DEVELOPMENT OF A MIDDLE EAR GAS EXCHANGE MODEL TO PREDICT THE EFFECTS OF PRESSURE VARIATIONS ON THE TYMPANIC MEMBRANE DURING FLIGHTS

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Abstract. *The middle ear pressure variation is one of the most important factors that causes discomfort on flight passengers and crew. The purpose of this work is to develop a mathematical model of the middle ear cavity pressure variation during typical flight's conditions. The middle ear cavity was modeled as a gas chamber that transfer gas to other physiological systems trough three different ways. One of them is through the Eustachian tube, that conects the tympanic cavity to the nasopharynx. The tympanic cavity also transfes gas through diffusion to the blood stream in the tympanic cavity and in the mastoid cavities system. A system of four non-linear ordinary differential equations was obtained which was resolved using the forth order Runge-Kutta numerical method. Parametric analysis of the active airflow resistance of the Eustachian tube, tympanic membrane compliance and mastoid cavities volume were conducted because it was found in the literature that these parameters varies among population. Simulation's results, for a typical cabin pressure variation curve, showed that people with higher airflow resistances of the Eustachian tube may experiment higher gradients of pressure in their tympanic membranes. They also showed that the model is not much sensitive to the tympanic membrane compliance parameter and that higher mastoid volumes increases gas diffusion between the middle ear and blood stream.*

Keywords*: tympanic membrane, middle ear, cabin pressure, mathematical modeling.*

1. INTRODUCTION

The middle ear pressure variation during flights is an important factor of discomfort for passengers and crew because pressure differences in the tympanic membrane causes its deformation. This pressure difference must be held at low values to avoid the tympanic membrane to deform causing discomfort, pain or rupture. However, it is necessary that the airplane cabin pressure varies during flight so the airplane structure is able to support the pressure difference between cabin and the atmosphere. The cabin pressure of an airplane varies at high rates during flights, especially during the ascent and descent stages of flight.

The aim of this work is to develop a mathematical model of the gas exchange in the middle ear to predict the effect of pressure variation during typical flights on the tympanic membrane. This model considers the main ways of gas exchange in the middle ear: through the Eustachian tube and through diffusion to the blood stream in the mastoid cavities system mucosa and in the tympanic cavity mucosa.

1.1. The Human Ear

The human ear is basically formed by three parts: external ear, middle ear and inner ear (DÂNGELO; FATTINI, 2000) (Figure 1). Among the three parts that forms the human ear, the middle ear is the one of most interest in this study. It consists of two air cavities physically connected but physiologically separated. The anterior part, tympanic cavity, contains the ossicles of human ear and the posterior part is formed of a system of air cells, also called mastoid air cell system (KANICK; DOYLE, 2005).

The tympanic cavity is connected to the nasopharynx through the Eustachian tube. The posterior part of the Eustachian tube is bony and rigid and is physically an extension of the tympanic cavity. The tympanic membrane separates the tympanic cavity from the external environment. Thus the tympanic membrane is the most affected structure because of the cabin pressure variation. For that reason, it is important to model the pressure gradient in the tympanic membrane.

According to Fay et al. (2005), the tympanic membrane is composed of a series of layers and two of these layers contain collagen fibers. One has fibers that run in a radial pattern while the adjacent layer's fibers run in the circumferential direction. According to Cheng et al. (2007), on average the tympanic membrane is 10 mm diameter and 0.08 mm thick. Several experiments were conducted with the purpose of obtaining the mechanical properties of the tympanic membrane. It was observed that the membranes have an elastic behavior but there were great differences in the results obtained by many authors (CHENG ET AL., 2007; FAY ET AL., 2005).

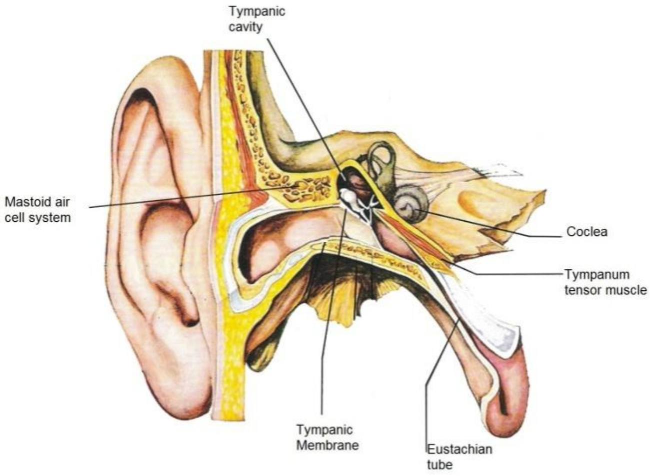

Figure 1. Human ear anatomy.

Just as the cabin pressure changes, the tympanic cavity pressure also changes. However, these changes are different in most cases. Therefore, the cabin pressure and the tympanic cavity pressure might be different. The human ear has some mechanisms that balance the air pressure and composition among the tympanic cavity and adjacent environments. The main known mechanisms are the gas exchange through the Eustachian tube, through gas diffusion with the blood and gas exchange with the mastoid air cell system.

1.2. Gas exchange through the Eustachian tube

The main tympanic cavity gas exchange mechanism is through the Eustachian tube, also known as ventilation or inhalation. The Eustachian tube is a physiologic tube extending from the middle ear to the nasopharynx. It is formed of bone, cartilage and fibrous tissue (ARMSTRONG; HEIM, 1934). According to SAE ARP1270 (2000), the bony part of the Eustachian tube, next to the tympanic cavity, is normally opened. Whilst the cartilaginous part, next to the nasopharynx, is normally collapsed by the compression of external tissues. Sadé e Ar (1997) say that the Eustachian tube is 3 to 4 cm long, formed by two cone like structures fused together by a narrow ring, the isthmus, which is a ring 1 to 2 mm long and 0.6 to 1.2 mm in diameter.

When the pressure difference between the tympanic cavity and the nasopharynx is greater than 3.4 kPa, it is enough to open the Eustachian tube passively, which closes again when the pressure difference falls to 1.4 kPa (GROTH et al. 1985). This may happen when the airplane is in the ascent stage of the flight. In the descent stage of the flight, the necessary pressure difference between the tympanic cavity and the nasopharynx must be greater than 5.9 kPa to open passively the Eustachian tube. This value is greater because of the physicals characteristics of the Eustachian tube. The Eustachian tube also opens actively when the tensor palati muscle is contracted during swallowing, yawning or other movements of the mandible. It opens for 0.20 to 0.25 seconds once every 1 to 2 minutes (SADÉ; AR, 1997; KANICK; DOYLE, 2005).

1.3. Gas exchange through the middle ear mucosa

The amount of gas within the tympanic cavity is also transferred through the process of diffusion through the middle ear mucosa. The middle ear mucosa is irrigated with the venous blood stream and lines the tympanic cavity walls and mastoid cavities walls. Thus, the gases presented in the tympanic cavity and mastoid cavities exchanges with the gases dissolved in the blood stream in order to obtain equilibrium. While the Eustachian tube remains collapsed the diffusion through middle ear mucosa is the main gas exchange mechanism in the middle ear.

2. MATHEMATICAL MODEL

The mathematical model was developed considering gas transfer through the Eustachian tube and through the mucosa of the mastoid cavities and tympanic cavity. Figure 2 shows a scheme of the physical model of the middle ear system containing the tympanic cavity, mastoid cavities and the Eustachian tube. The tympanic cavity was modeled as a gas reservoir with volume V_{TC}. The mastoid cavities system was also modeled as a gas reservoir with volume V_{MC}. The tympanic cavity and the mastoid cavities system are assumed to be connected without any resistance. The Eustachian tube was modeled as a gas flow resistance, R_{ET}. The tympanic membrane is considered to be elastic and its elasticity was modeled as a function of its compliance, C_{TM}, which is the relation between the pressure gradient through the membrane and the volume dislocated by this pressure gradient. The middle ear mucosa is well irrigated with blood and lines the tympanic cavity walls and the cavities of the mastoid system.

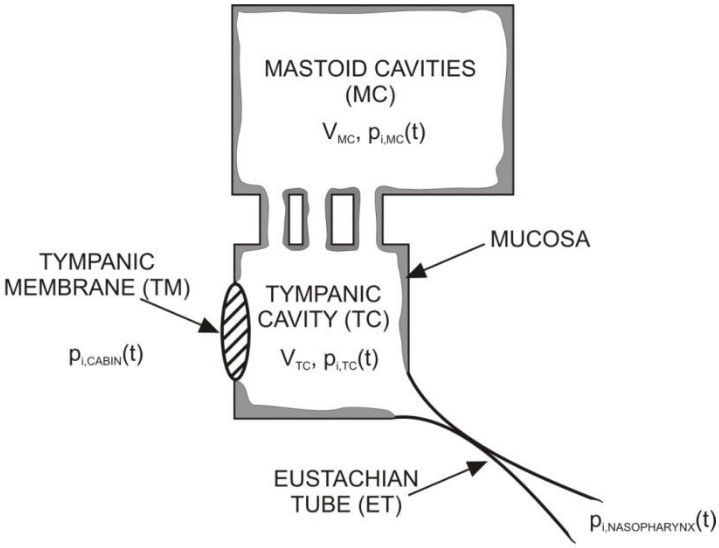

Figure 2. Schematic model of the middle ear system

The Eustachian tube can be considered as a resistance to the airflow between the tympanic cavity and the nasopharynx. When it is collapsed the airflow is stopped, but when it is opened, actively or passively, the airflow suffers a pressure drop passing through it, as shown in Eq. (1). This airflow resistance was experimentally measured by Cantekin et al. (1979). Their experiment showed an actively opened resistance of $12*10^8$ Pa.s/m^3 and passively opened resistances varying from $176*10^8$ Pa.s/m^3 to $30*10^8$ Pa.s/m^3. These values were obtained from a group of volunteers without any history of middle ear disease. The group of volunteers with any middle ear disease history presented an actively opened Eustachian tube resistance of $235*10^8$ Pa.s/m^3.

$$q_{ET} = \frac{(P_N - P_{TC})}{R_{ET}} \tag{1}$$

In Eq. (1), q_{ET} is the volumetric airflow through Eustachian tube, P_N is the total pressure in the nasopharynx, P_{TC} is gases total pressure in the tympanic cavity and R_{ET} is the airflow Eustachian tube resistance.

Kanick and Doyle (2005) used a diffusive model for the gas transfer through the tympanic cavity mucosa. The model assumes that the rate of pressure variation in the tympanic cavity is proportional to the pressure difference between the tympanic cavity and the blood stream, as shown in Eq. 1. This model will also be used to model the gas diffusion through the mastoid cavities' mucosa.

$$\frac{dP_{i,TC}}{dt} = k_{i,TC}\left(P_{i,B} - P_{i,TC}\right) \tag{2}$$

In Eq. (2), the subscript i represents each gas specie, $P_{i,TC}$ is the partial pressure of gas i in the tympanic cavity, $k_{i,TC}$ is the diffusion coefficient of gas i through the mucosa, $P_{i,B}$ is the partial pressure of gas i in the blood stream.

Applying conservation equations to the model, obtains the differential equation Eq. (3), which represents a non-linear system of ordinary differential equations. The four gases species considered in this system are oxygen, carbon dioxide, water vapor and nitrogen (plus other inert gases), resulting in four non-linear ordinary differential equations. The main hypotheses adopted in this model are that all gases has ideal gas behavior, partial pressures in the mastoid cavities are equal to tympanic cavity partial pressures and that the gases diffusion coefficients are the same in tympanic cavities' mucosa and mastoid cavities' mucosa.

$$\frac{dP_{i,TC}}{dt} = \frac{P_{TC}}{(V_{TC} + V_{MC})}\frac{\left(P_{i,N} - P_{i,TC}\right)}{R_{ET}} - \frac{P_{i,TC}}{(V_{TC} + V_{MC})}C_{TM}\left(\frac{dP_{TC}}{dt} - \frac{dP_{cabin}}{dt}\right) + k_{i,TC}\left(P_{i,B} - P_{i,TC}\right) \tag{3}$$

In Eq.(3), the subscript i represents each gas specie, $P_{i,TC}$ is the partial pressure of gas i in the tympanic cavity and in the mastoid cavities, P_{cabin} is the total cabin pressure, P_{TC} is the total tympanic cavity pressure, $k_{i,TC}$ is the diffusion coefficient of gas i through the mucosa, $P_{i,B}$ is the partial pressure of gas i in the blood stream.

3. RESULTS

The model's four differential equations system was solved using the fourth order Runge-Kutta numerical method in a C++ program code. The several parameters used in the model were obtained from many works previously published. Table 1 shows a resume of these parameters and their source reference.

Table 1. Parameters values used in simulations

Parameter	Description	Symbol	Value	Reference
Active ET(1) resistance	ET airflow resistance when actively opened	R_{ET}	$12 \cdot 10^8$ Pa.s/m^3	Cantekin et al. (1979)
Passive ET resistance	ET airflow resistance when passively opened	R_{ET}	$30 \cdot 10^8$ Pa.s/m^3	Cantekin et al. (1979)
ET opening pressure	Pressure difference necessary to passively open the ET	P_{OL}	3.5 kPa	Groth et al. (1985)
ET closing pressure	Minimum pressure difference necessary to maintain the ET opened after passively opening.	P_{CL}	1.4 kPa	Groth et al. (1985)
ET opening interval	Interval between active ET openings	t_{OP}	75 s	Sadé e Ar (1997), Kanick e Doyle (2005)
TM(2) compliance	Relation between TM volume displacement and the pressure difference that causes this displacement	C_{TM}	$3 \cdot 10^{-11}$ m3/Pa	Gaihede, Felding e Elbrond (1995b)
Gas diffusion coefficients through middle ear mucosa	Oxygen Carbon dioxide Water vapor Nitrogen and other inert gases	k_{O2} k_{CO2} k_{H2O} k_{N2}	0.008 min^{-1} 0.16 min^{-1} 0.32 min^{-1} 0.0008 min^{-1}	Kanick e Doyle (2005)
Tympanic cavity volume		V_{TC}	1 mL	Kanick e Doyle (2005)
Mastoid cavities volume		V_{MC}	10 mL	Mean value experimentally obtained from 19 volunteers

(1) Eustachian tube
(2) Tympanic Membrane

Besides the parameters shown in Table 1, the model also receives as input some time dependable variables. The nasopharynx pressure is a time dependable variable of the model and it's total pressure is assumed to be equal to the cabin pressure. However, gases molar fractions in the nasopharynx are different from the cabin. Blood stream gases partial pressures are others inputs in the model, it was considered that blood partial pressures are constant during simulations. Table 2 shows gases partial pressures in the nasopharynx and in the blood stream at sea level.

Table 2. Gases partial pressures at sea level in the nasopharynx and in the venous blood stream

Gas Species	Partial Pressure (kPa) Nasopharynx (Kanick; Doyle, 2005)	 Venous Blood (Sade; Ar, 1997)
Oxygen	14.9	5.1
Carbon Dioxide	4.3	5.9
Water Vapor	6.3	6.3
Nitrogen and other gases	75.8	76.6
Total	101.3	93.9

The cabin pressure variation curve is one of the most important time dependable variables. At each instant, the model reads the cabin pressure and its change rate and calculates the total pressure in the tympanic cavity. Figure 3 shows a typical flight cabin pressure curve used in the simulations.

The dotted line in the graphic represents the cabin pressure variation curve during a typical flight and all three stages of the flight can be seen in the graphic. The first stage is the ascent stage represented by the decreasing of the cabin pressure. The second stage is the cruising stage, represented by the stability of the cabin pressure in a lower pressure. The third stage is the descent stage, represented by the increasing cabin pressure. The continuous line plotted in the graphic represents the tympanic cavity pressure variation curve, which was calculated from the mathematical model. While the Eustachian tube remains closed, the tympanic cavity pressure tends to equalize with the blood stream pressure. This phenomenon is better observed during the cruising stage of the flight, where there are fewer cabin pressure variations. When the Eustachian tube opens, however, the tympanic cavity's total pressure tends to equalize with the nasopharynx total pressure which is equal to the cabin pressure. During the ascent and descent stages of the flight the pressure difference between tympanic cavity and cabin reaches the highest values in the flight.

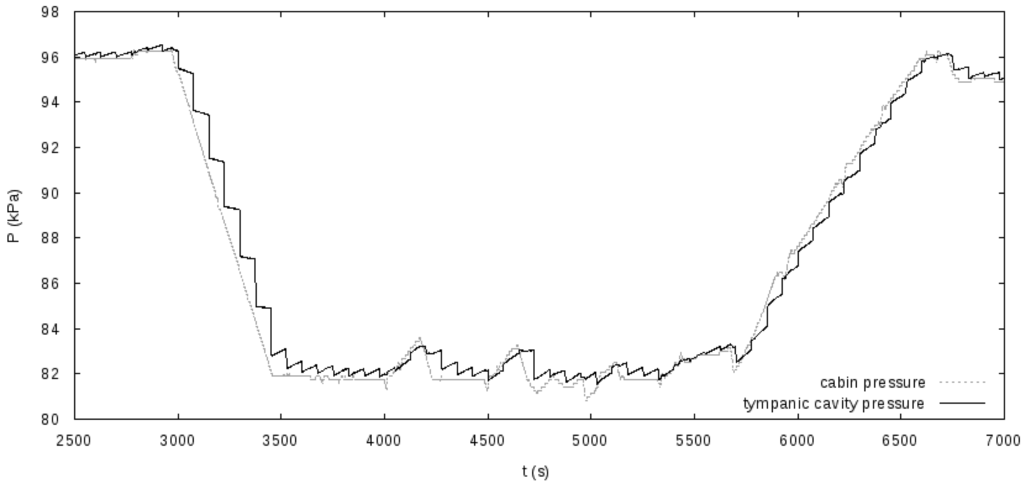

Figure 3. Typical flight cabin pressure curve (EMBRAER).

The active Eustachian tube resistance varies much among individuals. Figure 4 shows a graphic with four lines of the tympanic cavity pressure calculated considering four different active Eustachian tube airflow resistance. These values were selected from the work of Cantekin et al. (1979). It can be observed that as high is the active Esutachian tube resistance higher is the pressure difference in the tympanic membrane. Although Cantekin et al. (1979) observed that the active Eustachian tube resistance is constant among individuals without any history of middle ear disease, they also observed that among other individuals this parameter's value varies from less than $12*10^8$ Pa.s/m^3 up to $235*10^8$ Pa.s/m^3, maximum value observed. The graphic shows that the mathematical model is very sensible to this parameter and that individuals with Eustachian tube malfunctioning may suffer more than others during flights.

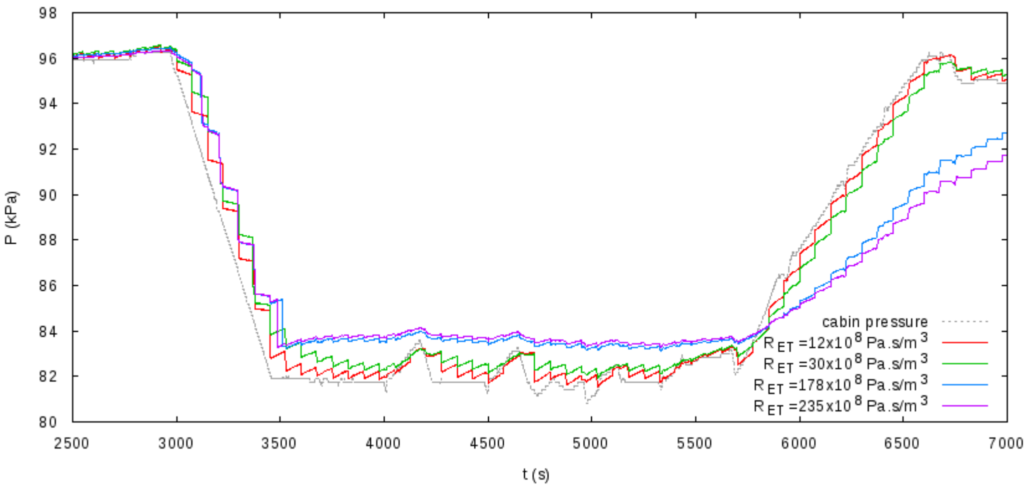

Figure 4. Parametric analysis of the active Eustachian tube airflow resistance.

The tympanic membrane compliance is another important parameter that varies among individuals. It is inversely related to the tympanic membrane stiffness, which means that the lower the tympanic membrane compliance is, more rigid it is. Gaihede et al. (1995) experimentally measured the tympanic membrane compliance in 45 volunteers without any history of middle ear disease. Their results showed that the tympanic membrane compliance varied from $1.2*10^{-11}$ m^3/Pa to $9.8*10^{-11}$ m^3/Pa, with a mean value of, approximately, $3.0*10^{-11}$ m^3/Pa. In order to evaluate the sensibility of the mathematical model to this parameter, the tympanic cavity total pressure was calculated considering four different values of the tympanic membrane compliance. Figure 5 shows a graphic with these simulations results, which shows that the developed model has a low sensibility to the tympanic membrane compliance parameter. In other words, the variation in the tympanic membrane compliance among individuals is not determinant in their sensibility to external pressure variations.

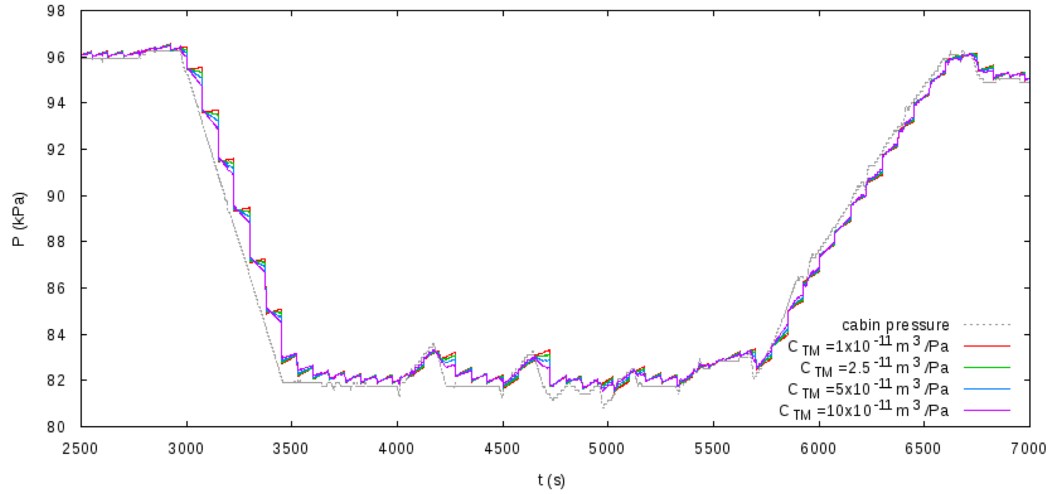

Figure 5. Parametric analysis of the tympanic membrane compliance.

The mastoid cavities total volume was also simulated considering different values. According to Kanick and Doyle (2005), the total mastoid cavities volume may vary, among individuals, from 0 mL to 15 mL. Because of that range of variation, four different values of the mastoid cavities volume were used in the model in order to evaluate its sensibility to this parameter, a value of 30 mL was used to consider individuals with larger mastoid cavities volumes. Figure 6 shows the graphic considering these four cases.

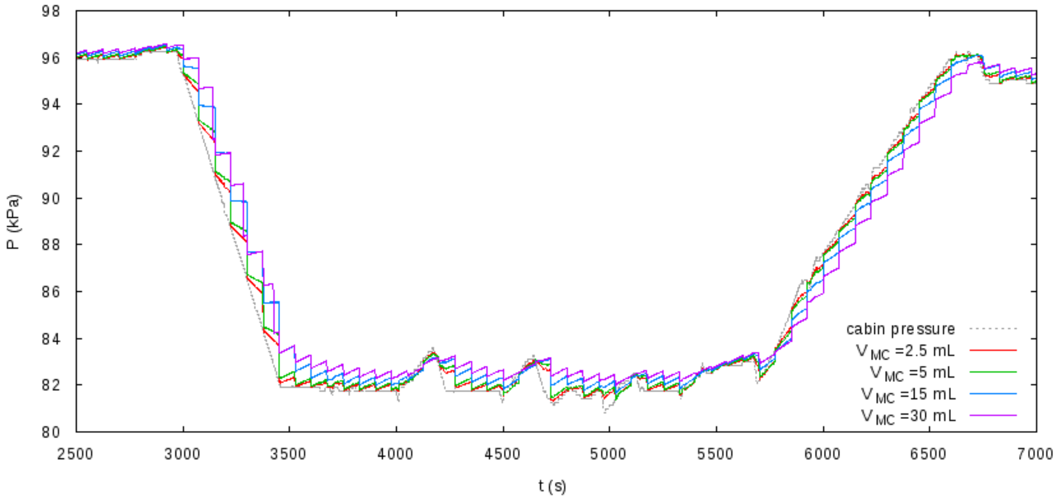

Figure 6. Parametric analysis of the total mastoid cavities volume.

The graphic shows that the larger is the volume of mastoid cavities larger is the pressure difference between the tympanic cavity pressure and cabin pressure. It happens because larger volumes implies in more gas diffusion through mastoid cavities mucosa and the tympanic cavity pressure changes approximating to blood stream pressure faster than in cases where volumes are smaller.

3. CONCLUSIONS

The mathematical model of the tympanic cavity pressure variation during typical flights conditions was successfully developed. The model considered the gas exchange through the Eustachian tube and through diffusion to the blood stream.

The parametric analysis showed that the model is very sensitive to the active Eustachian tube airflow resistance. Higher values of the resistance result in higher pressure gradients in the tympanic membrane. Eustachian tubes with higher airflow resistances were observed in a group of volunteers with a history of middle ear disease.

The tympanic membrane compliance analysis showed that the model is less sensitive to this parameter than to the Eustachian tube resistance, concluding that any value of this parameter can be adopted in the model in the range of 10^{-11} to 10^{-10} m^3/Pa.

The mastoid cavities volume analysis showed that higher volumes mean more gas diffusion through the mucosa. Consequently, the tympanic cavity's pressure tends to equalize to the blood stream pressure while the Eustachian tube remains collapsed. The model also showed to be less sensitive to this parameter than to the Eustachian tube resistance.

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