THERMODYNAMIC MODELING OF ENERGY SEPARATION PROCESS INTO A RANQUE-HILSCH VORTEX TUBE

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Abstract. *The Ranque-Hilsch Vortex Tube is a simple, no moving components, small and lightweight, low cost, easy-to-operate and maintenance free device for gas expansion to simultaneously produce heating and cooling. This paper presents a thermodynamic model for the Ranque-Hilsch Vortex Tube steady state functioning and evaluates its potential for green heating or green cooling.*

Keywords*: Ranque-Hilsch Vortex Tube, thermodynamic modeling, green refrigeration.*

1. INTRODUCTION

Accompanied by the world financial markets sudden implosion and massive reduction of trade and industrial production, the current crisis evolves to a new crisis of available technologies for exploring and transforming the existing natural resources into manufactured goods and services. Paralleling the current crisis, the end of the oil era dramatically increases the need for energy efficient “green technologies” to use widely available on the Earth alternative and renewable clean energy sources.

Large-scale efficient conversion of solar and wind energy, ocean waves and tides, and biomass or geothermal energy into electricity, mechanical energy, heating or cooling for industrial use, or into electricity, hot/cold air energy, hot/cold water energy to meet air-conditioning, cooking or personal hygiene demands for domestic applications could not only alleviate the serious environmental problems of today but also to ensure the energy supplying for future generations.

It is in this context that the teaching of mechanical engineering should assume it’s very important and strategic role to educate the future able-for-innovation professionals as the holders of necessary knowledge for developing new energy technologies for efficient conversion of energy derived from alternative and renewable energy sources.

This paper focuses on the “green refrigeration technology” known as the Vortex Tube or Hilsch-Ranque Tube technology and the need for thermodynamic models suitable for undergraduate class presentation.

The Ranque-Hilsch Vortex Tube is an interesting device to produce the refrigerating effect when a limited amount of cooling is required in special circumstances: a reliable small size equipment and freedom from electrical supplies. It has been described first in 1933 and then patented in 1934 by Georges Ranque a French scientist. Later, in 1946, a larger interest in this non-conventional method for producing cooling has been generated by a widely read article of Rudolph Hilsch. Since it is a simple, compact, light and quiet device, and does not use refrigerants, during the last decades, the need for less polluting technologies and more ecological technical solutions, generated a careful examination of the Vortex tube as an attractive alternative to improve the energy efficiency for some industrial applications (Stanescu *et al.*, 1998, 2009, 2012), cooling equipment in NCM machines, non conventional refrigeration (Radcenco, 1990), etc.

Based on thermodynamics fundamentals, this paper presents a simple thermodynamic model formulated by the method of thermodynamic cycles, to approach the refrigerating effect of the Ranque-Hilsch Vortex Tube (RHVT) functioning.

2. RANQUE-HILSCH VORTEX TUBE FUNDAMENTALS

Although it has been used industrially, the Ranque-Hilsch Vortex Tube has remained partly misunderstood until now. A number of theories have been developed to understand and explain the physical phenomena inside the Vortex tube (Rocha *et al.*, 1997). The theory of D. C. Fulton (see the R433 Vortex Tube Refrigerator Technical Specification, P. A. Hilton Ltd., England, 1996.) relies on the conservation of angular momentum to explain the separation of the compressed gas entering the Vortex tube, into a cold gas stream and a hot one. Based on the First and Second Laws of Thermodynamics, Petrescu *et. al.* (1995) studied comparatively the Hilsch-Ranque Vortex Tube expansion processes and the adiabatic reversible expansion performances. A comprehensive study of various aspects of Vortex Tubes´ design has been presented by Radcenco (1990), through an exergy analysis.

Figure 1 shows the Ranque-Hilsch Vortex Tube schematically. It is built by two concentric pipes of different diameters joined at one end. The two cylindrical chambers into the pipes are separated by a disk with a central orifice. The other end of the larger pipe is equipped with an outlet valve to control the gas stream discharge, while the gas stream flowing through the smaller pipe discharges freely. Next to the junction and equally spaced around the larger pipe periphery are nozzles arranged to discharge tangentially into the cylinder.

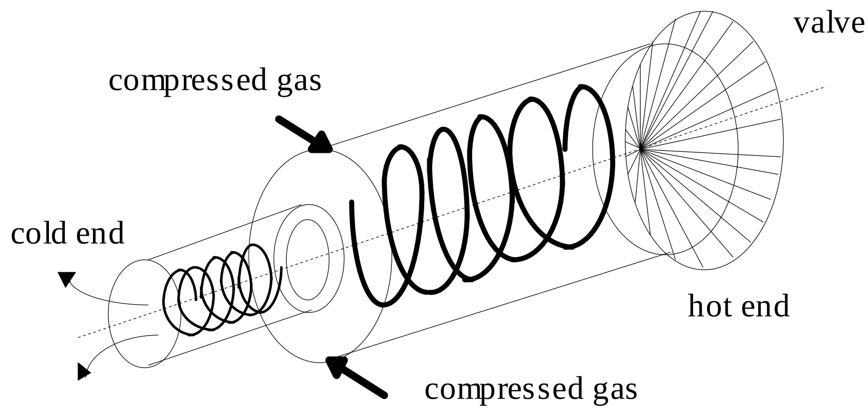


Figure 1. Schematic view of the Ranque-Hilsch Vortex Tube.

The core of the forced vortex, created into the tube when supplying compressed gas to the nozzles, is cold and is extracted from the end of smaller chamber. The periphery of the vortex is hot and is extracted from the end of the larger chamber, controlled by the valve.

3. THERMODYNAMIC MODEL

The main goal of this paper is to present a simple thermodynamic model, formulated based on the method of thermodynamic cycles, to approach the refrigerating effect of the RHVT.

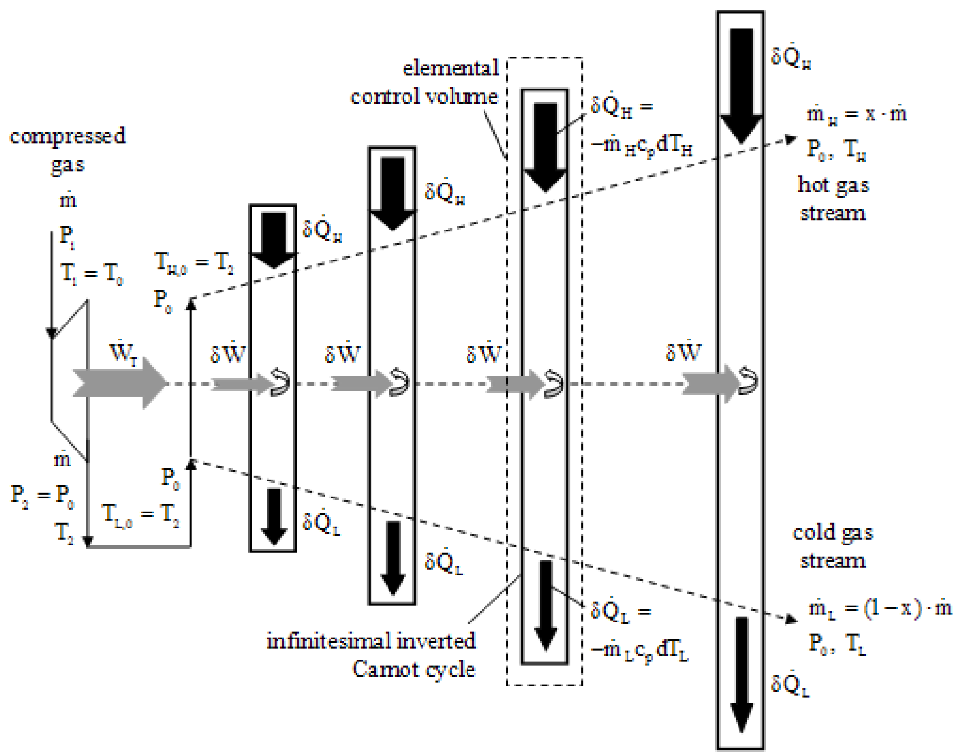


Figure 2. RHVT thermodynamic model based on infinitesimal inverted Carnot cycles.

We approach the refrigerating effect based on an equivalent physical mechanism for the internal energy redistribution into the rotating mass of gas. Assuming the steady state operation, the internal energy redistribution is modeled as a heat transfer process between the two gas-streams along the larger cylindrical chamber within the Vortex Tube as shown in Fig. 2.

Powered by a reversible adiabatic gas turbine converting into mechanical power the internal energy of the compressed gas delivered to the Ranque-Hilsch Vortex Tube, an ensemble of elemental refrigeration plants functioning accordingly to infinitesimal inverted Carnot-like cycles, remove energy from the cold gas stream (the core of the rotating gas column) and transfers it to the hot gas stream (the annular periphery of the rotating gas column).

4. MATHEMATICAL MODEL

The mathematical model is represented by the energy conservation and the second law equations for each one of the infinitesimal inverted Carnot-like cycles in Fig. 2 and the second law equation for each one of the adiabatic elemental control volumes crossed by both, the hot and cold gas streams, and containing one elemental reversible refrigeration plant in Fig. 2:

$$\oint \delta Q = \oint \delta W \tag{1}$$

$$\oint \frac{\delta Q}{T} \leq 0 \tag{2}$$

$$\delta \dot{S}_{gen} = \frac{dS}{dt} - \sum_i \frac{\dot{Q}_i}{T_i} + \sum_{out} \dot{m}s - \sum_{in} \dot{m}s \geq 0 \tag{3}$$

where $\delta\dot{S}_{ger}$ is the entropy generation rate, dS/dt represents the rate of entropy accumulation inside the control volume, $\dot{Q}_i / T_i$ gives the entropy transfer rate and the last two terms in Eq. (3) accounting for the net entropy flow rate out of the elemental control volume. Now, rewriting Eqs. (1) and (2) in terms of heat fluxes and mechanical power yields

$$\delta \dot{Q}_L + \delta \dot{Q}_H = \delta \dot{W} \tag{4}$$

$$\frac{\delta \dot{Q}_L}{T_L} + \frac{\delta \dot{Q}_H}{T_H} \leq 0 \tag{5}$$

where $\delta\dot{Q}_L = -(1-x)\dot{m}c_p dT_L \geq 0$ represents the heat flux removed by the elemental refrigerating plant from the cold gas stream and $\delta\dot{Q}_H = -x\dot{m}c_p dT_H \leq 0$ gives the heat transfer interaction between the elemental refrigerating plant and the hot gas stream. The ratio between the hot gas mass flow rate leaving the Vortex Tube and the mass flow rate of compressed gas delivered to the Vortex Tube is written $x = \dot{m}_H / \dot{m}$.

Dimensionless forms of Eqs. (3), (4) and (5) are written down by assuming steady state functioning and dimensionless temperature $\overline{T} = T/T_0$, dimensionless mechanical power $\overline{W} = \dot{W}/(\dot{m}c_p T_0)$ and dimensionless entropy generation $\overline{S}_{gen} = \dot{S}_{gen}/(\dot{m}c_p)$

$$\frac{d\overline{W}}{d\overline{T}_H} = -(1-x)\frac{d\overline{T}_L}{d\overline{T}_H} - x \tag{6}$$

$$\frac{d\overline{T}_L}{d\overline{T}_H} \geq -\frac{x}{1-x}\frac{\overline{T}_L}{\overline{T}_H} \tag{7}$$

$$\frac{d\overline{S}_{ger}}{d\overline{T}_H} = \frac{x}{\overline{T}_H} + \frac{1-x}{\overline{T}_L}\frac{d\overline{T}_L}{d\overline{T}_H} \tag{8}$$

Based on the observation that the first derivative $d\overline{T}_L / d\overline{T}_H < 0$, in order to model the simultaneous gas heating up and cooling down processes inside the Vortex Tube we rewrite Eq. (7) as

$$\frac{d\overline{T}_L}{d\overline{T}_H} = -\frac{\xi x}{1-x}\frac{\overline{T}_L}{\overline{T}_H} \tag{9}$$

where $0 < \xi \le 1$, with $\xi = 1$ for infinitesimal Carnot cycles and $0 < \xi < 1$ for internally irreversible Carnot-like cycles.

5. NUMERICAL RESULTS
5.1. ODE system numerical integration method

To calculate temperatures of the cooled and respectively the heated gas stream, $\overline{T}_L$ and $\overline{T}_H$, we integrate based on the fourth-order Runge-Kutta technique the next parametric ODE system

$$\frac{d\overline{W}}{d\overline{T}_H} = x\left(\xi\frac{\overline{T}_L}{\overline{T}_H} - 1\right) \tag{10}$$

$$\frac{d\overline{T}_L}{d\overline{T}_H} = -\frac{\xi x}{1-x}\frac{\overline{T}_L}{\overline{T}_H} \tag{11}$$

$$\frac{d\overline{S}_{ger}}{d\overline{T}_H} = \frac{(1-\xi)x}{\overline{T}_H} \tag{12}$$

with the following initial conditions $\left[\overline{W} \quad \overline{T}_L \quad \overline{S}_{ger}\right]^T = \left[0 \quad \overline{P}_1^{(1-k)/k} \quad 0\right]^T$ where the dimensionless pressure is given by $\overline{P} = P/P_0$.

As shown in Fig. 2 we consider initially $\overline{T}_{L,0} = \overline{T}_{H,0} = \overline{P}_1^{(1-k)/k}$ and assume the gas expansion through the turbine to be adiabatic and reversible. Numerical integration is performed until $\overline{W} = -\overline{W}_T = \overline{P}_1^{(1-k)/k} - 1$. Values of $\overline{T}_L$, $\overline{T}_H$ and $\overline{S}_{ger}$ are then considered to represent the solution and then employed to evaluate the Ranque-Hilsch Vortex Tube potential for cooling, $\overline{Q}_L = \dot{Q}_L/(\dot{m}c_pT_0) = (1-x)(\overline{T}_L - 1)$ and for heating $\overline{Q}_H = \dot{Q}_H/(\dot{m}c_pT_0) = x(\overline{T}_H - 1)$. The heating potential is evaluated by considering the thermal energy possible to be removed from the hot gas stream until it reaches the ambient temperature T_0. Conversely, the cooling potential of the Vortex Tube is given by the amount of thermal energy the cold stream may remove from a certain system until the gas heats up and reaches the ambient temperature T_0.

5.2. System of algebraic equations semi-analytical method

The semi-analytical solution of the system of ordinary differential equations (10), (11) and (12) is obtained by firstly observing that variables are separated into Eq. (11) and then determining:

$$\frac{\overline{T}_L}{\overline{T}_{L,0}} = \left(\frac{\overline{T}_H}{\overline{T}_{H,0}}\right)^{-\frac{\xi x}{1-x}} \tag{13}$$

Based on this result we may calculate $\overline{T}_L/\overline{T}_H$

$$\frac{\overline{T}_L}{\overline{T}_H} = \left(\frac{\overline{T}_H}{\overline{T}_{H,0}}\right)^{-\frac{\xi x}{1-x}-1} \tag{14}$$

and then read Eq. (10) as

$$d\overline{W} = x\left[\xi\left(\frac{\overline{T}_H}{\overline{T}_{H,0}}\right)^{-\frac{\xi x}{1-x}-1} - 1\right]d\overline{T}_H \tag{15}$$

Now, by integrating Eqs. (12) and (15) between the state 2, at the gas turbine outlet, and the final states when the gas leaves the Vortex Tube we may write the following equations to determine the hot gas temperature $\overline{T}_H$

$$x\overline{T}_H^{\left(\frac{\xi x}{1-x}+1\right)} - \overline{T}_H^{\left(\frac{\xi x}{1-x}\right)} + (1-x)\overline{P}_1^{\frac{1-k}{k}\left(\frac{\xi x}{1-x}+1\right)} = 0 \qquad (16)$$

and to evaluate the entropy generation during the Vortex Tube steady state functioning

$$\overline{S}_{ger} = (1-\xi)x\ln(\overline{T}_H / \overline{T}_{H,0}) \qquad (17)$$

Once the hot gas temperature $\overline{T}_H$ has its numerical value determined it is now possible to calculate the cold gas temperature $\overline{T}_L$ based on Eq. (14). Values of $\overline{T}_L$, $\overline{T}_H$ are then employed to evaluate the Vortex Tube potential for heating $\overline{Q}_H$ or cooling $\overline{Q}_L$.

5.3. Discussion of the numerical results

Table 1 shows comparatively the numerical results of the ODE system integration and the semi-analytical solution. Since both, the numerical results of the ODE system integration and the semi-analytical solution in Table 1 agree well, the next step calculates based only on the semi-analytical method the cooling potential $\overline{Q}_L = (1-x)(\overline{T}_L - 1)$ for different ratios between the hot gas mass flow rate and the mass flow rate of compressed gas delivered to the Ranque-Hilsch Vortex Tube, x, and for different degrees of irreversibility ξ. Consistency of the proposed thermodynamic approach is numerically checked by monitoring the entropy generation $\overline{S}_{gen} \geq 0$.

Table 1. Numerical results ODE integration and the semi-analytical solution.

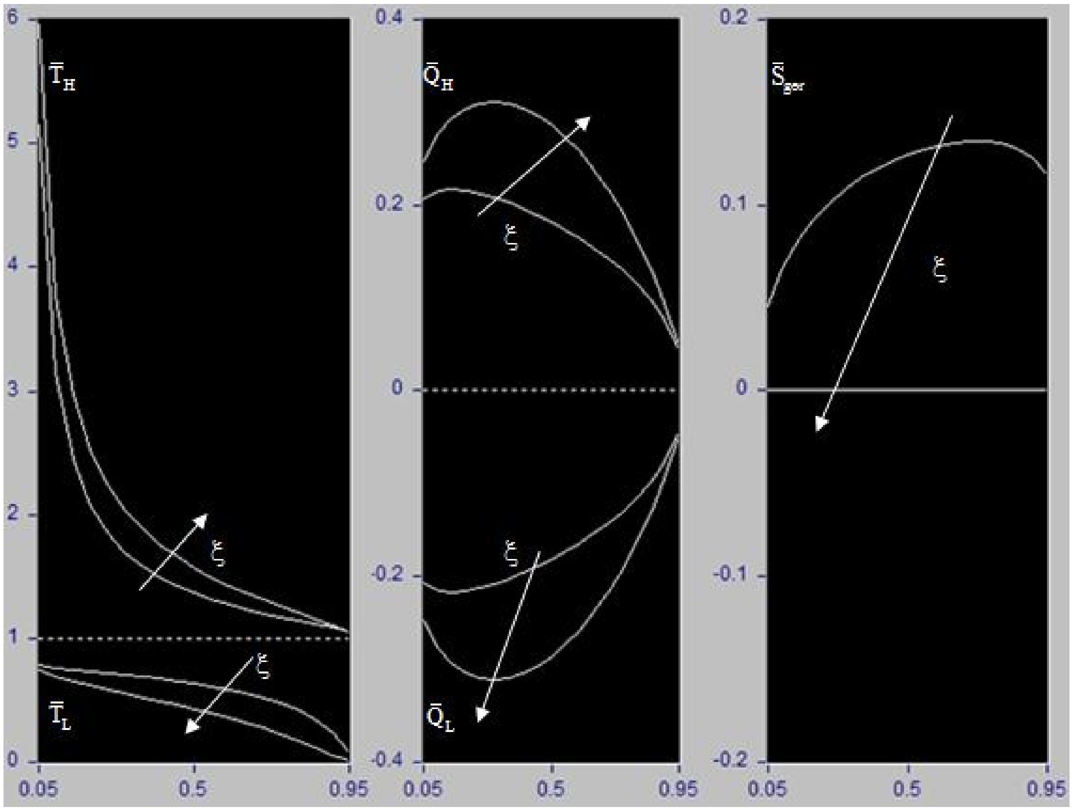
	Numerical integration*	Semi analytical solution*	Relative error
$\overline{T}_H$	1.37100	1.37200	-7.3×10^{-4}
$\overline{T}_L$	0.44310	0.44270	9.0×10^{-4}
$\overline{S}_{ger}$	0.06163	0.06167	-6.5×10^{-4}
$\overline{Q}_L = -\overline{Q}_H$	0.22276	0.22290	-6.3×10^{-4}

* $x = 0.6$, $\xi = 0.8$, $k = 1.4$, $\overline{P}_1 = 2$, $\overline{T}_{H,0} = \overline{T}_{L,0} = 0.8203$, $\overline{S}_{ger,0} = 0$, $\overline{W}_0 = 0$

Figure 3a graphically presents the effect of thermodynamic irreversibility on the temperatures of the hot and cold gas leaving the Vortex Tube, while Figs. 3b and 3c show the effects of ξ on the Vortex Tube potential for heating $\overline{Q}_H$ or cooling $\overline{Q}_L$ and the generation of entropy $\overline{S}_{gen} \geq 0$. It is worth noting that the thermodynamic model presented in this paper captures well the effect of thermodynamic irreversibility on the Vortex Tube performance. In Fig. 3a it can be observed variations with x and ξ of temperatures $\overline{T}_H$ and $\overline{T}_L$. The higher values of $\overline{T}_H$, respectively the lowest values of $\overline{T}_L$ correspond to $\xi = 1$ (the reversible model). Curves in Figure 3b indicate the existence of an optimal cooling functioning regime of the RHVT in order to maximize its cooling/heating potential $\overline{Q}_{L,max} = \overline{Q}_{H,max} = -0.3111$ for $x = 0.3$, $\xi = 1$ and $\overline{P}_1 = 2$.

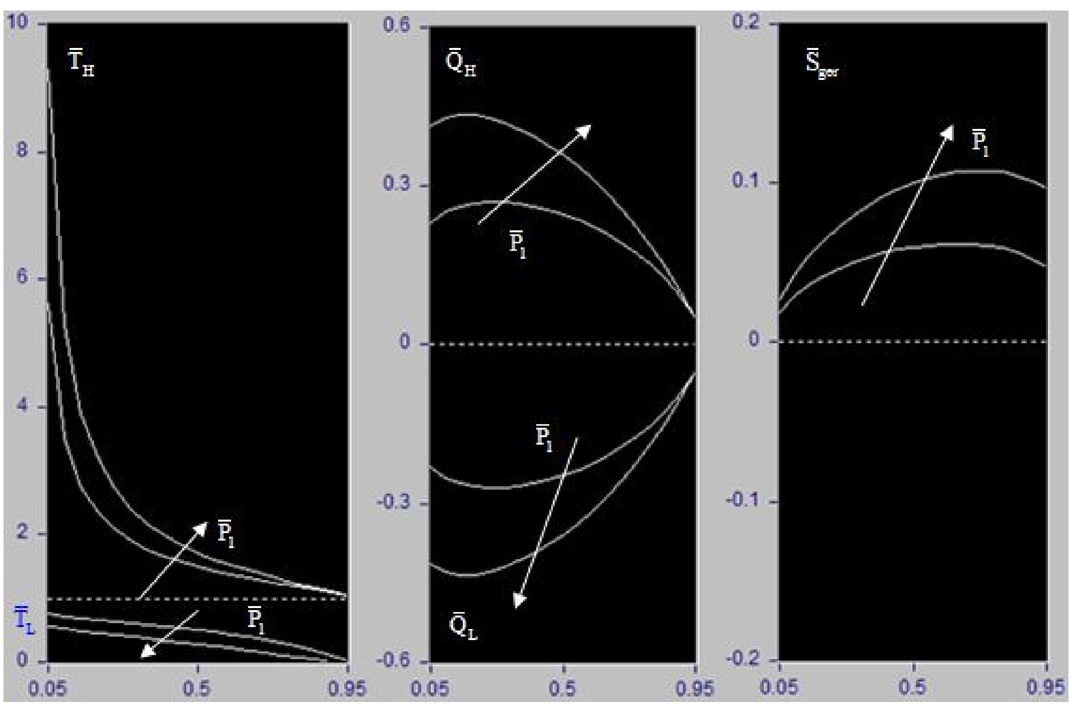
It is graphically shown in Fig. 4a the influence of thermodynamic irreversibility and the compressed gas pressure $\overline{P}_1$ on the hot and cold gas temperatures. Figs. 4b and 4c present the effects of $\overline{P}_1$ on the Vortex Tube potential for heating $\overline{Q}_H$ or cooling $\overline{Q}_L$ and the generation of entropy $\overline{S}_{gen} > 0$. The thermodynamic model presented in this paper also captures well the effect of thermodynamic irreversibility and the compressed gas pressure $\overline{P}_1$ on the Vortex Tube performance.

In Fig. 4a it can be observed variations with x and $\overline{P}_1$ of temperatures $\overline{T}_H$ and $\overline{T}_L$. The higher values of $\overline{T}_H$ and the lowest values of $\overline{T}_L$ correspond to the higher value of the compressed gas pressure $\overline{P}_1$. Figure 4b also suggests the existence of some optimal functioning regime in order to maximize the cooling/heating Vortex Tube potential $\overline{Q}_{L,max} = \overline{Q}_{H,max} = -0.4350$ for $x = 0.15$, $\xi = 0.8$ and $\overline{P}_1 = 5$.



(a) (b) (c)

Figure 3. The x and ξ variation of (a) the hot and cold gas temperatures $\overline{T}_H$ and $\overline{T}_L$, (b) the Vortex Tube potential for heating $\overline{Q}_H$ or cooling $\overline{Q}_L$ and (c) the entropy generation $\overline{S}_{gen}$ ($\xi = 1$, $\xi = 0.5$, $\overline{P}_1 = 2$).



(a) (b) (c)

Figure 4. The x and $\overline{P}_1$ variation of (a) the hot and cold gas temperatures $\overline{T}_H$ and $\overline{T}_L$, (b) the Vortex Tube potential for heating $\overline{Q}_H$ or cooling $\overline{Q}_L$ and (c) the entropy generation $\overline{S}_{gen}$ ($\xi = 0.8$, $\overline{P}_1 = 2$, $\overline{P}_1 = 5$).

6. CONCLUSIONS

A thermodynamic model suitable for undergraduate teaching of the green refrigeration based on the Vortex Tube steady state functioning and to evaluate its potential for green heating or green cooling is presented.

We approach the Vortex Tube refrigerating effect based on an equivalent physical mechanism for the internal energy redistribution into a rotating column of gas. Assuming the steady state operation, the internal energy redistribution is modeled as a heat transfer process between the two gas-streams along the larger cylindrical chamber within the Vortex Tube. Powered by a reversible adiabatic gas turbine converting into mechanical power the internal energy of the compressed gas delivered to the Vortex Tube, an ensemble of elemental refrigeration plants functioning accordingly to infinitesimal inverted reversible/irreversible Carnot-like cycles, remove energy from the cold gas stream and transfers it to the hot gas stream.

To evaluate the Vortex tube cooling/heating potential, two numerical methods are employed based on the fourth-order Runge-Kutta technique and a semi-analytical method. Next we calculate for different ratios between the hot gas mass flow rate and the mass flow rate of compressed gas delivered to the Vortex Tube, and for different degrees of irreversibility, the cooling/heating potential based on the semi-analytical method. Consistency of the proposed approach is checked by monitoring the entropy generation.

The thermodynamic model presented in this paper captures well the effect on the Vortex Tube performance of the thermodynamic irreversibility and the compressed gas pressure and suggests the existence of an optimal cooling functioning regime of the Vortex Tube in order to maximize its cooling/heating potential.

7. ACKNOWLEDGEMENTS

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