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MECHANICAL MODEL FOR CAVITATING FLOW IN HYDRAULIC PIPELINES

Alexandre Hastenreiter Assumpção, alexhast123@yahoo.com.br [1]
Felipe Bastos de Freitas Rachid, rachid@vm.uff.br [2]

[1,2]Laboratory of Gas and Liquid Transport, Departament of Mechanical Engineering – Graduate Program in Mechanical Engineering Francisco Eduardo Mourão Saboya – TEM/PGMEC, Universidade Federal Fluminense , Rua Passos da Pátria 156, Niterói, RJ 24210-240, Brazil.

Abstract. *The purpose of this work is to present a mechanical model to describe the cavitating flow in hydraulic pipelines. Although the model is capable to describe the cavitation phenomenon in unsteady as well as steady states, the applications presented in this work are restricted to slack flow, which take place in steady states. The flow is assumed to be homogeneous and isothermal. The fluid is treated as a pseudo-mixture, comprising the liquid and the vapor phases. Both phases are assumed to be compressible and to coexist at every material point and time instant. The balance equations of mass for each of the phases are considered in the model, along with one balance equation of momentum for the mixture as a whole, within an onedimensional context. The phase change transformation is properly accounted for as an irreversible process. The main dimensionless groups are identified and their influence on the slack flow phenomenon quantified by means of numerical simulations. The obtained results show that model is capable to mimic coherently both the opening as well as the closure of the vapor cavity.*

Keywords*: Slack Flow, Cavitation, irreversible phase change transformation.*

1. INTRODUCTION

The cavitation phenomenon is caused by the formation of vapor phase in a liquid due to pressure reduction below the saturation pressure of liquid at given temperature. In hydraulic pipelines operating in steady states, cavitation can cause a standard flow called slack flow.

The pressure can become equal or less than saturation pressure of the liquid depending on the geometric characteristics of topographic profile of the pipe. In this situation, part of the liquid transforms into vapor forming a cavity at the upper region of the pipe. The cavity extension may vary significantly according to some flow conditions. This typical phenomenon of cavitating flow in steady states is usually called by slack flow.

The slack flow substantially affects the piezometric profile of the pipe. Figure 1 shows two piezometric profile of pipe operating in steady states, whose topographic profile is represented by black line. It can be shown that a pipeline slack line flow regions always begin at an elevation peak or on the downhill slop, but never begin on an uphill slope, and finish on the downhill slop (Nicholas, 1995).

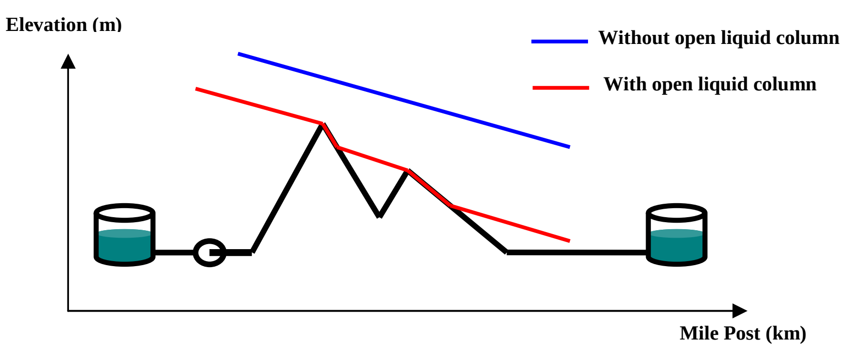


Figure 1. Piezometric profile of the pipe with and without open liquid column

Particulary in the oil pipelines, the occurrence of slack flow increases significantly the required time to detect a leak as compared with a same pipeline operating in tight flow condition. It is particulary important in the oil pipelines which are constrained to operate with leak detection systems.

In spite of relevance of approached topic, there are little available works in technical literature proposing model for steady cavitating flow, except for Nicholas (1995). On the other hand, because of catastrophic damage caused by non steady cavitation, there are a lot of models about unsteady cavitating flow such as the models of Wiggert & Sandquist (1979), Kessal & Amaounche (2001), Shu (2003), Freitas Rachid (2003), Liu et al (2005) and, finally, Xie et al (2006).

Although the models previously cited describe nicely the unsteady cavitating flow, most of them are not able to describe the opening and closure liquid column phenomenon. One of objectives of this work is to develop a mechanical model that describes the cavitation phenomenon in unsteady as well as steady states, even though the examples presented are exclusively in steady states. In this model, the phase change is regarded as an irreversible process and the friction factor used is one proposed by Nicholas (1995) that considers the slack flow as well as the tight flow condition.

2. BASIC ASSUMPTIONS

The cavitation is a phenomenon restricted to the small and discrete regions of fluid flow. On this basis, it can assume that the liquid and vapor phases do not slip one over the other. Consequently, the velocity of liquid and vapor phases are equals in all domain of flow, in other words

$$v_l = v_v = v \tag{1}$$

$$T_l = T_v = T \tag{2}$$

So, the fluid can be regarded a pseudo-mixture of the two compressible phases (liquid and vapor) with average properties that coexist in the same material point and time instant, along the length of pipe. To take it into account, it is considered an internal variable α, α □ (0,1), called by vapor volume fraction, defined as being the ratio of the volume of vapor and the total volume of the fluid. Thus, the mass density of the fluid can be expressed as

$$\rho = (1-\alpha)\,\rho_l + \alpha\rho_v \tag{3}$$

in which ρ_l and ρ_v stand for the mass densities of the liquid and vapor phases, respectively (Freitas Rachid, 2003; Assumpção & Freitas Rachid, 2008).

In the context of the thermodynamic, the previous condition is not true in regions of flow in which there is no phase change because it admits the existence of thermodynamic equilibrium between the liquid and vapor phases. However, this assumption simplifies substantially the implementation of model in computational terms, because it always allows resolving the same set of equations for the same set of dependent variables in whatever the section of pipe.

It can be assumed that the pressures of liquid and vapor phases are equal whereas the effects of interfacial stress are not regarded ($p_l = p_v = p$). In spite of there is heat transfer between fluid and environment, there is no heat transfer between the liquid and vapor phases because the phases share same temperature.

3. CONSTITUTIVE EQUATIONS

The constitutive equations are necessary to complete the formulation of model specifying the mechanical behaviors of fluid and pipe. In the follow sections, the stress – strain relations for pipe and the equations of states for fluid will be presented. The thermodynamic theory about the change phase process and friction factor proposed by Nicholas (1995) will be presented too.

3.1. Relationship between area and pressure

The relationship between the cross section area of pipe and fluid pressure in thin-walled pipes subject to the isothermal transformations and small deformations is given by

$$A = A_0\left(1 + \xi\,\frac{pD_0}{eE}\right) \tag{4}$$

The anchorage factor ξ is equal 1 if the pipe has expansion joint. If the pipe is supported axially, the factor ξ is equal $1-\nu^2$.

3.2. Equations of states

The presented model is able to simulate the steady flow of any pure substance. However, water is used in the simulated cases. So, the equations of states presented describe the thermodynamic behaviors of liquid and vapor phases of water.

It is admitted that the vapor water behave as a perfect gas and the liquid behave according to the linear relation between pressure and mass density. Although these assumptions can seem restrictive enough, they allow simplifying the problem formulation without changing the model results. Using the cubic equations of states for both liquid and vapor phases, Assumpção & Freitas Rachid shown that the EOS more accurate do not change significantly the results concerning the pressure, void fraction and rate of energy dissipation variations in the phase change process of water. Based on this result, it can be postulated the following equation of states for vapor phase:

$$p_v = \rho_v a_v^2 \tag{5}$$

in which a_v is the isothermal sonic velocity in vapor phase and ρ_v is the mass density of phase vapor.

The behavior of phase liquid is described by linearized equation of states, fitted from the Fine & Millero equation of states (1973). This equation describes reliably the behavior of liquid water for certain gauges pressures (0 a 100 bar) and temperatures ranges (0 a 100°C) (Fines & Millero, 1973).

$$p_l = a_l^2 \left(\rho_l - \rho_l^0\right) \tag{6}$$

The Eq. (6) represents a secant line that intersects the points (p_{sat}, ρ_{ls}) e (p_0, ρ_l^0) in which ρ_{ls} is a mass density of water at the saturation pressure p_{sat} and in a certain temperature, p_0 is the absolute pressure zero and ρ_l^0 is the mass density of water at pressure p_0. It can be shown that the angular coefficient a_l^2, which represents the square of sonic velocity in liquid phase, and the linear coefficient ρ_l^0 can be calculated by the following equations:

$$a_l^2 = \frac{p_{sat}}{\rho_{ls} - \rho_l^0} \tag{7}$$

$$\rho_{ls} = \frac{1}{\upsilon_0 \left[1 - \left(\frac{p_{sat} - p_{atm}}{B}\right)\right]} \tag{8}$$

$$\rho_l^0 = \frac{B}{\upsilon_0 \left(B + p_{atm}\right)} \tag{9}$$

3.3. Friction Factor in slack flow

In steady states, the shear stress τ in wall pipe can be calculated in terms of the flow velocity v and the friction factor f of Darcy – Weisbach (Streeter & Wylie, 1993) by the expression:

$$\tau = f \frac{\rho v^2}{8} \tag{10}$$

The friction factor in filled column flow can be calculated by empirical expressions or diagrams and chart as, for example, Moody chart. In general, the friction factor is a function of Reynolds number of liquid flow, Re_L, and roughness of the pipe wall, ε. On the other hand, in slack flow, the friction factor f_m is a function of Reynolds number of mixture liquid – vapor, Re, roughness of the pipe wall and void fraction α.

In order to obtain an expression that characterizes an equivalent friction factor able to satisfy both flow conditions, Nicholas (1995) proposed a variation of the Manning equation for calculus of head loss in slack flow based on the similarities between slack flow and open channel flow. In the context of this work, the equation proposed for Nicholas (1995) in terms of shear stress in the pipe wall is given by

$$\tau = f_m \frac{\rho v^2}{8} \tag{11}$$

$$f_m = \frac{f}{\left(1 - \frac{\operatorname{sen}(2\omega)}{2\omega}\right)^{4/3}} \tag{12}$$

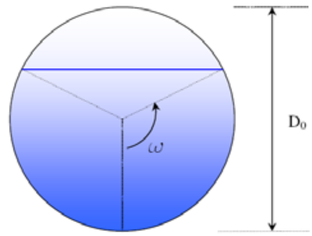


Figure 1. Cross section of pipe operating in slack flow condition

3.4. Thermdynamic Theory

Most of models for cavitating flow proposed in the technical literature has a common characteristic under the thermodynamic point of view. The classic models consider that the phase change is accounted for as a reversible process in which the pressure remains constant and equal to the saturation pressure in a certain flow temperature (Nicholas, 1995; Kessal & Amaouche , 2000; Shu, 2003).

Although the hypothesis of reversible process is appropriate in modeling of certain problems, it is not adequate to apply it in problems involving large energy dissipation. The cavitation phenomenon is a typical example. If regarded as reversible process in homogeneous unsteady flow, the sonic propagation velocity jumps locally making the search for numerical solutions is more complex even though using robust numerical techniques. In the same way that in homogeneous steady flow, the models based on hypothesis of reversibility of cavitation phenomenon are able to describe the opening liquid column, but do not describe the closure.

In order to describe consistently the problem and to overcome the difficulties presented previously, Freitas Rachid (2003) proposed a thermodynamically consistent model for cavitating flows of compressible fluids in which the cavitation phenomenon is accounted for as irreversible process. It can be demonstrated analytically that, under the hypothesis of homogeneous flow, the thermodynamic force associated the rate of mass transfer in the cavitation phenomenon (Γ) is directly proportional to the Gibbs free energy of liquid and vapor phases difference by means of positive constant β (Freitas Rachid, 2003).

In the context of Thermodynamic of irreversible processes, it can be shown that the irreversible cavitation is an intermediate case between the two reversible cases. Changing parametrically the values of positive constant β, it can be verified that for $\beta \geq 10^{-6}$ kg²/m³ J s the phase change occurs reversibly in which the pressure remains constant and equal to saturation pressure. On the other hand, for $\beta \leq 10^{-12}$ kg²/m³ J s, the phase change also occurs reversibly, but the pressure fluctuates around the saturation pressure, arranging the expansions and contraction of vapor in the mixture. Finally, the values of positive constant β ($\sim 10^{-8}$ kg²/m³ J s) describe the irreversible phase change in which the pressure varies around the saturation pressure (Freitas Rachid, 2003).

The Gibbs free energy of the phases can be calculated by means of the following thermodynamic relationships:

$$g_l = \frac{1}{1-\alpha}\frac{\partial \Psi}{\partial \rho_l} \tag{13}$$

$$g_v = \frac{1}{\alpha}\frac{\partial \Psi}{\partial \rho_v} \tag{14}$$

in which ψ is the Helmholtz free energy per unit mass of mixture that can be calculated as (Freitas Rachid, 2003; Assumpção & Freitas Rachid, 2008):

$$\Psi = \hat{\Psi}(\rho_l, \rho_v, \alpha, T) = (1-\alpha)\rho_l \Psi_l(\rho_l, T) + \alpha \rho_v \Psi_v(\rho_v, T) \tag{15}$$

Ψ_l and Ψ_v are the Helmholtz free energy per unit mass of liquid and vapor, respectively, and can be calculated by integration of the equations of states (Freitas Rachid, 2003):

$$p_l = \rho_l^2 \frac{\partial \Psi_l}{\partial \rho_l} \tag{16}$$

$$p_v = \rho_v^2 \frac{\partial \Psi_v}{\partial \rho_v} \tag{17}$$

Based on the preceding assumptions and regarding the phase change as isothermal transformation, it can be derived the following expression for the Gibbs free energy difference:

$$g_l - g_v = a_l^2 \ln\left(\frac{p + a_l^2\rho_l^0}{p_{sat} + a_l^2\rho_l^0}\right) + a_v^2 \ln\left(\frac{p_{sat}}{p}\right) + a_l^2 a_l^2 \rho_l^0 \left(\frac{1}{p_{sat} + a_l^2\rho_l^0} - \frac{1}{p + a_l^2\rho_l^0}\right) \tag{18}$$

Based on preceding results described and the hypothesis of mixture of phases, it can be derived the follow constitutive equation for the mass rate of phase change:

$$\Gamma = \begin{cases} 0, & \text{se } p > p_{sat} \text{ e } \alpha < \alpha^* \\ \beta(g_l - g_v), & \text{otherwise} \end{cases} \tag{19}$$

in which p_{sat} is the saturation pressure of liquid at given temperature T, β is a material positive constant and α^* is a limit value of void fraction below which it can be admitted the coexistence of liquid and vapor in equilibrium, as the pressures are larger than the saturation pressure, without mass transfer between the phases. The Eq. (19) establishes the necessary condition for opening as well as closure liquid column. The rate of energy dissipation associated the phase change is given by

$$d = \begin{cases} 0, & \text{se } p > p_{sat} \text{ e } \alpha < \alpha^* \\ \beta(g_l - g_v)^2, & \text{otherwise} \end{cases} \tag{20}$$

Unlike the models proposed by Freitas Rachid (2003) and Assumpção & Freitas Rachid (2008) that admit the existence of pure liquid ($\alpha = 0$) and pure vapor ($\alpha = 1$), in the model proposed, liquid and vapor coexist in equilibrium at every point of domain such as α □ (0,1). This hypothesis is not true under the thermodynamic view point because liquid and vapor do not coexist in equilibrium whether $p \neq p_{sat}$, but simplifies enough the formulation of problem. The great advantage is to resolve the same set of equations for a same set of dependent variables (p, v, α), whatever the value of α. Despite of advantage, it must be calculated the value of α^* in Eq. (19).

In regions of filled column flow, the mass fraction of vapor in the mixture C is constant

$$\frac{\alpha \rho_v}{\rho} = C = \text{constant} \tag{21}$$

Since the initial conditions $\alpha_0 = \alpha\ (s = 0)$ e $p_0 = p\ (s = 0)$ are known, the constant C can be univocally determined by means of the expression

$$C = \frac{\alpha_0 (P_0 / a_v^2)}{\alpha_0 (P_0 / a_v^2) + (1 - \alpha_0)\left[(P_0 / a_l^2) + \rho_l^0\right]} \tag{22}$$

So, it can be expressed the void fraction in regions of filled column flow as a function of pressure:

$$\alpha = \hat{\alpha}(p) = \frac{a_1 p + b}{a_2 p + b} \tag{23}$$

in which a_1, a_2 and b are positives constants ($a_2 > a_1$):

$$a_1 = C a_v^2 \tag{24}$$

$$a_2 = C a_v^2 + (1 - C) a_l^2 \tag{25}$$

$$b = C \rho_l^0 a_v^2 a_l^2 \tag{26}$$

It can be shown that the $\alpha = \hat{\hat{\alpha}}(P)$ is a function strictly decrescent of pressure. So, if the condition $\alpha_0 = \alpha\ (s = 0)$ is such as $\alpha_0 = \alpha(s=0) > \alpha_{min}$, the value of α^* can be determined by means of equation

$$\alpha^* = \hat{\hat{\alpha}}(p_{sat}) = \frac{a_1 p_{sat} + b}{a_2 p_{sat} + b} \tag{27}$$

The condition $\alpha_0 > \alpha_{min}$ is sufficient to ensure the condition $\alpha^* > \alpha_0$ since the pressure $p_0 = p(s=0)$ is greater than the saturation pressure. Consequently, the vapor contracts and expands according to pressure variation: if the pressure decreases, the vapor expands; if the pressure increases, the vapor contracts.

Based on preceding assumptions, it can be verified that the conditions $p > p_{sat}$ e $\alpha < \alpha^*$ presented in Eq. (19) set up consistently the conditions of opening and closure of column liquid. The parameter α^* represents the maximum void fraction experienced by mixture as the liquid column begin to open.

4. MECHANICAL MODEL

The purpose of this model is to describe coherently the steady cavitating flow in hydraulic pipelines and the opening and closure of liquid column. Taking into account the basic assumptions and the constitutive equation, Eq. (4), (5) and (6), it can be shown that the balance momentum and mass for liquid and vapor can be written in follow dimensionless forms:

$$\frac{dp^+}{ds^+} = \frac{-\dfrac{\rho(\rho_l - \rho_v)}{\rho_l \rho_v}\Gamma^+ H_R v^+ - f\left(v^+\right)^2 - \dfrac{\rho}{\rho_{ls}}\dfrac{\sin(\theta)}{\tilde{F}_r^2}}{1 - M^2\left[1 + \dfrac{\xi D_0 \rho a^2}{eE}\right]} \tag{28}$$

$$\frac{dv^+}{ds^+} = -\frac{f\ v^+}{2} - \frac{\sin(\theta)}{2v^+\tilde{F}_r^2} - \frac{1}{2v^+}\frac{dP^+}{ds^+} \tag{29}$$

$$\frac{d\alpha}{ds^+} = \frac{\rho_{ls}\Gamma^+ H_R}{\rho_v 2v^+} + \frac{f\ \alpha}{2} + \frac{\alpha}{2v^{+2}}\left[\frac{\sin(\theta)}{\tilde{F}_r^2} + \left(1 - M^2\left(\frac{\rho a^2}{\rho_v a_v^2} + \frac{\xi D_0 \rho a^2}{eE}\right)\right)\frac{dP^+}{ds^+}\right] \tag{30}$$

in which p^+, v^+ e α are the dimensionless variables and are given by

$$p^+ = \frac{p - p_{sat}}{\frac{1}{2}\rho_{ls} v_0^2} \tag{31}$$

$$s^+ = \frac{s}{D_0} \tag{32}$$

$$v^+ = \frac{v}{v_0} \tag{33}$$

and the terms Γ^+, a e M are dimensionless mass rate of phase change, the sonic propagation velocity in mixture liquid – vapor and the Mach number

$$\Gamma^+ = \begin{cases} 0, \quad \text{se } p^+ > 0 \text{ e } \alpha < \alpha^* \\ \dfrac{(g_l - g_v)}{(g_l - g_v)_{ref}}, \ \text{caso contrário} \end{cases} \tag{34}$$

$$a = \sqrt{\frac{1}{\rho}\frac{\rho_l a_l^2 \rho_v a_v^2}{(1-\alpha)\rho_v a_v^2 + \alpha \rho_l a_l^2}} \tag{35}$$

$p = \hat{p}(s)$, $v = \hat{v}(s)$ e $\alpha = \hat{\alpha}(s)$ are dependent variables and all of them are functions of pipe length (s). The values of parameters D_0, e, g, ξ, E, a_l, a_v and $\theta = \hat{\theta}(s)$ are defined since the fluid, the pipe material and topographic profile are specified. The other variables ρ_l, ρ_v, ρ, a, Γ e τ are calculated by means of constitutive equations.

To characterize the actions of thermodynamics forces associated for phase change, it is defined the following dimensionless number:

$$H_R = \frac{2D_0\beta\left(g_l - g_v\right)_{ref}}{\rho_{ls}v_0} \tag{36}$$

in which $(g_l - g_v)_{ref}$ represents the absolute value of Gibbs free energy of liquid and vapor phases difference calculated in pressure much larger than the saturation pressure at flow temperature. The dimensionless number H_R can be interpreted as being a ratio between the thermodynamic and inertial forces.

4. RESULTS

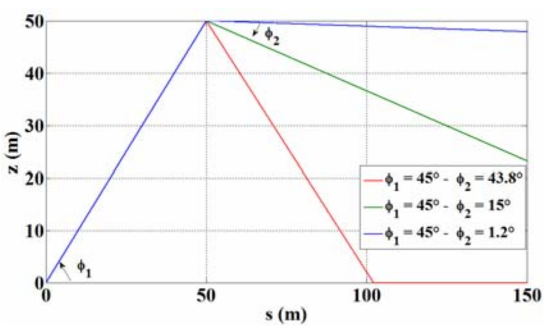
The proposed model is formed by three ordinary differential equations in the variables $p = \hat{p}(s)$, $v = \hat{v}(s)$ e $\alpha = \hat{\alpha}(s)$. Although there are many numerical methods available to solve ordinary differential equations, herein we have used the Gear's method – an implicit, multistep, adaptive, of variable order (1st at 5th order) method since the ordinary differential equations are stiff.

To evaluate the ability and reliability of model, the follow case is simulated. Consider a pipe of diameter D_0 and length L = 150 m by which water at temperature 20°C flows. At this temperature, the values of p_{sat}, ρ_l^0, a_l e a_v are respectively equals to 2.34 kPa, 998.154 kg/m³, 1477 m/s and 361 m/s. The pipe is constructed in steel (E = 207 GPa and ν = 0.27) and has expansion joints (ξ = 1). The initial conditions are known and will be presented in appropriate moment as well as the diameter D_0 and topographic profile of pipe.

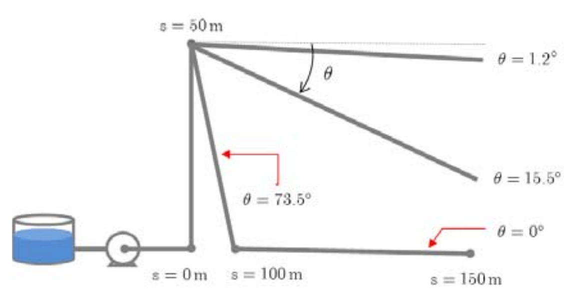
It can be shown that the necessary condition for slack flow is characterized by existence of an elevation peak or on the downhill slop (Nicholas, 1995). Based on it and knowing that the activation mechanism of phase change is mainly due to pressure, it can be done a preliminary analysis of Eq. (28) identifying some relevant aspects.

The three terms present in numerator of Eq. (28) represent the three forces that act on the fluid. The first term represents the thermodynamic force associated for phase change process. The second and third terms represent the friction force between the pipe wall and fluid and gravitational forces, respectively. The friction force acts on fluid in the sense of decrease the pressure. The thermodynamic force and gravitational force act according to signal of terms Γ^+ e sin (θ), respectively.

To choose appropriately the topographic profile that characterizes the opening liquid column phenomenon, it is possible to establish that if, in the moment of opening liquid column, $\Gamma^+ = 0$ e sin (θ) > 0, so the larger opening will occur as θ = π/2. The closure of column liquid will be observed in three cases in which the topographic profiles are represented in Fig. 2.



(a)



(b)

Figure 2. (a) Topographic profile of pipe: peak elevation and slopping hill (b) actual arrangement of pipeline

It is appropriate to define a new Froude number in order to regard the influence of elevation gradient of downward side pipe:

$$F_r = \frac{v_0}{\sqrt{2D_0 g\left|\sin(\theta)\right|}} \tag{37}$$

To analyze systematically the influence of each dimensionless number in the opening and closure liquid column phenomenon, three distinct typical values diameters, initial velocity, slope angle of topographic profile and constant β will be regarded. The tables 1 and 2 present the values for set of input data for each value of F_r and H_R simulated.

Table 1. Values of D_0, v_0, ϕ_2 e β used in the computation of $F_{r\,min}$, $F_{r\,inter}$, $F_{r\,max}$

	D_0 **(mm)**	v_0 **(m/s)**	ϕ_2	β $\left(kg^2/m^3\,Js\right)$
$F_{r\,min}$	1000	1.0	45°	10^{-8}
$F_{r\,inter}$	500	1.5	15°	10^{-8}
$F_{r\,max}$	200	2.0	1.2°	10^{-8}

Table 2. Values of D_0, v_0, ϕ_2 e β used in the computation of $H_{R\,min}$, $H_{R\,inter}$, $H_{R\,max}$

	D_0 **(mm)**	v_0 **(m/s)**	ϕ_2	β $\left(kg^2/m^3\,Js\right)$
$H_{R\,min}$	200	2.0	1.2°	10^{-15}
$H_{R\,inter}$	500	1.5	1.2°	10^{-12}
$H_{R\,max}$	1000	1.0	1.2°	10^{-8}

The numerical results for each simulated case in the proposed model are so presented. In all cases, the initial void fraction is $\alpha_0 = 10^{-8}$ (Wylie & Streeter, 1993). The other initial condition and value of α^* are given in Tab. 3.

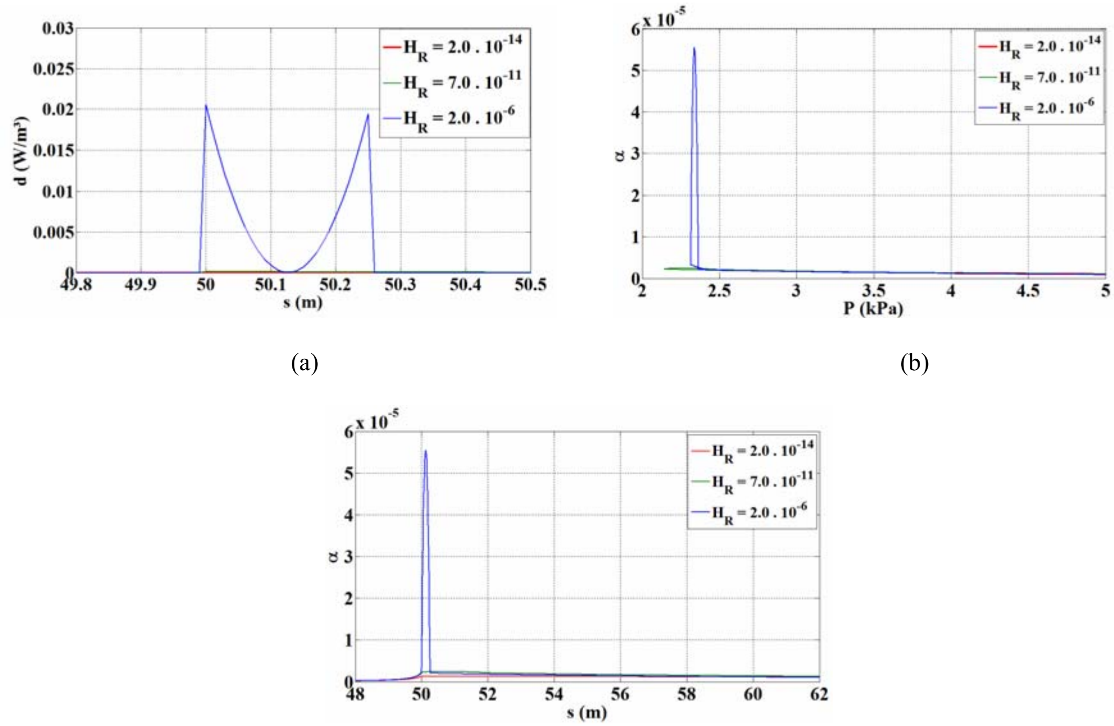
Table 3. Initial pressure and velocity conditions in each case simulated

	p_0 **(kPa)**	v_0 **(m/s)**	α^*
$F_{r\,min} = 0.226$	492.6	1.0	$2.10.\ 10^{-6}$
$F_{r\,med} = 0.925$	494.07	1.5	$2.11\ .\ 10^{-6}$
$F_{r\,max} = 6.98$	501.68	2.0	$2.14\ .\ 10^{-6}$
$H_{R\,min} = 2.0 \cdot 10^{-14}$	503.50	2.0	$2.15\ .\ 10^{-6}$
$H_{R\,med} = 7.0 \cdot 10^{-11}$	494.00	1.5	$2.11\ .\ 10^{-6}$
$H_{R\,max} = 2.0 \cdot 10^{-6}$	492.45	1.0	$2.10\ .\ 10^{-6}$

The dimensionless number F_r and H_R play a fundamental role in the description of opening and closure liquid column phenomenon.

The dimensionless number H_R is substantially influenced by value of β. This parameter has a preponderant role in the computation of H_R and the phenomenon description. High values of β characterize irreversible phase change processes and, at the same time, high values of H_R. So, it can be observed that high values of H_R describe the irreversible phase change processes. The figures 3a – 3b present respectively the plots of energy dissipation and pressure versus void fraction in the slack flow region for each values of H_R. In the Fig. 3b, it can be observed the hysteresis phenomenon present in any irreversible physic mechanism.

The opening and closure liquid column phenomenon are more realistically described when the phase change is regarded an irreversible processes. In the Fig. 3c, the opening and closure liquid column sections can be identified as $H_R = 2.0\ .\ 10^{-6}$. On the other hand, in the other values of H_R, it is only possible to determine the opening liquid column section.

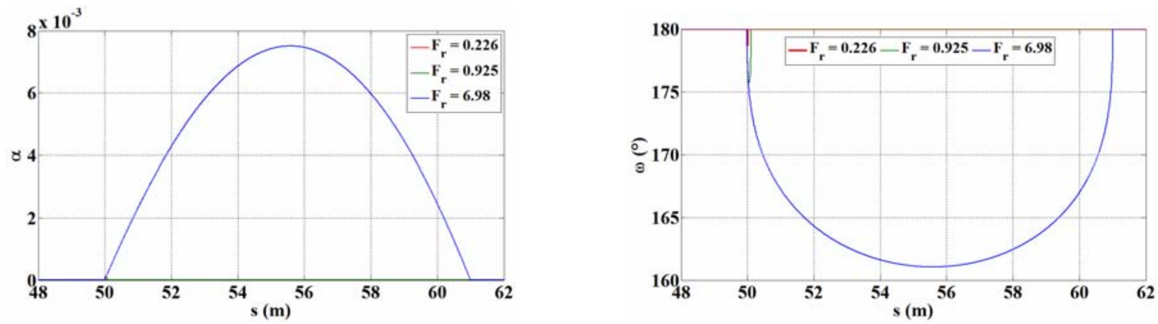


(a) (b)

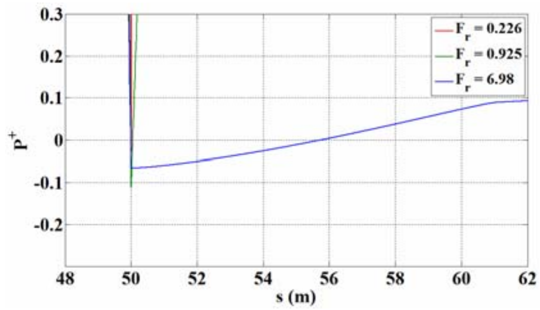
(c)

Figure 3. (a) Rate of energy dissipation in the slack flow region (b) Pressure versus void fraction and hysteresis phenomenon (c) void fraction as a function of pipe length for different values of H_R

The dimensionless number H_R is a measure of influence of thermodynamic force on the pressure gradient and the way as thermodynamic force acts on pressure gradient depends on the signal of Γ^+. If $\Gamma^+ > 0$, in other words, liquid transform into vapor, the thermodynamic force acts in the sense of open the liquid column. On the other hand, if $\Gamma^+ < 0$, vapor transform into liquid, it acts in the sense of close the liquid column. After the section $s = \xi_i$, the column liquid begin to open due to the friction force and thermodynamic force ($\Gamma^+ > 0$). However, in general, the section $s = \xi_i$ is situated in peak elevation or slopping hill (Nicholas, 1995). In all cases, the gravitational force acts in the sense of close the column liquid becoming the pressure gradient positive. Since the magnitude of gravitational force is larger than the thermodynamic force, the pressure stop to decrease and begin to increase in a section $s = \xi_m$, $\xi_m \in (\xi_i , \xi_f)$. Thereafter, vapor begins to go back the liquid state, in other words, $\Gamma^+ < 0$. In this condition, the thermodynamic force acts in the sense of close the liquid column and restore the flow in filled liquid column.



(a) (b)



(c)

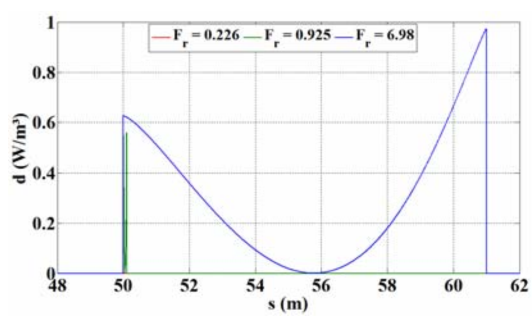
Figure 4. (a) Void fraction in the slack flow region (b) wet angle (c) Cavitation number as a function of pipe length for different values of Fr

The figures 5a, 5b and 5c show that the length of slack flow region depends on the magnitude of Froude number. Based on Eq. (37), the Froude number can be considered as a ratio between inertia and gravitational forces.

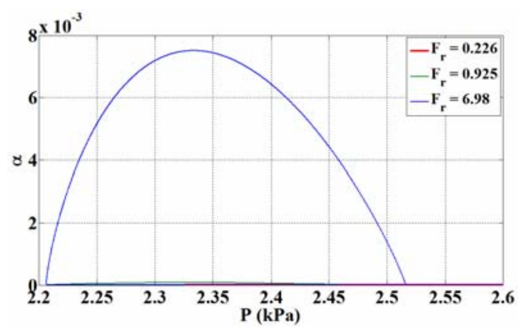
The influence of gravitational force on the pressure gradient depends on the signal of pipe elevation gradient. Since the section of open liquid column occurs on elevation peak or slopping hill, the pipe elevation gradient is always negative. Based on Eq. (28), the gravitational force always will contribute in the sense of become the positive pressure gradient and close the liquid column.

The length of slack flow region depends on the magnitude of Froude number. Large length of slack flow corresponds to high Froude number. In these cases, the gravitational, friction and thermodynamic forces have the same order of magnitude. From section $s = \xi_i$ to $s = \xi_m$, the thermodynamic and friction forces act in the sense of open the liquid column and gravitational force acts in the sense of close the liquid column. Due to the reasons presented previously, from section $s = \xi_m$ to $s = \xi_f$, the thermodynamic force acts in an effort to become the positive pressure gradient and close the liquid column. When the Froude number is much small, the liquid column opens and soon after closes since the order of magnitude of gravitational force is much large compared as other forces.

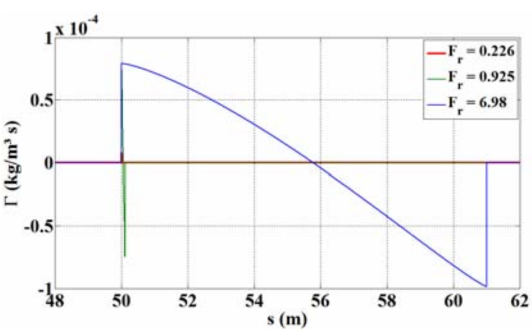
The figures 5a and 5b show plots of energy dissipation and hysteresis for each Froude number. Although the phase change is described as an irreversible process (in other words: $\beta = 10^{-8}$ kg²/m³ J s), the amount of energy dissipation during the processes are different.



(a)



(b)



(c)

Figure 5. (a) Rate of energy dissipation in the slack flow region (b) Hysteresis phenomenon (c) mass rate of phase change as a function of pipe length for different values of Fr

In the Fig. 5c the mass rate of phase change in slack flow region is represented graphically for each Froude number. It can be observed that the plot of mass rate of phase change as function of pipe length is not symmetric in slack flow region. It show that part of vapor mass generated during the open liquid column do not return to liquid state during and after closure liquid column.

5. CONCLUSIONS

The purpose of this work is to present a mechanical model to describe the cavitating flow in hydraulic pipelines. Although the model is capable to describe the cavitation phenomenon in unsteady as well as steady states, the applications presented in this work are restricted to slack flow, which take place in steady states. The flow is assumed to be homogeneous and isothermal. The fluid is treated as a pseudo-mixture, comprising the liquid and the vapor phases. Both phases are assumed to be compressible and to coexist at every material point and time instant. The balance equations of mass for each of the phases are considered in the model, along with one balance equation of momentum for the mixture as a whole, within an onedimensional context. The phase change transformation is properly accounted for as an irreversible process. The main dimensionless groups are identified and their influence on the slack flow phenomenon quantified by means of numerical simulations. The obtained results show that model is capable to mimic coherently both the opening as well as the closure of the vapor cavity.

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