

IHCP TECHNIQUE BASED ON GREEN'S FUNCTIONS AND DYNAMIC OBSERVERS APLIED TO ORTHOGONAL CUTTING

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Abstract. *The temperature fields generated in the cutting processes is subject of much research. The studies of these thermal fields in machining are very important for the development of new technologies aimed to increase tool life and to reduce production costs. Cutting temperatures have strongly influenced both the tool life and the metallurgical state of machined surfaces. Since the direct temperature measurements at the tool-piece interface are very complex this work proposes the estimation of the temperature and the heat flux at the tool-piece interface using the inverse heat conduction problem technique. The aim of the present paper is to develop a three-dimensional inverse algorithm in transient conditions for heat flux and machining temperature estimation. The thermal model is obtained by the numerical solution of the transient 3D heat diffusion equation that considers both the tool and tool holder assembly. The thermal model is developed by using the finite elements method in ANSYS[®] code with a mesh of 149.150 elements and SOLID70 element as shown. The inverse solution is performed by using the technique based on Green's functions and dynamic observers. Simulated cases were investigated and presented excellent results.*

Keywords: *Heat Flux, Machining processes, FEM, IHCP, Dynamic Observer, Green's Function*

1. INTRODUCTION

Temperature knowledge at the cutting interface during a machining process is very important. Cutting temperatures have strongly influenced both the tool life and the metallurgical state of machined surfaces. Several studies can be largely found at the literature regarding this subject.

Numerical simulation methods applied to machining process have been increasing during the last decade. Nowadays numerical simulation techniques are widely required to predict and optimize machining processes, combining cutting parameters in order to obtain better machined surfaces, or even to reduce temperature during a machining process. Several authors have proposed numerical and experimental techniques trying to predict temperature field at the tool and piece during machining process. Since the direct temperature measurements at the tool-piece interface are very complex the inverse solution represents a good approach.

Using numerical analysis Jen & Gutierrez (2000) investigated the tool temperature distribution considering different thermal properties. For this purpose the authors developed a three-dimensional transient thermal model with a FEM (Finite Element Method) formulation. For the solution the authors considered a known heat flux at the cutting interface and neglected convection effects and contact resistance. In this study the tool geometry influence under the temperature was evaluated. J. Yvonnet et al (2006) proposed an inverse technique to indentify the generated heat flux at the cutting interface and the heat transfer coefficient between tool and the environment during a typical orthogonal cutting process.

Carvalho (2006) developed a three-dimensional numerical model of a tool-holder set using Finite Difference Method. The heat flux at the cutting interface was estimated by using golden section that is an optimization technique with polynomial approach. The author used this procedure to estimated heat flux generated during a turning process. It was possible to analyze the high temperature gradients and its bad effects on tool life. The technique is suitable to solve the inverse problem but presents some restriction with the time step.

Sousa(2006) proposed an inverse technique based on Dynamic observer and Green's function to estimate heat flux generated at the cutting interface during a drilling process. The direct model consists on a three-dimensional piece been drilled, the model was developed using Finite Volume Method. In this case the author was focused on the resulting thermal field at the piece. The inverse technique was applied with success to solve inverse drilling problem. The study showed that the temperature and heat flux at the tool-piece interface during a drilling process can be estimated if a suitable experimental apparatus was designed avoiding problems of sensitivity. Sousa has shown that the inverse technique is very simple to be implemented and has a great flexibility since it is possible to build the direct problem using any method or computational language.

This work proposes a thermal analysis of a turning process. The tool-holder set thermal field is here identified by using FEM (Finite Elements Method) and the inverse technique is based on Dynamic Observers and Green's function. The main goal is to obtain the tool-holder set temperature distribution by estimating the heat flux at interface work-piece. Simulated cases were investigated and the inverse technique is ready to be applied in a real cutting situation.

2. FUNDAMENTALS

Direct model that was solved using FEM is represented by a tool-holder set as shows Fig. 1.

2.1. Thermal model: direct problem

The direct problem is represented by a tool -holder set as shown in Fig. 1. It is considered here a cemented carbide tool and a titanium holder. Thermal properties for the holder and tool are shown in Table 1.

The thermal model geometry was built using Auto-Cad[®] in order to obtain the mesh to be used in the FEM (ANSYS[®]). Heat flux, $q''(t)$, that appears during a machining process is generated in the contact area, S_1 as represented in Fig. 1. Thermal model also consider convection effects around the tool-holder set.

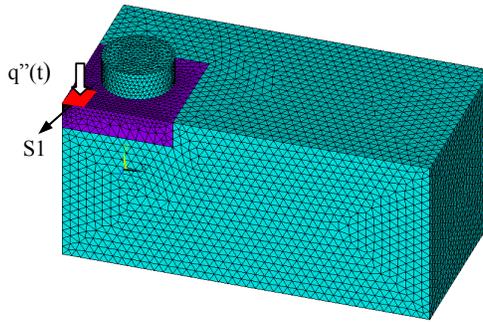


Figure 1. Three-dimensional thermal machining problem

The thermal model is obtained by the numerical solution of the transient and three-dimensional heat diffusion equation, that can be described by

$$\frac{\partial^2 T(x,y,z,t)}{\partial x^2} + \frac{\partial T(x,y,z,t)}{\partial y^2} + \frac{\partial^2 T(x,y,z,t)}{\partial z^2} = \frac{1}{\alpha} \frac{\partial T(x,y,z,t)}{\partial t} \quad (1)$$

In the region R ($0 < x < a$, $0 < y < b$, $0 < z < c$) and $t > 0$, subjected to the boundary conditions in the cutting interface:

$$-k \frac{\partial T(x,y,0,t)}{\partial z} = q(x,y,0,t) \text{ on } S_1 (x_o \leq x \leq x_1, y_o \leq y \leq y_1) \quad (2)$$

$$-k \frac{\partial(x,y,0,t)}{\partial z} = h[T(x,y,0,t) - T_\infty] \text{ on } S_2 (x_1 \leq x \leq a, y_1 \leq y \leq b) \quad (3)$$

$$-k \frac{\partial(x,y,0,t)}{\partial z} = h[T(x,y,0,t) - T_\infty] \text{ on } S_3 (a \leq x \leq d, y_o \leq y \leq b) \quad (4)$$

$$-k \frac{\partial(x,y,0,t)}{\partial z} = h[T(x,y,0,t) - T_\infty] \text{ on } S_4 (x_o \leq x \leq a+d, y_o \leq y \leq b) \quad (5)$$

$$k \frac{\partial(x,0,z,t)}{\partial y} = h[T(x,0,z,t) - T_\infty] \text{ on } S_5 (x_o \leq x \leq a, z_o \leq z \leq c) \quad (6)$$

$$k \frac{\partial(x,0,z,t)}{\partial y} = h[T(x,0,z,t) - T_\infty] \text{ on } S_6 (a \leq x \leq d, z_1 \leq z \leq c) \quad (7)$$

$$-k \frac{\partial(x,b,z,t)}{\partial y} = h[T(x,b,z,t) - T_\infty] \text{ on } S_7 (x_o \leq x \leq a, z_o \leq z \leq c) \quad (8)$$

$$-k \frac{\partial(x, b, z, t)}{\partial y} = h[T(x, b, z, t) - T_\infty] \text{ on } S_8 (a \leq x \leq d, z_0 \leq z \leq c) \quad (9)$$

$$k \frac{\partial(0, y, z, t)}{\partial x} = h[T(0, y, z, t) - T_\infty] \text{ on } S_9 (y_0 \leq y \leq b, z_0 \leq z \leq c) \quad (10)$$

$$-k \frac{\partial(a, y, z, t)}{\partial x} = h[T(a, y, z, t) - T_\infty] \text{ on } S_{10} (y_0 \leq x \leq b + d, z_0 \leq z \leq c) \quad (11)$$

$$-k \frac{\partial(a + d, y, z, t)}{\partial x} = h[T(a + d, y, z, t) - T_\infty] \text{ on } S_{11} (y_0 \leq y \leq b, z_0 \leq z \leq c) \quad (12)$$

$T(x, y, z, 0) = T_0$ for $t > 0$ in the region R .

In the Equation 1 where S_i represents the different surfaces to be considered in thermal model and can be better identified in section 3 (Fig. 5). If the value of the heat flux, $q''(x, y, \theta, t)$, is known, the Eq. (1) represent the direct problem related to the inverse problem studied.

The thermal model has considered that the thermal properties of both tool and holder materials are constant. The respective values are shown in Tab. 1.

Table 1. Thermal Properties of workpiece (cemented carbide) and tool holder (titanium).

Materials	k[W/mK]	ρ [kg/m ³]	c_p [J/kgK]	ϵ [J/Km ² s ^{1/2}]
titanium	21	4650	645	7936,26
cemented carbide	43,1	14600	200	11218,4

2.2. Inverse Technique

The inverse problem solution technique based on Green's functions and dynamic observers with global transfer Function, Sousa (2009) can be divided in two distinct steps: i) obtaining of the transfer function model G_H ; ii) obtaining of the heat transfer functions G_Q and G_N and the building algorithm identification. The transfer function model, G_H , is obtained from the equivalent dynamic systems theory and using Green's functions. The G_Q and G_N are obtained by following the procedure presented by Blum and Marquardt (1997).

Transfer Function Model identification (G_H). The solution of Eq. (13) can be given in terms of Green's function as by Beck et al. (1992)

$$T(x, y, z, t) = \int_{\tau=0}^t [G_H(x, y, z, t/\tau) q_I(\tau)] d\tau \quad (13)$$

where

$$G_H(x, y, z, t - \tau) = \frac{\alpha}{k} \int_0^x \int_0^z G_H^+(x, y, z, t/x', y', z', \tau) \Big|_{y'=y_H} dx' dz' \quad (14)$$

and $G(x, y, z, t - \tau)$ represents the Green's function of the thermal problem given by Eq.(14).

The Green's function is available for the homogeneous version associated with the problem defined by Eq. (1). Although the analytical Green's function is available and exists, Beck et al. (1992), it will not be used in this work. By the contrary, the solution of the problem defined by Eq. (1) will be performed numerically using finite element methods.

Equation (14) reveals that an equivalent thermal model can be associated with a dynamic model. It means, a response of the input/output system can be associated to Eq. (15) in the Laplace domain as the convolution product (Özisik, 1993)

$$T(x, y, z, t) = G_H(x, y, z, t - \tau) * q_I(\tau) \quad (15)$$

This dynamic system can be represented as shown in Fig. (2). Equation (15) can also be evaluated in the Laplace domain as a single product

$$\bar{T}(x, y, z, s) = \bar{G}_H(x, y, z, s) \bar{q}_I(s) \quad (16)$$

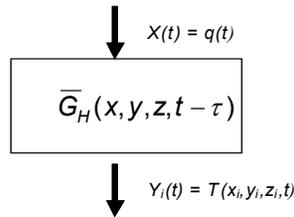


Figure 2. Dynamic thermal model system

The complete G_H identification is obtained using an auxiliary problem. The heat transfer function $\bar{G}_H(x, y, z, s)$ can, then, be obtained through the auxiliary problem which is a homogenous version of the problem defined by Eq. (13) for the same region with a zero initial temperature and unit impulsive source located at the region of the original heating. It means, the auxiliary thermal problem can be described as.

Similarly, the auxiliary thermal problem solution can be derived using Green function and the convolution properties as

$$T^+(x^+, y^+, z^+, t^+) = G_H(x^+, y^+, z^+, t^+ - \tau^+) * 1 \quad (17)$$

The Laplace transform of 1 is

$$L[1] = \frac{1}{s}, \text{ then} \quad (18)$$

$$\bar{T}^+(x^+, y^+, z^+, s) = \bar{G}_H(x^+, y^+, z^+, s) \frac{1}{s} \quad (19)$$

If the dynamic system is linear, invariant, and physically invariable, the response function $\bar{G}_H(x, y, z, s)$ is the same, independently of the pairs input/output and can be obtained by

$$\bar{G}_H(x, y, z, s) = s \bar{T}^+(x, y, z, s) \quad (20)$$

In order to complete the $\bar{G}_H(x, y, z, s)$ identification, the $\bar{T}^+(x, y, z, s)$ must be obtained at a specific position $r_i = (x_i, y_i, z_i)$.

A simple and efficient procedure is proposed here to obtain $T^+(r_i, s)$: if Eq. (20) represents a cross correlation function of the two functions of stationary random process s and $\bar{T}^+(x, y, z, s)$, then $\bar{G}_H(x, y, z, s)$ will be independent of the absolute time t and will depend only on the time separation t_a .

In this case, provided that the function $T^+(r_i, s)$ can be fitted by a polynomials function in the sampled interval $[0, t_a]$ as

$$T^+(r_i, t) = a_1 t + a_2 t^2 + a_3 t^3 + \Lambda \quad (21)$$

where a_i are the polynomial coefficients. Then, taking the Laplace transform of Eq. (21) gives

$$\bar{T}^+(r_i, s) = \frac{a_1}{s} + \frac{a_2}{s^2} + \frac{a_3}{2s^3} + \frac{a_4}{6s^4} + \Lambda + \frac{a_n}{(n-1)s^n} \quad (22)$$

Thus, from Eq. (20) G_H can be written as

$$\bar{G}_H(r_i, s) = s \bar{T}^+(r_i, s) = s \left[\frac{a_1}{s} + \frac{a_2}{s^2} + \Lambda + \frac{a_n}{s^n} \right] \quad (23)$$

$$\text{or } \bar{G}_H(r_i, s) = a_1 + \frac{a_2}{s} + \frac{a_3}{2s^2} + \frac{a_4}{6s^3} + \Lambda + \quad (24)$$

It can be observed that from the theory of partial fractions, if $\bar{G}_H(r_i, s)$ is expressible in partial fractions as in Eq. (24) its inversion is readily obtained by using the Laplace transform table. An advantage of this procedure is that the same procedure can be used indistinctly by one, two or three-dimensional models, provided the only active boundary condition is the unknown heat source.

The Global Transfer function is obtained using more than one thermocouple information. Figure 3 presents a single-input/two-output system. It means this figure shows an equivalent dynamic system to a thermal problem given by Fig. 1 considering two thermocouples for different positions (1, 2). It is assumed here that noise terms $n_1(t)$ and $n_2(t)$ are uncorrelated with each other and with the input signal $q(t)$.

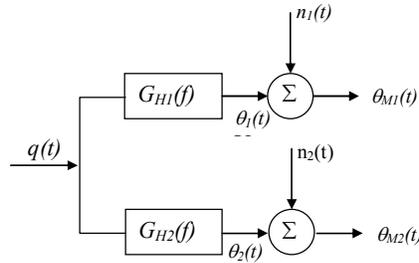


Figure 3. Single-input/two-output system.

In this case Eq.(13) gives

$$\theta(x_i, y_j, t) = \int_{\tau=0}^t G_H(x_i, y_j, t-\tau) q(\tau) d\tau \quad (25)$$

or in Laplace domain

$$\bar{\theta}(x_1, y_1, s) = G_H(x_1, y_1, s) q(s) \quad (26)$$

$$\bar{\theta}(x_2, y_2, s) = G_H(x_2, y_2, s) q(s) \quad (27)$$

Adding Eq.(26) to Eq.(27)

$$\bar{\theta}_1 + \bar{\theta}_2 = [G_{H1}(s) + G_{H2}(s)] q(s) \quad (28)$$

As Equation (28) has a similar representation of Eq. (16) the same estimators can be used to represent the single-input/two-output system. The only change that must be made is to consider the sum of temperature and the sum of heat transfer function in the system. As mentioned, the use of more than two thermocouples is immediate.

At this point, the G_Q and G_N functions need to be obtained in order to complete the inverse algorithm.

Obtaining of estimators Transfer Functions G_O and G_N . The thermal model can be represented by a dynamic system given by a block diagram shown in Fig. 4 (Blum and Marquardt, 1997):

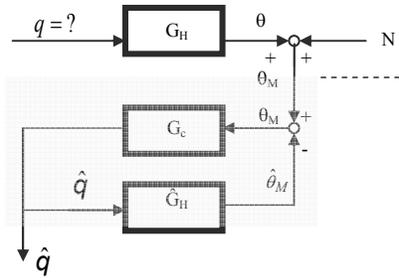


Figure 4. Frequency-domain block diagram.

It can be observed from the block diagram that:

i) the unknown heat flux $q(s)$ is applied to the conductor (reference model), G_H , and results in a measurement signal θ_M corrupted by noise N ,

$$\theta_M = \theta + N = G_H q + N \quad (29)$$

ii) The estimate value \hat{q} is computed from the output data θ_M . Thus, the estimator can be represented in a closed-loop transfer function of the feedback loop (Fig. 4) as

$$\hat{q} = \frac{G_C}{1 + G_C G_H} \theta_M \quad (30)$$

that characterizes the behavior of the solution algorithm.

Substituting Eq. (29) in Eq. (30) we obtain:

$$\hat{q} = \frac{G_C G_H}{1 + G_C G_H} q + \frac{G_C}{1 + G_C G_H} N \quad (31)$$

or

$$\hat{q} = G_Q q + G_N N \quad (32)$$

where

$$G_Q = \frac{G_C G_H}{1 + G_C G_H} \text{ and } G_N = \frac{G_C}{G_H} = \frac{G_C}{1 + G_C G_H} \quad (33)$$

The transfer function G_Q is chosen to have the behavior of type I chebychev filter and G_N is identified by Eq. (33) provided that G_H is obtained. Thus, from Eq. (30) the resulting algorithm can then be given by

$$\hat{q}(s) = G_N(s) \times \theta_M(s) \quad (34)$$

It can be observed in Eq. (33) that if the algorithm estimates the heat flux correctly, G_Q is equal to unity, $G_Q = 1$, and the frequency ω is within the pass band. In this case, the noise transfer function G_N is equal to the inverse transfer function of the heat conductor (G_H^{-1}).

According to Blum and Marquardt the observer is essentially an on-line scheme. In this case, estimation of the heat flux at the current time step is based on current and past temperature measurements only. In this case, any on-line estimator involves a phase shift or lag. To remove this lag Blum and Marquardt proposed a filtering procedure that can be resumed in the use of two discrete-time difference equations

$$q(k) = \sum_{i=0}^{n_n} b_i Y_M(k-i) - \sum_{i=1}^{n_n} a_i q(k-i) \quad (35)$$

and

$$\hat{q}(k) = \sum_{i=0}^{n_n} b_i q(k-i) - \sum_{i=1}^{n_n} a_i \hat{q}(k-i) \quad (36)$$

The coefficients a_i and b_i that appear in Eqs. (35) and (36) are obtained using Eq. (33). In this case, the inverse procedure is concluded with the $\bar{G}_H(x, y, s)$ identification.

3. RESULTS AND DISCUSSION

The inverse technique based on Green's functions and dynamic observers with global transfer function is applied to solve a simulated case. Simulated tests are important to evaluate the inverse technique algorithm and to investigate possible problem of sensitivity.

The great advantage of this procedure, for example, is to avoid additional complications, such as biases in the direct problem and to use the results to design the best experimental apparatus defining important variable as sensor locations or time during of the experiment. Simulated data can also be used to establish the sensitivity of the problem and to anticipate possible inverse estimation problems.

This section presents a simulated test case considering a tool-holder set with a cemented carbide tool and titanium holder which thermal properties are presented in Tab. 1.

Simulated temperature data distributions for the direct problem are generated using the solution of Eq. (1), considering a known heat flux evolution $q''(t)$. Random errors are then added to these temperatures. The temperatures with error are then used in the inverse algorithm to reconstruct the imposed heat flux.

The experimental simulated temperatures are calculated from the following equation:

$$Y(L, t) = T(L, t) + \varepsilon_j \quad (37)$$

where ε_j is a random number. The parameter ε_j assumes values of 0, and within $\pm 1^\circ\text{C}$.

3.1. Simulated test

Figure 3 shows the tool-holder set represented by a titanium holder and a cemented carbide tool, with thermal properties as shown in Tab. 1. Thermal effects of the turning process are given by imposing a known heat flux in a rectangular area S_1 (Fig. 5) and assuming the other superficial surfaces insulated. Simulated data for six different positions are calculated by solving the direct problem. The positions denoted as 1 to 6 are described in Tab. 2.

Figures 6 and 7 present the heat flux imposed and the calculated temperatures added with a random error (Eq. 37) on position 1 to 6.

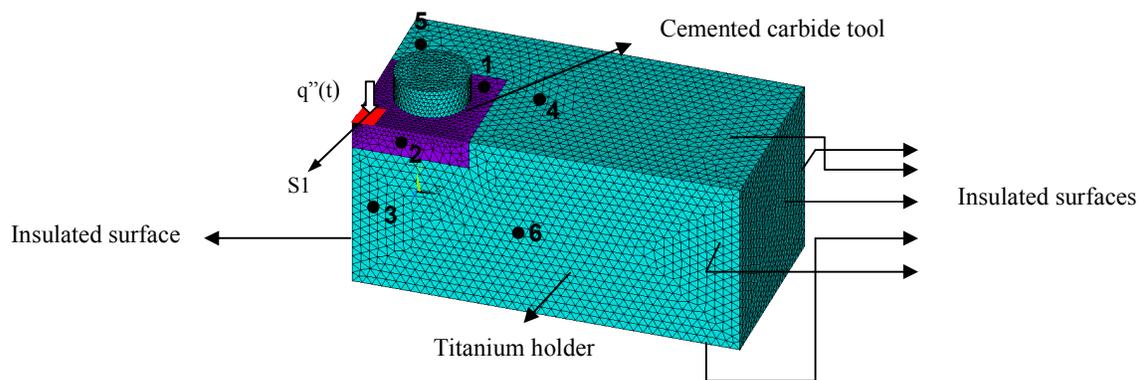


Figure 5. Tool-holder set and heat flux location for direct problem.

Table 2 – Simulated Thermocouples location.

Coordinates [m]	T1	T2	T3	T4	T5	T6
x	0,0112	0,0063	0,0089	0,0178	0,0038	0,0232
y	0,0200	0,0184	0,0116	0,0200	0,0200	0,0116
z	0,0089	0,0200	0,0200	0,0147	0,0147	0,0200

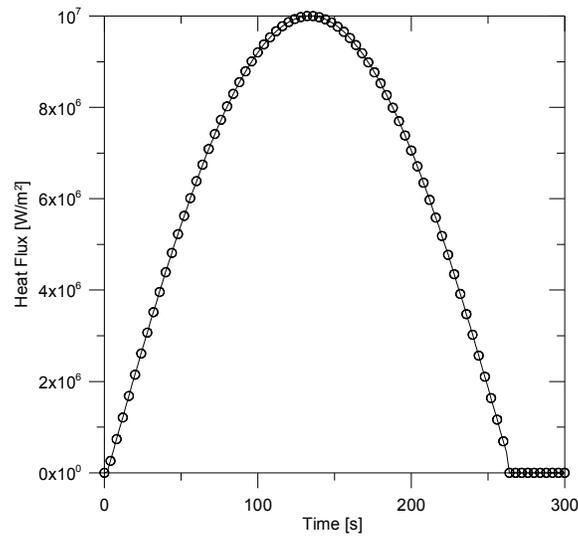


Figure 6. Heat Flux imposed, $q''(t)$, direct problem.

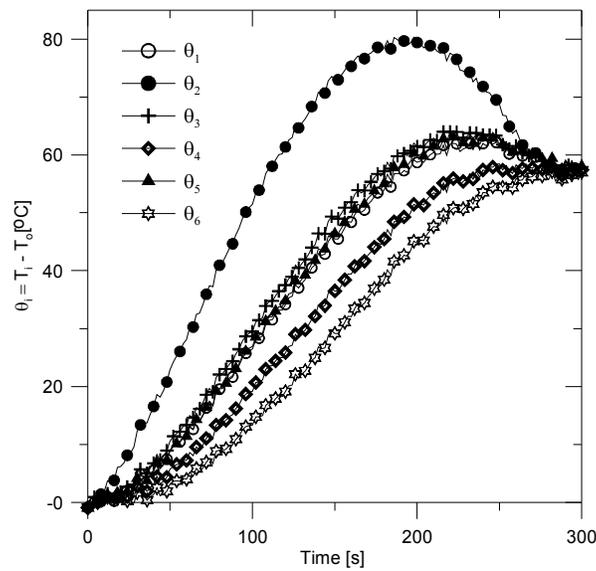


Figure 7. Temperature evolution for thermocouples 1 to 6 (Table 2).

Heat flux estimative at the cutting interface is then calculated by the inverse technique using three of the thermocouples information, T_2 , T_3 and T_6 , it means that the global heat transfer function is derived by using only the thermocouples at the positions 2, 3 and 6. Figure 8 shows the estimated heat flux and a comparison between the estimated and imposed heat flux. It can be observed from the estimation results that the inverse technique is able to reproduce the heat flux generated at the cutting interface. Since the heat flux is identified, the temperature evolution during the simulated drilling process can be calculated. Figure 9 shows the complete temperature field calculated using the estimated heat flux for selected times.

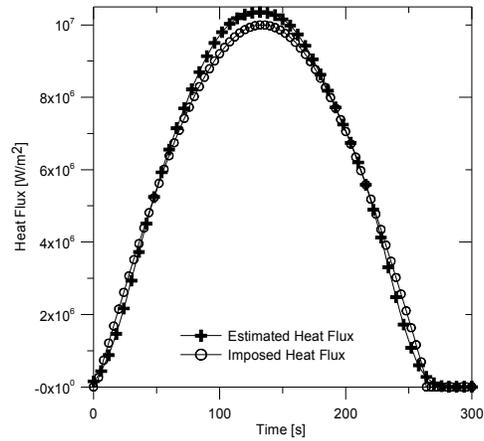


Figure 8. Estimated heat flux versus imposed heat flux

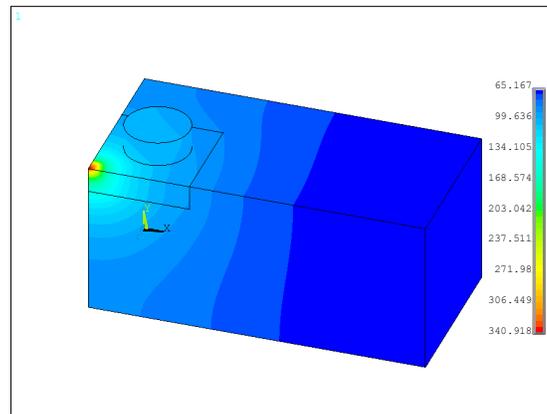


Figure 9. Resulting temperature field during a machining process at different elapsed machining times.

Temperature control is important to prevent piece and tool damage due to the excessive temperature. Since heat flux at the cutting interface has been reconstructed it is possible to estimate the maximum temperature at the piece solving the direct problem.

Figure 10 shows a comparison between the calculated and the measured temperatures for thermocouples 1 and 4. It is observed a good accordance between the experimental and calculated temperatures with a maximum deviation of 2,5°C during the heating time as shows Fig. 11 that presents absolute deviation.

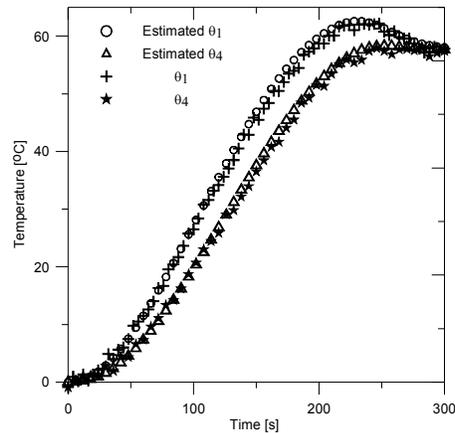


Figure 10. Calculated and the measured temperatures for thermocouples 1 and 4

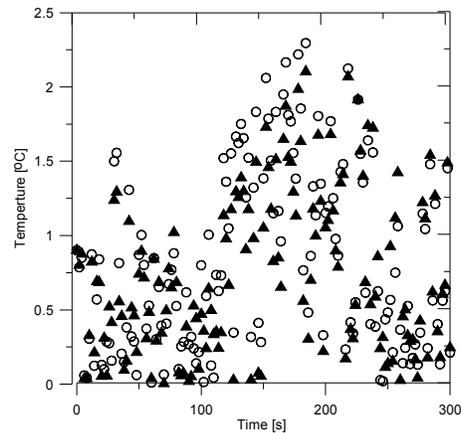


Figure 11. Absolute deviation between temperatures

3. CONCLUSIONS

The technique based on Green's function and dynamic observers is applied here to solve with success a turning inverse problem. This study shows that the inverse technique represents a very good way to be applied in cutting thermal problems. The great advantage of this technique is to avoid iteration numerical procedures. As the geometry of cutting process (tool and tool holder) are very complicated the thermal problem must be solved using a suitable numerical methods, that in this case was the finite element methods. In this sense the technique based on Green's function and dynamic observers can be used without difficulty since the sensitivity problem can be solved just once. As simulated tests can anticipate experimental difficulties, the results have show that the inverse procedure proposed here is ready to be applied in a real cutting problem.

4. ACKNOWLEDGMENTS

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