partial(\rho u)}{\partial t} + \frac{\partial(\rho uu)}{\partial z} + \frac{\partial(\rho vu)}{\partial r} = -\frac{\partial p}{\partial z} + \mu \left(\frac{\partial^2 u}{\partial z^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial r^2}\right) + \rho g$$
(6)

$$\frac{\partial(\rho v)}{\partial t} + \frac{\partial(\rho vv)}{\partial z} + \frac{\partial(\rho vv)}{\partial r} = -\frac{\partial p}{\partial r} + \mu \left(\frac{\partial^2 v}{\partial z^2} + \frac{1}{r}\frac{\partial v}{\partial r} + \frac{\partial^2 v}{\partial r^2} - \frac{v}{r^2}\right)$$
(7)

$$c_{p}\left[\frac{\partial(\rho T)}{\partial t} + \frac{\partial(\rho vT)}{\partial z} + \frac{\partial(\rho vT)}{\partial r}\right] = k\left[\frac{\partial^{2} T}{\partial z^{2}} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^{2} T}{\partial r^{2}}\right]$$
(8)

where ρ is the density, u and v are the axial and radial velocity, T is the temperature, μ is the dynamic viscosity, k is the thermal conductivity, cp is the specific heat at constant pressure, p is the pressure. In the above equations, ρ is considered as an unknown in all terms.

An initial condition of zero-velocity field and isothermal temperature ($T_o = 70$ °C) field are used. For the momentum equation, impermeability and no-slip conditions are adopted on every wall, and zero shear stress at the symmetry line. For the energy equation, heat flux is used as boundary condition at all tank walls (side, bottom and top).

5. NUMERICAL APPROACH

The numerical solution is obtained by integrating the Eqs. (5-8) in Volume Finite method, as described in Patankar (1980) and Ferziger and Peric (1997). Pressure-velocity coupling is obtained by the SIMPLEC method. The Power Law scheme is used for the interpolation in the control volume faces. The convergence criterion requires the normalized mass flow residue for any grid volume to be lower or equal to 10-4 kg/s. A correction block is used to calculate the velocity and temperature components. The numerical grid (z_i , r_i) for the spatial coordinates (z, r) is generated as Davidson (1990) and presented in Eqs. (9) and (10).

$$z_{i} = L\left\{-0.5 \tanh\left[\alpha_{z}\left(2\frac{i-2}{n-2}-1\right)\right]/\tanh(-\alpha_{z})+0.5\right\},\tag{9}$$

with $2 \le i \le n$, where n is the number of volumes and α_z the grid refinement coefficient in the axial direction. In the radial direction, grid refinement is needed only at the sidewall and is obtained by

$$r_{j} = R\left\{ \tanh\left[\alpha_{r}\left(\frac{j-2}{m-2}\right)\right] / \tanh\left(-\alpha_{r}\right) \right\},\tag{10}$$

with $2 \le i \le m$, where m is the number of volumes and α_r the grid refinement coefficient in the radial direction. In this work, $\alpha_r = 3.0$ and $\alpha z = 2.7$ are used. A schematic of the vertical cylindrical storage tank is shown in Fig. 1(a).

During the simulations, the oil properties are considered variables. The changes in specific heat, dynamic viscosity, Prandtl number and density can be observed in Fig. 2(a-b), respectively. These figures show through symbols the values presented by Fox and McDonald (2004), curve fitting and adjustment equation which provides the value of each property according to the mean temperature. As can see in these figures, the physical properties of the oil vary widely with temperature. So, for each time step, it is calculated the oil average temperature which is also used in the equations of Figs. 2(a-d) to obtain the property values of the oil.



Figure 2. Oil properties: (a) specific heat; (b) dynamic viscosity; (c) Prandtl number and (d) density.

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The problem studied starts with a uniform temperature field, and is subjected to a cooling process in all tank walls. Cases with more intense cooling rates ($\overline{U} = 5.0 \text{ W m}^2 \text{K}^{-1}$) are submitted to time step and grid dependence analysis. Grid dependence analysis compares meshes of 40 x 60, 60 x 90 and 90 x 120 volumes, in the radial and axial directions, respectively. In addition, time steps of 0.5, 1.0 and 2.0 s are tested. Results for these analyses are presented in Fig. 3(a) as temperature profiles at z/H = 0.5, after temperature profiles at z/H = 0.5, after one hour of cooling, for the three grids tested. This figure shows that the different grid (40 x 60, 1 60 x 90 and 90 x 120) reproduces practically the same results, so a grid of 160 x 90 volumes is used in all simulated cases. The influence of the time step can be seen in Fig. 3(b), which shows transient average temperature results for three time steps (0.5, 1.0 and 2.0 s). As seen in the figure, the results for different time steps are essentially unchanged. Therefore it is decided to simulate all cases, including those with less intense cooling with a 1.0 s time step.



Figure 3. Grid and time step dependence: (a) Temperature profiles, in z / H = 0.5, obtained for different computational meshes; (b) Transient average temperature for different time steps.

As the cooling processes are applied in all walls (top, base and lateral) of the tank, thermal and dynamic boundary layers develop in these regions. So, the mesh is refined near all walls, with a refinement corresponding to placing 16 volumes elements inside of the boundary layers. Figures 4 and 5 shows thermal and hydrodynamic boundary layers for the most critical case, $U = 5.0 \text{ W m}^{-2} \text{ K}^{-1}$. These figures show the complete formation of the thermal and hydrodynamic boundary layers, including the region of vertical shear among the upstream and downstream flow, which indicates that the distribution volume is also suitable.

Figures 4(a-d) shows velocity fields for cooling time (t) of 1, 6, 9 and 12 h, respectively. In these figures we can identify the hydrodynamic boundary layer (HBL) in the vertical wall of the tank. At the beginning of the cooling process (Fig. 4a), the HBL starts at the top of the tank, passes through a developed region and ends with a reduced thickness at the base of the tank. Continuing with the cooling process, thermal stratification forms at the base of the tank. In those areas that have already stratified, the heat transfer process is almost exclusively diffusive. This reduces the length of the vertical HBT with time, as can be seen by comparing Figs. 4(b-d). Still in Fig. 4(a-b), two convective currents, with opposite directions, are observed at the top of the tank. These convective cells are typical of the Rayleigh-Bernard process and occur due to cooling of the fluid near the upper tank wall.

Figures 5(a-d) shows the transient evolution of the dimensionless temperature (θ) contour. In these figures no temperature variations are observed along most of the radius. The greater thermal gradients are observed in the thermal boundary layer (TBL), in the same region in which the HBT is develops.

Results presented in Fig. 3(a-b), 4(a-d) and 5(a-d) indicate that the mathematical model, computational grid refinement and numerical methods used are capable of representing the natural convection process with the desired detail, described in the last paragraphs. Moreover, the numerical code was validated experimentally in previous author works by same authors. Thus, the results presented here can be considered experimentally validated and numerically verified.



Figure 4. Transient evolution of the fluid inside of the storage tank. Velocity fields for Ra^{*} = 2.7.108, $\hat{U} = 3$ and A_c= 2: (a) t = 1 h; (b) t = 6 h; (c) t = 9 h and (d) t = 12 h.



Figure 5: Transient evolution of the fluid inside of the storage tank. Dimensionless temperature (θ) contour for Ra*=2.7.108, $\hat{U} = 3$ and Ac = 2: (a) t = 1 h; (b) t = 6 h; (c) t = 9 h and (d) t = 12 h.

6. DIMENSIONAL ANALYSIS

Dimensional analysis (DA) is used to obtain a correlation for the cooling processes of tanks. DA consists of three steps: (a) make a list of relevant variables; (b) convert these dimensional quantities into dimensionless numbers (DN); (c) find a physically sound relationship of these DN without help of any governing equations (Ruzicka, 2008). The internal thermal free convection process is a function of: gravitational force; density gradient, heat transfer to the environment, tank geometry and material properties. So, the following variables are available: thermal conductivity (k), characteristic dimension (Lc = Ω / A_{Tot}), temperature difference (T = T_m - T_{amb}), thermal diffusivity ($\alpha = k/\rho c_p$), internal heat transfer coefficient (h), thermal expansion coefficient (β), gravity acceleration (g), overall heat transfer coefficient (\overline{U}), heat flux (q'') and diameter of the tank (D). Then, the internal heat transfer coefficient can be written as: $h = f(k, Lc, \Delta T, \alpha, D, \overline{U}, g, q'', \beta)$.

To obtain the dimensionless form of heat transfer coefficient, the Buckingham theorem is applied to these variables, as described by Ruzicka (2008). It is choose [M], [L], [T] and [θ] as the required fundamental dimensions, and k, Lc, ΔT and α as repeating parameters. The others (h, D, \overline{U} , g, q" and β) are set as the remaining parameters, including h as dependent parameter. To find the relationship between the independent parameter and the others, these are written in the form of power monomials, based on dimensional homogeneity of physical parameters (Ruzicka, 2008; Fox and McDonald, 2004), resulting in six dimensionless groups Π :

$$\Pi_1 = \frac{\overline{h}L_c}{k}; \ \Pi_2 = \frac{D}{L_c}; \ \Pi_3 = \frac{\overline{U}L_c}{k}; \ \Pi_4 = \frac{L_c^3}{\varepsilon^2}; \ \Pi_5 = \frac{q^{"}L_c}{k\Delta T}; \ \Pi_6 = \Delta T\beta.$$
(11)

The dimensionless parameter Π_1 includes the dimensional parameter dependent h. In this case, and to finalize the application of the Buckingham theorem, the dimensionless parameter Π_1 must be written in terms of the other five dimensionless parameters: $\Pi_1 = f(\Pi_2, \Pi_3, \Pi_4, \Pi_5, \Pi_6)$, where f is an unknown function. Substituting the dimensionless parameters in the last function and recognizing we have:

$$\overline{Nu} = f(A_c, \hat{U}, Pr, Ra^*).$$
⁽¹²⁾

The following dependence for Nu with the relevant parameters identified has been proposed:

$$Nu = C_1 A_c^{C_2} \hat{U}^{C_3} (\Pr \operatorname{Ra}^*)^{C_4}, \tag{13}$$

where C_1 , C_2 , C_3 and C_4 are coefficients that are obtained through nonlinear regression with the DataFit software. For this equation, this software presented the following coefficients: $C_1 = 1.86$, $C_2 = 1.11$, $C_3 = 0.174$ and $C_4 = 0.155$, resulting in this equation:

$$\overline{Nu} = 1.86A_c \,\hat{U}^{0.174} \,(Pr, Ra^*)^{0.155}.$$
(15)

The comparison between the Nusselt number numerically obtained with those obtained using Eq. (15) results in a correlation coefficient R2 of about 0.95. Fig. 6 shows the relation between the numerical Nusselt number, obtained through differential and the Nusselt number obtained by the proposed correlation (Eq. 15), in which can be observed the linear relationship between the results. The linear relationship, together with the results of R2, shows the validity of this proposed correlation.



Figure 6. Numerical Nusselt number vs. correlation Nusselt.

7. CONCLUSION

In this work, the transient cooling process in cylindrical vertical tanks is numerically simulated using a Finite Volume method, starting with a homogeneous temperature field, with no-slip boundary conditions and impermeable walls. Thirty two configurations are employed for different tank volumes, aspect ratios and overall heat transfer coefficients. For each case simulated, the internal heat transfer coefficient is obtained. The Buckingham theorem is used to correlate the internal heat transfer coefficient with the main parameters that govern the cooling process in thermal reservoirs. The result is a function that combines the aspect ratio, the Nusselt, Prandtl and Rayleigh numbers. This correlation is checked with numerical simulations resulting in good agreement. Moreover, being a function of the average temperature, it can be a useful tool for the improvement of temperature profiles found from global methods.

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