HYDRODYNAMICS AND HEAT TRANSFER SIMULATION FOR TWO-PHASE INTERMITTENT FLOW IN HORIZONTAL PIPES

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Abstract. *Two-phase flows with heat transfer are found in many engineering applications. One of them is the conduction of oil and gas in the deep ocean, where exists a temperature gradient due to the difference between the temperature in the source and that from the surrounding environment. In liquid-gas two-phase flows, one of the most frequent patterns is the slug flow. This flow pattern is characterized by the alternate succession of two structures: an aerated slug and an elongated gas bubble, which constitutes a unit cell. In spite of the unit cell properties variation with time, it can be modeled as stationary if mean time values are used. In that context, the present work presents a mechanistic one-dimensional stationary model for the calculation of the main hydrodynamical and heat transfer parameters of slug flow. Based on mass, momentum and energy balances on the unit cell, an implicit algebraic equation system will be obtained. The solution for a unit cell is found through an iterative process and then propagated along the pipe, assuming that the pressure and temperature gradients are linear. As a result, geometric characteristics, phase velocities, pressure and temperature along the pipe can be known. From the temperature profile, the two-phase heat transfer coefficient can be calculated, which can be compared with some correlations found in the literature. Results show good agreement with the reported data in the literature.*

Keywords*: two-phase flow, intermittent flow, convection, heat transfer*

1. INTRODUCTION

The intermittent liquid-gas flow occurs over a wide range of gas and liquid flow rates. It is characterized by the alternate succession of two structures: an aerated slug and an elongated gas bubble, which constitute a unit cell. Each of the components of the unit cell has its characteristics changed across the time and space.

There are many hydrodynamic models based on the unit cell concept considering stationary flow. This approximation physically represents that one single unit cell is repeated along the time and space. Examples of these models are: Dukler and Hubbard (1975) for horizontal flow, Fernandes et al (1983) for vertical flow and Taitel and Barnea (1990) for horizontal, vertical and inclined flow. Nevertheless, these models are used just to evaluate one unit cell, ignoring its propagation on the whole pipe.

The study of the heat transfer in intermittent flow is a matter of importance as it has many industrial applications. One of them is the oil transfer in long production lines, where the pipes are exposed to harsh external conditions. This interaction causes heat exchanges between the two-phase mixture and the surrounding environment. As a result, the temperature of the fluids will vary along the pipeline producing changes in the in-situ properties of the fluids like the density or the liquid viscosity, directly related to the pressure drop. In addition, wax deposition or hydrates formation can occur, as these processes depend on the thermodynamic equilibrium, which is directly related with the temperature (Lima, 2009).

Most of the studies on the two-phase flow heat transfer mainly concerns to the calculation of the heat transfer coefficient. The majority of these studies proposes correlations found through experimentation. For example, Kim and Ghajar (2006) proposed an accurate correlation as a function of the fluid properties and the flow rates. Camargo (1991) proposed a mechanistic model for intermittent flow based on the hydrodynamic parameters.

On the other hand, few works have presented temperature simulation through energy balance. One of them is the unified model presented by Zhang et al (2006) where analytical solutions for the temperature are found for different two-phase patterns assuming that the hydrodynamic characteristics of the flow are known.

In that context, the objective of the present work is to propose a methodology to calculate the hydrodynamic and heat transfer parameters of two-phase intermittent flow. The flow is modeled through a stationary mechanistic model in a lagrangian frame of reference. Mass, momentum and energy balances are performed in order to obtain the governing equations. All the parameters are calculated for a unit cell and then propagated along the pipe considering the gas compressibility due to pressure and temperature changes. The liquid can be modeled as incompressible and the gas as ideal as long as the properties of the simulated fluids are far from the saturation region, thus, no phase change occurs.

The problem to be solved considers a two-phase mixture in the intermittent pattern flowing through a horizontal pipe with round cross section. The pipe is surrounded by environmental conditions, represented by an external heat transfer coefficient.

In the present work, the hydrodynamic equations are based on the stationary model of Taitel and Barnea (1990). Equations for the heat transfer are deduced using energy balance in each of the unit cell components. It results in an algebraic equations system. The algebraic model presented by Freitas et al (2008) is applied in the calculation process and the results are validated with data from the literature. Finally, results are discussed properly.

2. HYDRODYNAMIC MODEL

The hydrodynamic model is based on the stationary one-dimensional model presented by Taitel and Barnea (1990). The basic assumptions for the model development are: (i) periodic flow, (ii) Newtonian fluids with constant properties, (iii) guaranteed slug flow pattern at the entrance and exit of the pipe, (iv) ideal gas and incompressible liquid, (v) uniform distribution of the bubbles along the slug.

The physical model is shown in Figure 1a. The studied control volume is the unit cell, composed by a liquid slug (length L_S) and an elongated bubble (length L_B). The unit cell is moving at the translational velocity U_T and each of the components of the unit cell has a different velocity. It can be observed on the cross section of the slug (Figure 1b) that the liquid wets all the perimeter of the pipe, while in the bubble's cross section (Figure 1c) the liquid is stratified on the bottom of the pipe due to gravity.

The concept of superficial velocity (j) is defined as the volumetric flow of the phase per unit of area. Considering constant gas mass flow rate and the state equation of perfect gases, the superficial velocities of the gas (j_G) in two sections can be expressed as a function of pressure and temperature, as seen on Eq. (1):

$$j_L = \frac{\dot{m}_L}{\rho_L A} \quad ; \quad j_G = \frac{\dot{m}_G}{\rho_G A} \quad ; \quad j_G \frac{P_G}{T_G} = j_{Gs} \frac{P_{Gexit}}{T_{Gexit}} \tag{1}$$

where $\dot{m}$ is the mass flow rate, ρ the fluid density, P the pressure and T the temperature and A the pipe cross section area. Sub-indexes L and G refer to liquid and gas respectively and the sub-index *exit* indicate the exit of the pipe, taken as a reference.

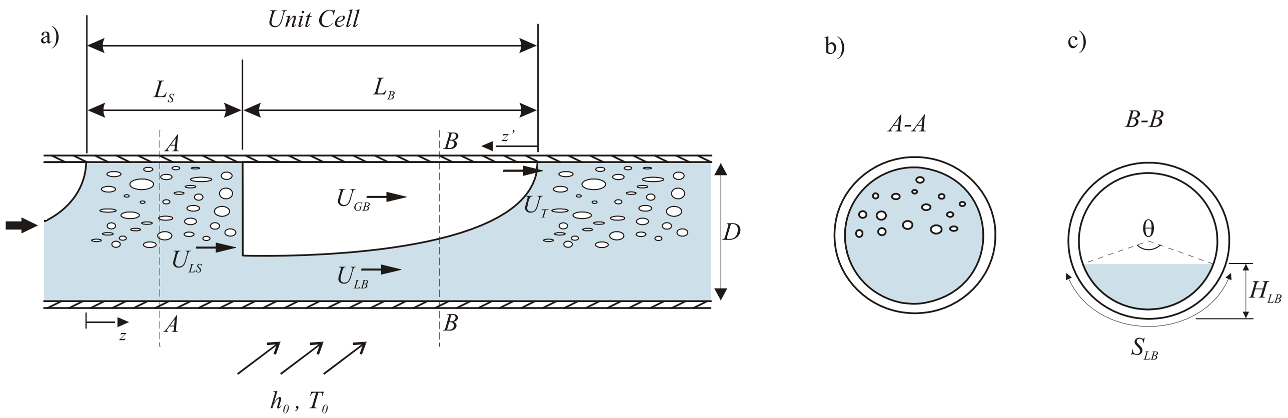

Figure 1: Physical model for the intermittent flow (a); Pipe cross sections in the slug (b); and elongated bubble (c).

The mixture velocity (J) in Eq. (2) is defined as the sum of the superficial velocities. In addition, J can also be expressed as a function of the velocities in the slug (U_{LS} and U_{GS}) just as reported by Shoham (2006).

$$J = j_L + j_G \quad ; \quad J = U_{LS} R_{LS} + U_{GS} \left(1 - R_{LS}\right) \tag{2}$$

where R_{LS} , R_{LB} and R_{GB} are the volume fraction of liquid in the slug, liquid film and gas bubble respectively.

Expressions for the liquid film and gas bubble velocities are deduced from mass balances for each phase in the entire unit cell. The reference system is non-inertial and moves with the velocity of the bubble nose, which is called translational velocity (U_T). Velocities for the liquid film (U_{LB}) and the elongated bubble velocities (U_{GB}) are given respectively by:

$$U_{LB} = U_T - \left(U_T - U_{LS}\right) \frac{R_{LS}}{R_{LB}} \quad ; \quad U_{GB} = U_T - \left(U_T - U_{GS}\right) \frac{\left(1 - R_{LS}\right)}{R_{GB}} \tag{3}$$

In order to calculate the bubble length, it is studied the hydrodynamics of the film which is modeled as channel flow with free surface according to Taitel and Barnea´s model (1990). They present Eq. (4) to define the geometry of the gas bubble:

$$\frac{dH_{LB}}{dz^*}=\frac{\tau_{LB}\frac{S_{LB}}{A_{LB}}-\tau_{GB}\frac{S_{GB}}{A_{GB}}-\tau_i S_i\left(\frac{1}{A_{LB}}+\frac{1}{A_{GB}}\right)}{-\rho_L\left(U_T-U_{LB}\right)\frac{\left(U_T-U_{LS}\right)R_{LS}}{R_{LB}^2}\frac{dR_{LB}}{dH_{LB}}-\rho_G\left(U_T-U_{GB}\right)\frac{\left(U_T-U_{GB}\right)\left(1-R_{LS}\right)}{\left(1-R_{LB}\right)^2}\frac{dR_{LB}}{dH_{LB}}} \tag{4}$$

where H_{LB} is the liquid height on the film, τ is the shear stress, S the wet perimeter, and LS, LB, GB and i the indexes for the liquid slug, liquid film, gas bubble and interface respectively.

Eq. (4) is numerically integrated to obtain bubble length (L_B), mean gas volume fraction ($\overline{R_{GB}}$) and equilibrium liquid film height. Note in Figure 1 that coordinate z^* is positive in the left direction, which is the reason why the slope must be negative. The integration should start in $H_{LB}\ (z^*=0)=D$, however, under some flow rate conditions, the slope is positive in $H_{LB}=D$. As the integration should not start until that slope is negative, a lower $H_{LB}(z^*=0)$ should be selected. Tests evidenced that a suitable point to start the integration is $H_{LB}(z^*=0)=0.9D$. Furthermore, a stop condition must be established in order to fix a finite bubble length. Liquid mass balance on the unit cell can be used, which is given by Eq. (5).

$$\dot{m}_L=U_{LS}AR_{LS}\rho_L\frac{L_S}{L_S+L_B}+\frac{1}{L_S+L_B}\int_0^{L_B}U_{LB}AR_{LB}\rho_L dz \tag{5}$$

Also, the unit cell frequency (*freq*) is used, which is defined as $freq = U_T/(L_S+L_B)$. Physically it represents the inverse of the time of a unit cell passage. Assuming that the frequency is known, the unit cell length in Eq. (5) can be substituted as a function of the translational velocity U_T and the frequency. In this sense, Eq. (4) is integrated until the liquid mass balance in Eq. (5) is satisfied. Frequency is evaluated with a constitutive equation, presented in the next section.

2.1 Constitutive Equations

In order to have the same number of variables and equations, it is necessary to use some constitutive equations. All the correlations in this section can be found in Omgba (2006). The first one is used to calculate the translational velocity, which is based on the correlation of Nicklin (1962). In this case, the coefficients proposed by Bendiksen (1980) are used.

$$U_T=c_0J+c_1\sqrt{gD}\quad\rightarrow\quad\frac{J}{\sqrt{gD}}\begin{cases}>3.5\rightarrow c_0=1.20 & c_1=0.00\\<3.5\rightarrow c_0=1.05 & c_1=0.54\end{cases} \tag{6}$$

where g is the gravity acceleration, D the pipe diameter.

The volume fraction of liquid or liquid holdup (R_{LS}), is calculated by the Malnes (1982) correlation:

$$R_{LS}=1-\frac{1}{1+\frac{83}{Fr\cdot Bo^{0.25}}}\qquad\begin{cases}Fr=J/\sqrt{gD}\\Bo=\left(\rho_L-\rho_G\right)gD^2/\sigma\end{cases} \tag{7}$$

where σ is the superficial stress coefficient, Fr is the Froude number and Bo is the Bond number.

A third constitutive equation is needed for the velocity of the dispersed bubbles (U_{GS}) in the liquid slug. Normally, this velocity is modeled as the superposition of the mixture velocity J and the velocity of a rising bubble in an infinite stagnant liquid (Omgba, 2006). The effects of the rising velocity are related to the gravity direction; however, as the pipe is horizontal, these effects can be ignored. That way, it can be assumed that the dispersed bubbles in the slug are moving with the mixture velocity. ($U_{GS} = J$).

For the unit cell frequency, Hill and Wood (1990) proposed a correlation which uses the flow parameters in stratified condition, found through mass flow rates using the Taitel and Dukler (1976) model.

$$freq = \frac{0.000761}{D}\left[\left(U_G - U_L\right)\frac{R_L}{1-R_L}\right]_{STRATIFIED} \tag{8}$$

where the parameters in brackets are evaluated in the stratified condition.

A constitutive equation must be used to calculate the frictional force. The shear stress is expressed as a function of the Fanning friction factor. Equations used are presented in Table 1.

Table 1: Constitutive equations for the frictional force (1)

Hydraulic Diameter (D_H)	**Reynolds Number** (Re)	**Friction Factor** (f)	**Shear Stress** (τ)
$D_{HF} = 4\frac{R_F A}{S_{LF}}$	$\mathrm{Re}_F = \frac{\rho_F U_F D_F}{\mu_F}$	$f_F = 0.001375\left[1+\left(2\cdot 10^4\frac{\varepsilon}{D}+\frac{10^6}{\mathrm{Re}_F}\right)\right]$	$\tau_F = \frac{f_F \rho_F U_F^2}{2}$

(1): The index F changes according to the element evaluated, being LS for liquid slug, LB for liquid film and GB for elongated gas bubble.

2.2 Pressure Drop and Pressure Gradient

The pressure drop is modeled according to the physical model proposed by Dukler and Hubbard (1975). It considers that the pressure is uniform along the gas bubble. Whereas the pressure in any cross section is uniform too, the pressure drop across the liquid film and gas bubble is zero. In the liquid slug, the region near the wake behind the gas bubble is called mixture region and is characterized by high turbulence. Great part of the pressure drop is concentrated in that mixture region. According to Taitel and Barnea (1990) the expression for the horizontal flow pressure drop in a unit cell, based on momentum balances in the liquid slug is given by:

$$\Delta P_U = \tau_{LS}\frac{S_{LS} L_S}{A} + \Delta P_{mix} \tag{9}$$

where ΔP_U is the pressure drop in a unit cell and slug and ΔP_{mix} is the pressure drop in the mixture region.

As observed on Eq. (9), the pressure drop in a unit cell is generated by the frictional force and the pressure loss in the mixture region. Taitel and Barnea (1990) showed that the pressure drop on the mixture region occurs due to the acceleration from the film velocity to the slug velocity and due to the hydrostatic force from the liquid height difference between the slug and the film. Taitel and Barnea (1990) also showed through momentum balance on the film that these two pressure losses can be expressed as the film frictional and gravitational weight terms. Considering that, the final expression for the pressure drop is given by:

$$\Delta P_U = \tau_{LS}\frac{S_{LS} L_S}{A} + \int_0^{L_B}\frac{\tau_{LB} S_{LB}}{A}dz \tag{10}$$

For this work, the pressure distribution in a unit cell is considered linear, so the pressure gradient λ_U^P is constant. In this way, the pressure can be calculated at any point using a linear equation found through the integration of Eq. (11).

$$\frac{dP_U}{dz} = \lambda_U^P = \frac{\Delta P_U}{L_S + L_B} \tag{11}$$

$$P_U(z) = P_{exit} - \lambda_U^P\left(L - z\right) \tag{12}$$

3. HEAT TRANSFER MODEL

The development of the heat transfer model is based on the first law of thermodynamics applied to control volumes in stationary regime (White, 2003)

$$\dot{Q} = \int_{SC} \left(i + \frac{1}{2}U^2 + e_p \right) \rho \vec{V_r} \cdot d\vec{A} \quad (13)$$

where i is the specific enthalpy, $\vec{V_r}$ is the relative velocity and $\dot{Q}$ is the heat externally transferred to the flow and e_p the potential energy. The mechanism of heat transfer is the forced convection. In addition, kinetic energy is small compared to the enthalpy and variation in potential energy is null due to the horizontal position of the pipe.

Considering the hypotheses explained before, the Eq. (13) is applied in each of the three differential control volumes specified in Figure 2: the elongated bubble, the slug and the liquid film. Despite the heat transfer, the state of the fluids is far from the saturation region, which is the reason why no phase change occurs. In addition, the expression for the enthalpy of an incompressible liquid and an ideal gas can be written as a function of the temperature (Moran and Shapiro, 2006), just as observed in Eq. (14):

$$i_L = C_L T_L \quad ; \quad i_G = Cp_G T_G \quad (14)$$

where C_L is the specific heat of the liquid and Cp_G the specific heat at constant pressure of the gas.

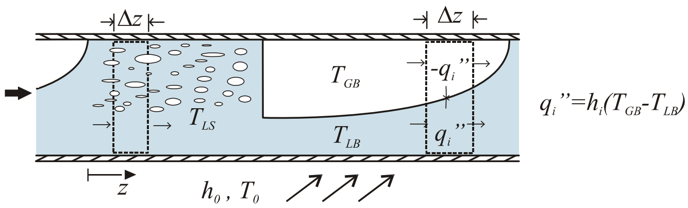

Figure 2: Stationary balance of energy on a unit cell.

Eq. (13) is applied to the liquid slug, considering that the only source of heat is the contact with the duct wall. Heat can be expressed by the Newton's law of cooling. Besides that, the term $\rho \vec{V_r} \cdot d\vec{A}$ applied to the slug physically represents the mass flow rate of the liquid. Considering the definition of derivative in the infinitesimal control volume, the following differential equation can be deduced:

$$\frac{dT_{LS}}{dz} = \frac{h_{LS}^G S_{LS}}{\dot{m}_L C_L} \left(T_0 - T_{LS} \right) \quad (15)$$

In the case of the elongated bubble and the liquid film, besides the heat provided by the duct wall, there is an amount of heat exchanged between the liquid and the gas in the interface. As the heat lost by one phase is gained by the other, the expression for the interface heat in the bubble is the negative of the interface heat in the film. In this sense, the heat transfer equations for the elongated bubble and the liquid film are respectively given by:

$$\dot{m}_G Cp_G \frac{dT_{GB}}{dz} = h_{SG}^G S_{GB} \left(T_0 - T_{GB} \right) - h_i S_i \left(T_{GB} - T_{LB} \right) \quad (16)$$

$$\dot{m}_L C_L \frac{dT_{LB}}{dz} = h_{LB}^G S_{LB} \left(T_0 - T_{LB} \right) + h_i S_i \left(T_{GB} - T_{LB} \right) \quad (17)$$

The solution of Eq. (15) for the slug temperature is easily obtained by direct integration. For convenience, the coordinate system is set to zero for each unit cell at the beginning of the slug. It is assumed that the temperature of the dispersed gas bubbles in the slug is the same as in the liquid. The boundary conditions are the following:

- Slug ($0 \le z \le L_S$) → $T_{LS}\ (z=0) = T_{LS0}$
- Film ($0 \le z \le L_B$) → $T_{LB}(z=0)=T_{LB0}=T_{LS}(z=L_S)$
- Bubble ($0 \le z \le L_B$) → $T_{GB}(z=0)=T_{GB0}=T_{LS}(z=L_S)$

On the other hand, equations (16) and (17) constitute a differential equations system, whose solution is obtained by mathematical analysis. Finally, explicit expressions for the temperature of each component of the unit cell are found. For the slug, the expression is:

$$T_{LS} = T_{LS0} - \left(T_{LS0} - T_{LSi}\right)\exp\left(-\frac{h_{LS}^G S_{LS}}{Cp_L \dot{m}_L} z\right) \tag{18}$$

For the liquid film and elongated bubble:

$$T_{LB} = \varphi_{LB1}\exp\left(r_1 z\right) + \varphi_{LB2}\exp\left(r_2 z\right) + \varphi_{LB3} \tag{19}$$

$$T_{GB} = \varphi_{GB1}\exp\left(r_1 z\right) + \varphi_{GB2}\exp\left(r_2 z\right) + \varphi_{GB3} \tag{20}$$

which constants are specified on Table 2.

Table 2: Constants for the analytic solution of the film and bubble temperatures.

$a_1 = \frac{h_{LB}^G S_{LB}}{\dot{m}_L Cp_L} + \frac{h_i S_i}{\dot{m}_L Cp_L}$	$a_2 = \frac{h_i S_i}{\dot{m}_G Cp_G}$	$b_1 = \frac{h_i S_i}{\dot{m}_L Cp_L}$	$b_2 = \frac{h_{GB}^G S_{GB}}{\dot{m}_G Cp_G} + \frac{h_i S_i}{\dot{m}_G Cp_G}$	$c_1 = \frac{h_{LB}^G S_{LB}}{\dot{m}_L Cp_L} T_0$	$c_2 = \frac{h_{GB}^G S_{GB}}{\dot{m}_G Cp_G} T_0$

$r_1 = \frac{-\left(a_1+b_2\right) - \sqrt{\left(a_1+b_2\right)^2 - 4\left(a_1 b_2 - a_2 b_1\right)}}{2}$	$\varphi_{GB3} = \frac{c_1 a_2 + c_2 a_1}{a_1 b_2 - a_2 b_1}$	$r_2 = \frac{-\left(a_1+b_2\right) + \sqrt{\left(a_1+b_2\right)^2 - 4\left(a_1 b_2 - a_2 b_1\right)}}{2}$
$\varphi_{GB1} = T_{GB0} - \frac{a_2 T_{LB0} - \left(r_1+b_2\right)\left(T_{GB0} - \varphi_{GB3}\right) + c_2 - b_2\varphi_{GB3}}{r_2 - r_1} - \varphi_{GB3}$		$\varphi_{GB2} = \frac{a_2 T_{LB0} - \left(r_1+b_2\right)\left(T_{GB0} - \varphi_{GB3}\right) + c_2 - b_2\varphi_{GB3}}{r_2 - r_1}$
$\varphi_{LB1} = \frac{\varphi_{GB1} r_1 + b_2 \varphi_{GB1}}{a_2}$	$\varphi_{LB2} = \frac{\varphi_{GB2} r_2 + b_2 \varphi_{GB2}}{a_2}$	$\varphi_{LB3} = \frac{1}{a_2}\left(b_2\varphi_{GB3} - c_2\right)$

3.1 Heat Transfer Coefficient

As presented before, the duct is surrounded by an external cooling flow. For the heat transfer coefficient, it must be considered the external convection, the conduction in the duct thickness and the internal convection between the duct wall and the two-phase mixture. Thus, it is used the concept of global heat transfer coefficient, based on thermal resistances (Incropera, 2008):

$$h_F^G = \frac{1}{\frac{1}{h_F} + \frac{D}{2k_c} Ln\frac{D_e}{D} + \frac{D}{D_e h_0}} \tag{21}$$

where k_c is the thermal conductivity of the pipe material, D_e is the external diameter, h_0 is the convective coefficient of the external flow and h_F is the heat transfer coefficient for each component of the unit cell. h_F is calculated as a one-phase coefficient. According to the experimental studies of Lima (2009), the expression that better adjusts to the one-phase flow behavior is the Gnielinski (1976) correlation:

$$h_F = \frac{\left(f_F/8\right)\left(\mathrm{Re}_F - 1000\right)\mathrm{Pr}_F}{1 + 12,7\left(f_F/8\right)^{0,5}\left(\mathrm{Pr}_F^{2/3} - 1\right)}\frac{k_F}{D_{HF}} \quad where \quad f_F = \left[0,079 \cdot Ln\left(\mathrm{Re}_F\right) - 1,64\right]^{-2} \tag{22}$$

This correlation shows better adjustment to the experimental data because it has an explicit dependence on the friction factor, which has considerable influence on the heat transfer coefficient (Bejan, 2004). In the case of the liquid slug, in order to simulate the higher turbulence occurring in this region, the heat transfer coefficient h_{LS} is increased by 30% (Camargo, 1991).

3.2 Two-Phase Heat Transfer Coefficient

Once the temperatures are known in a unit cell, it is possible to calculate a two heat transfer coefficient for the internal convection based on the Newton's cooling law. It is calculated one coefficient for each unit cell, considering the temperature drop from $z=0$ to $z=L_S+L_B=L_U$

$$h_{TP} = \frac{\dot{Q}}{\pi D.L_U.\Delta T} \qquad \begin{cases} \dot{Q} = \int_0^{L_S} h_{LS}^G S_{LS}\left(T_{LS} - T_0\right)dz + \int_0^{L_B}\left[h_{LB}^G S_{LB}\left(T_{LB} - T_0\right) + h_{GB}^G S_{GB}\left(T_{GB} - T_0\right)\right]dz \\ \Delta T = \dfrac{\left(T_{LBs} - T_{ws}\right) - \left(T_{LSe} - T_{we}\right)}{Ln\left(\dfrac{T_{LBs} - T_{ws}}{T_{LSe} - T_{we}}\right)} \end{cases} \tag{23}$$

where $\dot{Q}$ is the total heat transferred to the fluids, ΔT is the logarithmic mean temperature difference, T_{LBs} and T_{LSe} are the mean temperatures at the exit and at the entrance of the unit cell respectively, T_{ws} and T_{we} are the mean temperatures on the internal wall at the exit and at the entrance. Eq. (23) calculates the internal convection coefficient between the two-phase mixture and the duct wall. Temperature in the internal wall is given by Eq. (24):

$$T_w = T_F - \frac{h_F^G D}{h_F D_e}\left(T_F - T_0\right) \tag{24}$$

3.3 Unit cell mean temperature

The evaluation of the unit cell mean temperature is important to compare the results of the model with the experimental data. Zhang et al (2006) calculated this temperature as a function of the liquid temperatures. In addition, using the concept of frequency and translational velocity, the unit cell temperature is calculated based on the lengths:

$$T_U = \frac{\int_0^{t_F} T_{LB}dt + \int_0^{t_F} T_{LS}dt}{\left(L_S + L_B\right)/U_T} = \frac{\int_0^{L_B} T_{LB}dz + \int_0^{L_S} T_{LS}dz}{L_S + L_B} \tag{25}$$

The fact of using just the liquid temperature is based on the thermal capacity (ρCp) being considerably greater in the liquid than in the gas. For example, considering water (*ρ=1000 kg/m^3, Cp=4180 J/kg.K*) and air (*ρ=1.2 kg/m^3, Cp=1003 J/kg.K*) at normal conditions, the thermal capacity of the water is almost 4000 times greater than that for the gas.

3.4 Temperature gradient

Just as the assumption for the pressure, the concept of temperature gradient in a unit cell is also introduced for the temperature. It is simply defined as the difference between the temperature in the entrance of the slug and the temperature at the exit of the liquid film divided by the total unit cell length. This gradient is constant in a unit cell, since the variation of the mean unit cell temperature is linear.

$$\lambda_U^T = \frac{dT_U}{dz} = -\frac{T_{LS}\big|_{z=0} - T_{LB}\big|_{z=L_B}}{L_S + L_B} \tag{26}$$

$$T_U(z) = T_{entrance} - \lambda_U^T z \tag{27}$$

4. CALCULATION PROCEDURE

The calculation procedure is based on the algebraic hydrodynamic model of Freitas et al (2008). The solution starts with the determination of the flow hydrodynamics through the bubble design and pressure drop equations in (4) and (10) respectively. Then, from the hydrodynamics results, the heat transfer equations are applied to obtain the temperature profile. Input data necessary for the solution of the problem are: the properties of the fluids, the superficial velocities of each phase at the exit of the pipe (or mass flow rates), the pressure at the exit (P_{exit}), fluid temperature at the entrance and external conditions (external temperature T_0 and heat transfer coefficient h_0). The calculation procedure has two stages: calculation of the unit cell at the entrance and propagation along the pipe.

STAGE 1: Properties at the pipe entrance ($z = 0$).

a) Estimate a pressure gradient and temperature gradient at the pipe entrance (λ_U^P and λ_U^T).
b) Calculate the following flow parameters at the pipe entrance: j_G [Eq. (1)], J, U_T [Eq. (6)], R_{LS} [Eq. (7)], U_{LS} [Eq. (2)].

c) The bubble length L_B is obtained by the numerical integration of Eq. (4). Integrate until Eq. (5) is satisfied.
d) Obtain the liquid slug length L_S and the gas fraction on the bubble R_{GB},
e) Calculate the pressure drop through Eq. (10) and the new pressure gradient through (11)
f) Assume $T_{LS}(z=0)$, calculate the temperatures distribution using equations (18), (19) and (20). Calculate the mean temperature in the unit cell using (25) and the temperature gradient using (26). Recalculate the $T_{LS}(z=0)$ and repeat the process until the mean temperature calculated converge with the temperature given as input.
g) Using the new pressure and temperature gradients, recalculate the parameters until the gradients converge.

STAGE 2: Propagation of the properties along the pipe

a) With the gradients found at the entrance, evaluate the pressure and unit cell mean temperatures on the other points along the pipe using equations (12) and (27) respectively.
b) Obtain the unit cells in the evaluation points through b), c), d), e), f) and g) on Stage 1. That way, new gradients are obtained for each point.
c) Evaluate the flow parameters with b), c), d), e), f) and g) using the new gradients found in b). New pressure and temperature gradients are calculated.
d) Repeat c) until the gradients converge with 0.01%.

5. RESULTS

The proposed model is applied to an air-water flow cooled by an external water flow. Results are compared with experimental studies obtained from Lima (2009). In this experiment, the pressure and temperature were measured at the entrance and at the exit of the pipe. In addition, there was a transparent section of the pipe to visualize the flow pattern and to measure the geometric characteristics of the bubbles. The characteristics of one specific test of each study are specified in the table 3.

Table 3: Input values for the simulations.

Air and Water Flow			
Pipe Length [m]	6.07	Exit pressure [kPa]	171.0
Pipe diameter [mm]	52.00	External fluid temperature [K]	282.4
Exit gas superficial velocity [m/s]	0.2825	Entrance two-phase temperature [K]	307.7
Liquid superficial velocity [m/s]	1.378		

The results of the simulations using the proposed model are presented in the Table 4. Good agreement for the pressure drop and the translational velocity are observed. However, there is a considerable deviation for the slug and bubble length. Probably this occurred due to the stationary flow hypothesis, as the bubble length is directly related to the intermittency. Another reason for this to happen is that a stationary model cannot predict unsteady phenomena like coalescence. Just as one-phase flow, the pressure drop is directly influenced by the pipe's diameter; the smaller the diameter the grater is the pressure drop.

Table 4: Results of the simulations.

Air and Water	**Lima (2009)**	**Model**	**Error (%)**
Bubble Length at the exit [m]	0.340	0.213	-37.35
Slug Length at the exit [m]	0.82	1.018	24.14
Translational Velocity [m/s]	2.10	2.15	2.38
Pressure Drop [kPa]	2.718	2.470	9.10
Exit two-phase temperature [K]	304.60	304.24	-0.11
Heat Transfer Coefficient [w/m^{2}K]	7366	7121	-3.32

In Figure 3a the bubble geometry is observed. The bubble presents a curved profile at the nose and a quasi linear profile at the tail. Due to the high liquid superficial velocity, a low gas volume fraction is observed, as the liquid-gas interface is over the center of the pipe. In Figure 3b, the pressure distribution along the pipe considering six evaluation points is observed as the pressure presents a linear behavior. Also, it is observed that the model tends to slightly underestimate the pressure drop.

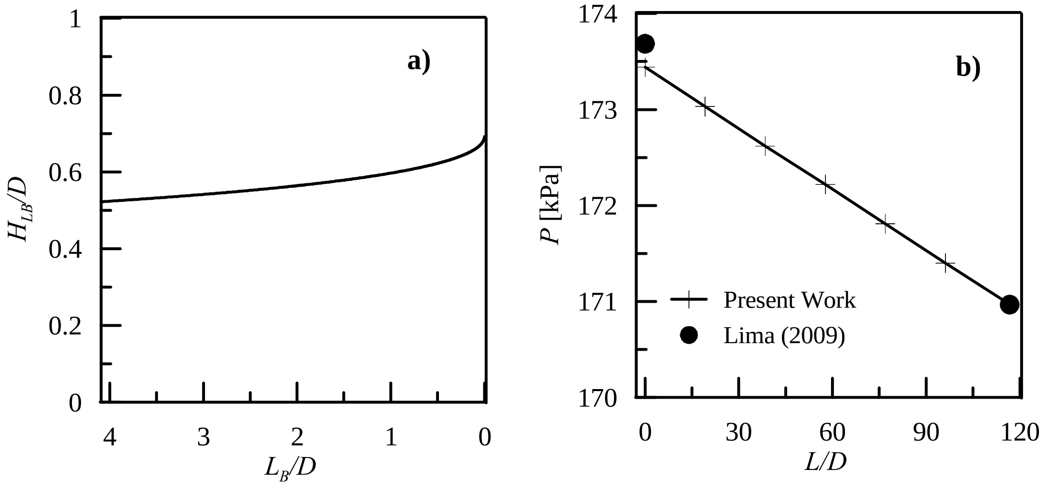

Figure 3. a) Geometric design of the exit bubble. b) Pressure distribution along the pipe length.

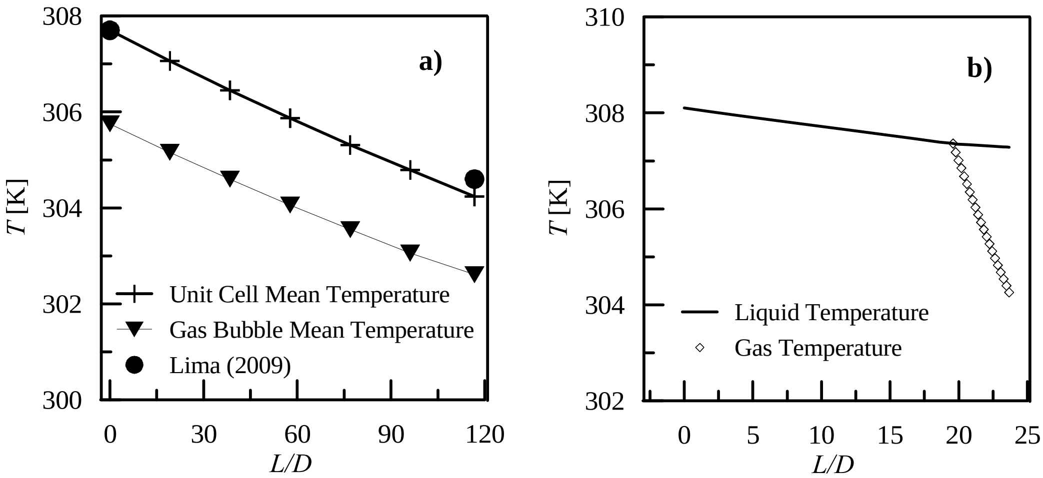

Figure 4. a) Mean temperatures along the pipe. b) Distribution of the temperatures along the entrance unit cell.

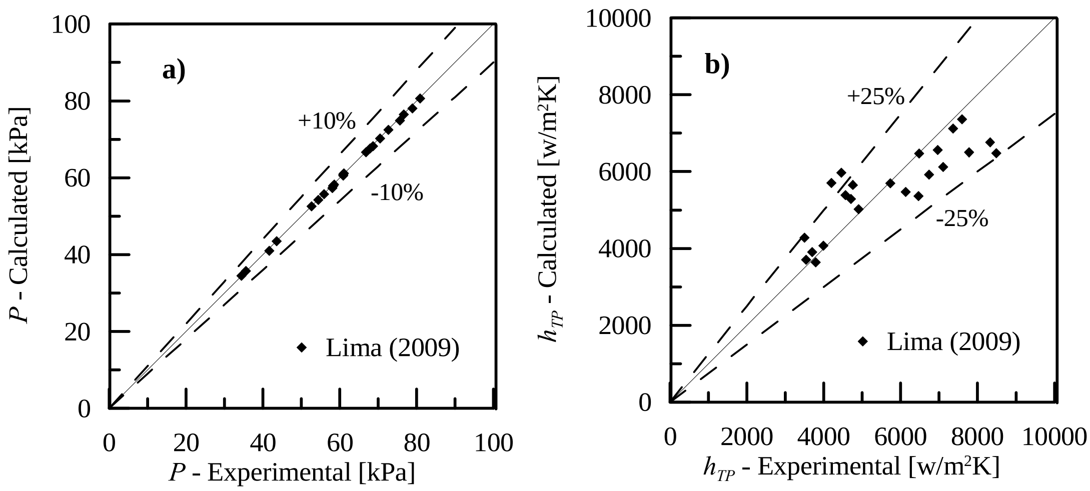

Figure 5: a) Calculated vs Experimental Gauge pressure at the entrance of the pipe. b) Calculated vs experimental two-phase heat transfer coefficient.

In Figure 4a the temperature distribution along the pipe is shown. The calculated mean unit cell temperature shows good agreement with the experimental data, which validates the expression in Eq. (25). It is observed that the mean temperature of the gas bubble is always considerably lower than that for the whole unit cell. In Figure 4b, the temperature profile of the unit cell at the exit is presented. Despite the temperature function being exponential, a linear behavior is observed which also validates the hypothesis in Eq. (27). The high temperature drop in the bubble region shows the influence of its low thermal capacity. This means that the gas requires less heat to modify its temperature, so despite the same amount of heat being provided, the gas temperature decreases more.

Lima (2009) reported 23 more tests with different rates of flow which were also evaluated in the model. In Figure 5a and 5b the gauge pressure at the entrance and the two-phase heat transfer coefficient are plotted against the experimental data. As seen on Figure 5a and 5b the calculated parameters gave good predictions. More than 92% of the data is confined in the 25% error range.

6. CONCLUSIONS

Hydrodynamics and heat transfer simulation on intermittent flow was presented. The model is based on stationary balances of momentum and energy. It calculates the parameters for a unit cell and then propagates the results along the pipe. As the model is composed just by algebraic equations its implementation is easy and its low computing time turns it a powerful tool to predict velocities, pressure drop and temperatures.

It is observed that the pressure has a quasi linear behavior; however, the pressure gradient is different for each unit cell. From the temperature profile, it is observed that temperature of the liquid phase is dominant. Although the equation for the liquid temperature is an exponential function, its distribution is quasi linear due to its high specific heat. As seen on the results, the hypothesis of constant pressure and temperatures gradients is correct.

Good agreement with experimental data is observed for the pressure drop, the temperatures and the heat transfer coefficient. However, it fails to predict bubble and slug lengths as they depend on the intermittency at the entrance. As the model considers periodic unit cells, the unsteady effects of intermittency are ignored.

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