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TRANSIENT HEAT CONDUCTION IN A SOLID CYLINDER WITH CONVECTIVE BOUNDARY CONDITIONS – PART II: INVERSE PROBLEM OF FUNCTION ESTIMATION

H. R. B. Orlande
Department of Mechanical Engineering, POLI/COPPE, Federal University of Rio de Janeiro, UFRJ, Cid Universitaria, Cx. Postal 68503, Rio de Janeiro, RJ, 21945-970, BRAZIL
helcio@serv.com.ufrj.br

A. Haag

F. P. J. de Barros

O. Wellele

P. M. P. da Silva

R. B. Storch

R. de Sá Padilha

***Abstract**. This paper presents the solution of an inverse heat transfer problem of function estimation. The physical problem considered here involves the heating of a solid cylinder by hot water in a temperature-controlled bath. This paper addresses the identification of the unknown boundary heat flux at the surface of the cylinder. Temperature measurements taken at selected positions within the cylinder are assumed available for the inverse analysis. The unknown heat flux is estimated by using the conjugate gradient method with adjoint problem. Results obtained with simulated and actual experimental measurements are presented and compared with those obtained with the prameter estimation approach described in Part I of this paper. This work was performed within the scope of a graduate course in the Department of Mechanical Engineering of COPPE/UFRJ. The last six authors of this paper are the students of this course; they are listed in alphabetical order, irrespective of their grade in the course.*

***Keywords**. Inverse problems, heat transfer, function estimation*

1. Introduction

Inverse heat transfer problems rely on temperature and/or heat flux measurements for the estimation of unknown quantities appearing in the analysis of physical problems in this field. As an example, inverse problems dealing with heat conduction have been generally associated with the estimation of an unknown boundary heat flux, by using temperature measurements taken below the boundary surface. Therefore, while in the classical direct heat conduction problem the cause (boundary heat flux) is given and the effect (temperature field in the body) is determined, the inverse problem involves the estimation of the cause from the knowledge of the effect.

The use of inverse analysis techniques represents a *new research paradigm,* Beck (1999). The results obtained from numerical simulations and from experiments are not simply compared *a posteriori*, but a close synergism exists between experimental and theoretical researchers during the course of the study, in order to obtain the maximum of information regarding the physical problem under picture.

Inverse problems are mathematically classified as *ill-posed*, whereas standard heat transfer problems are *well-posed,* Hadamard (1923). The solution of a well-posed problem must satisfy the conditions of existence, uniqueness and stability with respect to the input data. The existence of a solution for an inverse heat transfer problem may be assured by physical reasoning. On the other hand, the uniqueness of the solution of inverse problems can be mathematically proved only for some special cases. Also, the inverse problem is very sensitive to random errors in the measured input data, thus requiring special techniques for its solution in order to satisfy the stability condition (Tikhonov *et al*., 1977; Beck and Arnold, 1977; Alifanov, 1994; Beck *et al*., 1985; Alifanov *et al*., 1995; Dulikravich *et al*., 1986; Sabatier, 1978; Morozov, 1984; Murio, 1993; Trujillo, 1997; Hensel, 1991; Kurpisz *et al*., 1995; Desinov, 1999; Yagola *et al*., 1999; Ramm *et al*., 2000; Ozisik *et al*., 2000). A successful solution of an inverse problem generally involves its reformulation as an approximate well-posed problem and makes use of some kind of regularization (stabilization) technique. In several methods, the solution for the inverse problem is obtained in the least-squares sense.

Inverse problems can be solved either as a parameter estimation approach or as a function estimation approach. If some information is available on the functional form of the unknown quantity, the inverse problem can be reduced to the estimation of few unknown parameters. On the other hand, if no prior information is available on the functional form of the unknown, the inverse problem can be regarded as a function estimation approach in an infinite dimensional space of functions.

This paper deals with the solution of a function estimation problem involving the heating of a solid cylinder in a temperature-controlled water bath. The inverse problem is concerned with the estimation of the boundary heat flux at the surface of three different cylinders made of Teflon and aluminum, by using temperature measurements taken within the body. For the solution of the present inverse problem we make use of the conjugate gradient method with adjoint problem (Alifanov et al, 1995; Ozisik and Orlande, 2000). The basic steps of this method include: (i) Direct Problem; (ii) Inverse Problem; (iii) Sensitivity Problem; (iv) Adjoint Problem; (v) Gradient Equation; (vi) Iterative Procedure; (vii) Stopping Criterion; and (viii) Computational Algorithm. The physical problem of interest in this paper and its mathematical formulation are presented below, together with details of the steps of the conjugate gradient method with adjoint problem.

2. Physical Problem and Mathematical Formulation

The physical problem under picture in this work involves the heating of cylindrical body, immersed in a temperature controlled water bath. The body is assumed to be initially at the uniform temperature T_0. For $t > 0$, the body is heated by convection with the surrounding water. The resultant heat flux to the body is assumed to be uniform over its surface and given by a time-dependent function $q(t)$. The cylinder diameter is $2b$ and its thickness is $2L$, as illustrated in Fig. (1). The cylinder thermophysical properties are assumed to be constant during the time elapsed for the cylinder to reach equilibrium with the surrounding water.

By taking into account axial and longitudinal symmetries, the mathematical formulation of the heat conduction problem in the cylindrical body is given in dimensionless form by:

$$\frac{\partial \theta}{\partial \tau} = \frac{\partial^2 \theta}{\partial R^2} + \frac{1}{R}\frac{\partial \theta}{\partial R} + \frac{1}{K^2}\frac{\partial^2 \theta}{\partial Z^2} \tag{1.a}$$

$$\left.\frac{\partial \theta}{\partial R}\right|_{R=0} = 0 \tag{1.b}$$

$$\left.\frac{\partial \theta}{\partial R}\right|_{R=1} = Q(\tau) \tag{1.c}$$

$$\left.\frac{\partial \theta}{\partial Z}\right|_{Z=0} = 0 \tag{1.d}$$

$$\left.\frac{\partial \theta}{\partial Z}\right|_{Z=1} = Q(\tau) \tag{1.e}$$

$$\theta(R,Z,0) = 1 \tag{1.f}$$

where the following dimensionless groups have been defined

$$\theta(R,Z,\tau) = \frac{T(r,z,t) - T_o}{T_\infty - T_o} \tag{2.a}$$

$$\tau = \frac{\alpha\, t}{b^2} \tag{2.a}$$

$$R = \frac{r}{b} \tag{2.c}$$

$$Z = \frac{z}{l} \tag{2.d}$$

$$K = \frac{L}{b} \tag{2.e}$$

$$Q(\tau) = \frac{q(t)L}{k(T_\infty - T_o)} \tag{2.f}$$

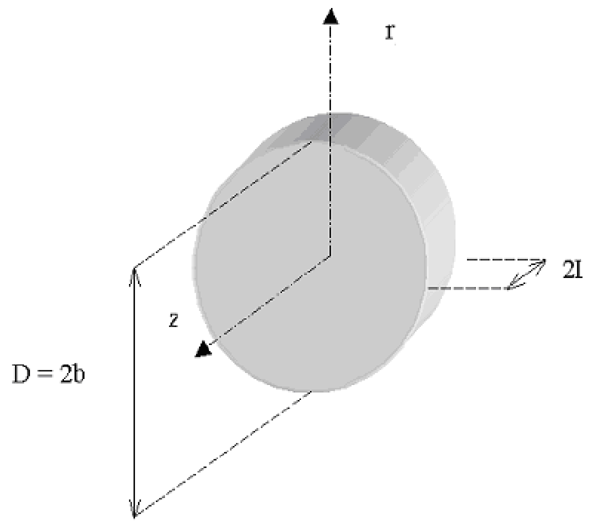

Figure 1. Body geometry

3. Direct Problem and Inverse Problem

In the *direct problem* associated with the mathematical formulation of the physical problem just described, the transient temperature field in the body is determined by assuming the physical properties, initial and boundary conditions and the geometrical characteristics of the body as known.

The *inverse problem* of interest for this work deals with the identification of the boundary heat flux $Q(\tau)$, by using transient temperature measurements taken within the body. For the solution of the inverse problem, all the other quantities appearing in the mathematical formulation of the physical problem are assumed to be known with high degree of accuracy. On the other hand, the temperature measurements may contain random errors.

We solve such inverse problem by making no *a priori* assumption regarding the functional form of the unknown heat flux, except for the functional space that it belongs to. Therefore, the solution of such inverse problem is obtained via a *function estimation approach* in an infinite dimensional space of functions condition (Tikhonov *et al*., 1977; Beck *et al*., 1977; Alifanov, 1994; Beck *et al*., 1985; Alifanov et *et al*., 1995; Dulikravich *et al*., 1986; Sabatier, 1978; Morozov, 1984; Murio, 1993; Trujillo, 1997; Hensel, 1991; Kurpisz *et al*., 1995; Desinov, 1999; Yagola *et al*., 1999; Ramm *et al*., 2000; Ozisik and *et al*., 2000). The Hilbert space of square integrable functions in the time domain of interest is selected as the functional space for the unknown (Alifanov *et al*., 1995; Ozisik *et al*., 2000).

It should be noticed the conceptual difference between the present inverse problem and that addressed in Part I of this paper, where the boundary heat flux was estimated by the identification of a constant convective heat transfer coefficient at the surface of the body and by the measurement of the water temperature. For the parameter estimation problem addressed in Part I of this paper, the thermal conductivity and the volumetric heat capacity were also considered as unknown parameters. However, for the cases examined, the analysis of the sensitivity coefficients revealed that only the thermal conductivity could be accurately identified.

For the solution of the present function estimation problem we assume that the cylinder thermal conductivity and volumetric heat capacity are known with high degree of accuracy. Therefore, the effects of uncertainties of such quantities on the inverse problem solution are not taken into account. Methods to deal with uncertainties on these parameters, which were assumed as known for the inverse analysis, in addition to the temperature measurements, constitutes a new research direction. For details, the reader is referred to Wang *et al*. (2004).

The function estimation considered here is solved through the minimization of an objective functional, involving the difference between measured and estimated temperatures. As for the companion paper, the following statistical hypotheses are assumed valid, Beck *et al*. (1977): the errors in the measured variables are additive, uncorrelated, normally distributed, with zero mean and known constant standard-deviation; only the measured variables appearing in the objective function contain errors; and there is no prior information regarding the values and uncertainties of the unknowns. For simplicity in the mathematical analysis, we also assume that the temperature measurements are continuous in the time domain.

The objective functional is given by:

$$S[Q(\tau)] = \sum^{2} \int_{\tau=0}^{\tau_f} \left\{ Y_m(\tau) - \theta\left(R_m, Z_m, \tau, Q(\tau)\right) \right\}^2 d\tau \tag{3}$$

The conjugate gradient method is used for the minimization of functional expressed by Eq. (3). The implementation of such method requires two auxiliary problems, known as the sensitivity problem and the adjoint problem. The derivations of these problems are discussed next.

4. Sensitivity Problem

The *sensitivity problem* can be obtained by assuming that the temperature $\theta(R,Z,\tau)$ is perturbed by an amount $\Delta\theta(R,Z,\tau)$, when the unknown heat flux $Q(\tau)$ is perturbed by $\Delta Q(\tau)$. By replacing $\theta(R,Z,\tau)$ by $[\theta(R,Z,\tau) + \Delta\theta(R,Z,\tau)]$ and $Q(\tau)$ by $[Q(\tau) + \Delta Q(\tau)]$ in the direct problem given by equations (1) and subtracting the original direct problem from the resulting expressions and neglecting second order terms, the following *sensitivity problem* is obtained:

$$\frac{\partial \Delta\theta}{\partial \tau} = \frac{\partial^2 \Delta\theta}{\partial R^2} + \frac{1}{R}\frac{\partial \Delta\theta}{\partial R} + \frac{1}{K^2}\frac{\partial^2 \Delta\theta}{\partial Z^2} \tag{4.a}$$

$$\left.\frac{\partial \Delta\theta}{\partial R}\right|_{R=0} = 0 \tag{4.b}$$

$$\left.\frac{\partial \Delta\theta}{\partial R}\right|_{R=1} = \Delta Q(\mathrm{t}) \tag{4.c}$$

$$\left.\frac{\partial \Delta\theta}{\partial Z}\right|_{Z=0} = 0 \tag{4.d}$$

$$\left.\frac{\partial \Delta\theta}{\partial Z}\right|_{Z=1} = \Delta Q(\tau) \tag{4.e}$$

$$\Delta\theta(R, Z, 0) = 0 \tag{4.f}$$

The sensitivity function $\Delta\theta(R,Z,\tau)$ gives the directional derivative of $\theta(R,Z,\tau)$ in the direction of the perturbation $\Delta Q(\tau)$, (Alifanov *et al*., 1995; Ozisik *et al*., 2000).

5. Adjoint Problem

To develop the *adjoint problem*, we introduce a *Lagrange multiplier* $\lambda(R,Z,\tau)$. We multiply equation (1.a) by $\lambda(R,Z,\tau)$ and integrate the resulting expression over the spatial and time domains. The expression obtained in this manner is added to the functional $S[Q(\tau)]$ given by equation (3) in order to obtain the following extended functional:

$$\begin{aligned} S[Q(\tau)] = {} & \sum_{m=1}^{2} \int_{\tau=0}^{\tau_f} \left\{ Y_m(\tau) - \theta\left(R_m, Z_m, \tau, Q(\tau)\right) \right\}^2 d\tau + \\ & + \int_{\tau=0}^{\tau_f} \int_{Z=0}^{1} \int_{R=0}^{1} \lambda(R,Z,\tau) \left[\frac{\partial \theta}{\partial \tau} - \frac{\partial^2 \theta}{\partial R^2} - \frac{1}{R}\frac{\partial \theta}{\partial R} - \frac{1}{K^2}\frac{\partial^2 \theta}{\partial Z^2} \right] R\, dR\, dZ\, d\tau \end{aligned} \tag{5}$$

Expressions for the variation $\Delta S[Q(\tau)]$ of the functional $S[Q(\tau)]$ can be developed by assuming that $\theta(R,Z,\tau)$ is perturbed by $\Delta\theta(R,Z,\tau)$ when $Q(\tau)$ is perturbed by $\Delta Q(\tau)$. The variation $\Delta S[Q(\tau)]$ gives the directional derivative of $S[Q(\tau)]$ in the direction of the perturbation $\Delta Q(\tau)$, (Alifanov et al, 1995, Ozisik and Orlande, 2000). By replacing $\theta(R,Z,\tau)$ by $[\theta(R,Z,\tau) + \Delta\theta(R,Z,\tau)]$, $Q(\tau)$ by $[Q(\tau) + \Delta Q(\tau)]$and $S[Q(\tau)]$ by $\{S[Q(\tau)] + \Delta S[Q(\tau)]\}$ in equations (1.a-f), subtracting from the resulting expressions the original equations (1.a-f), performing some lengthy but straightforward manipulations and letting the terms containing $\Delta\theta(R,Z,\tau)$ to go to zero (Alifanov *et al.*, 1995, Ozisik *et al.*, 2000), the following *adjoint problem* is obtained:

$$-\frac{\partial\lambda}{\partial\tau^*}+\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial\lambda}{\partial R}\right)+\frac{1}{K^2}\frac{\partial^2\lambda}{\partial Z^2}+2\sum_{m=1}^{2}\left[\theta\left(R_m,Z_m,\tau^*,Q(\tau^*)\right)-Y_m\left(\tau^*\right)\right]=0 \tag{6.a}$$

$$\left.\frac{\partial\lambda}{\partial R}\right|_{R=0}=0 \tag{6.b}$$

$$\left.\frac{\partial\lambda}{\partial R}\right|_{R=1}=0 \tag{6.c}$$

$$\left.\frac{\partial\lambda}{\partial Z}\right|_{Z=0}=0 \tag{6.d}$$

$$\left.\frac{\partial\lambda}{\partial Z}\right|_{Z=1}=0 \tag{6.e}$$

$$\lambda(R,Z,0)=0 \tag{6.f}$$

where $\tau^* = \tau - \tau_f$ (6.g.)

6. Gradient Equation

In the limiting process used above to obtain the adjoint problem, the following integral term is left:

$$\Delta S\left[Q(\tau)\right]=\int_{\tau=0}^{\tau_f}\int_{Z=0}^{1}\lambda\left(1,Z,\tau\right)\Delta Q(\tau)\,dZ\,d\tau+\int_{\tau=0}^{\tau_f}\int_{R=0}^{1}\lambda\left(R,1,\tau\right)\Delta Q(\tau)\,R\,dR\,d\tau \tag{7}$$

By invoking the hypothesis that the unknown function $Q(\tau)$ belongs to the space of square-integrable functions in the domain $0 < \tau < \tau_f$, we can write (Alifanov *et al.*, 1995; Ozisik *et al.*, 2000):

$$\Delta S\left[Q(\tau)\right]=\int_{\tau=0}^{\tau_f}\nabla S\left[Q(\tau)\right]\Delta Q(\tau)\,d\tau \tag{8}$$

where $\nabla S[Q(\tau)]$ is the gradient of the functional $S[Q(\tau)]$.

From the comparison of equations (7) and (8), we conclude that

$$\nabla S\left[Q(\tau)\right]=\int_{Z=0}^{1}\lambda\left(1,Z,\tau\right)dZ+\int_{R=0}^{1}\lambda\left(R,1,\tau\right)R\,dR \tag{9}$$

which is the *gradient equation* for the functional.

7. Iterative Procedure

The iterative procedure of the conjugate gradient method, as applied to the estimation of the unknown function $Q(\tau)$ through the minimization of the functional, Eq. (3), is given by (Alifanov *et al.*, 1995; Ozisik *et al.*, 2000):

$$Q^{k+1}(\tau) = Q^k(\tau) - \beta^k d^k(\tau) \tag{10}$$

where the superscript k denotes the number of iterations, β^k is the search step size and $d^k(\tau)$ is the direction of descent.

The direction of descent $d^k(\tau)$ is a conjugation of the gradient direction with previous directions of descent. It is given by (Alifanov *et al*., 1995; Ozisik *et al*., 2000):

$$d^k(\tau) = \nabla S[Q^k(\tau)] + \gamma^k d^{k-1}(\tau) \tag{11}$$

where γ^k is conjugation coefficient.

Different versions of the conjugate gradient method can be found in the literature depending on the form used for the computation of the direction of descent given by equation (11) (Alifanov *et al*., 1995; Ozisik *et al*., 2000; Colaco *et al*., 1999). For linear estimation problems such as the one under picture, where the sensitivity problem does not depend on the unknown function, Fletcher-Reeves', Polak-Ribiere's and Powel-Beale's versions of the conjugate gradient method are theoretically identical, Colaco *et al.* (1999). In this paper we use Fletcher-Reeves' version of the method, where the conjugation coefficient is taken as:

$$\gamma^k = \frac{\int_{\tau=0}^{\tau_f} \{\nabla S[Q^k(\tau)]\}^2 \, d\tau}{\int_{\tau=0}^{\tau_f} \{\nabla S[Q^{k-1}(\tau)]\}^2 \, d\tau} \qquad \text{with} \quad \gamma^k = 0 \quad \text{for} \quad k = 0 \tag{12}$$

The step size β^k is determined by minimizing the functional $S[Q^{k+1}(\tau)]$) with respect to β^k (Alifanov *et al*., 1995, Ozisik *et al*., 2000). We obtain:

$$\beta^k = \frac{\sum_{m=1}^{2} \int_{\tau=0}^{\tau_f} \left\{\theta\left(R_m, Z_m, \tau; Q^k(\tau)\right) - Y(\tau)\right\} \Delta\theta\left(R_m, Z_m, \tau; d^k(\tau)\right) d\tau}{\sum_{m=1}^{2} \int_{\tau=0}^{\tau_f} \Delta\theta\left(R_m, Z_m, \tau; d^k(\tau)\right)^2 d\tau} \tag{13}$$

where $\Delta\theta(R, Z, \tau; d^k(\tau))$ is the solution of the sensitivity problem given by equations (4.a-f), obtained by setting $\Delta Q^k(\tau) = d^k(\tau)$.

8. Stopping Criterion

The iterative procedure of the conjugate gradient method, given by equations (10-13) with the gradient computed from equation (9), is applied for the estimation of the unknown function $Q(\tau)$, until a stopping criterion is satisfied. We stop the iterative procedure of the conjugate gradient method when the functional given by equation (3) becomes sufficiently small, that is,

$$S[Q^{k+1}(\tau)] < \varepsilon \tag{14}$$

If the measurements are assumed to be free of experimental errors, we can specify ε as a relative small number. However, actual measured data contain experimental errors, which will result in an unstable inverse problem solution as the estimated temperatures approach those measured. Such difficulty can be alleviated by utilizing the *Discrepancy Principle* (Alifanov *et al*., 1995; Ozisik *et al*., 2000) to stop the iterative process and to provide the conjugate gradient method with the needed regularization for a stable solution.

In the discrepancy principle, we assume that the inverse problem solution is sufficiently accurate when the difference between estimated and measured temperatures is of the order of magnitude of the standard deviation (σ) of the measurements. The tolerance ε is then obtained from equation (3) as

$$\varepsilon = 2\,\sigma^2 \tau_f \tag{15}$$

The estimation of the boundary heat flux with the conjugate gradient method can be suitably arranged in a straightforward computational algorithm, which is omitted here for the sake of brevity. Details of such computational algorithm can be readily found in Ozisik *et al*. (2000).

9. Experiments

The experiments involving the heating of a cylindrical body in a temperature-controlled water bath were conducted in the *Laboratory of Heat Transmission and Technology* (LTTC) of PEM/COPPE. In order to examine the effects of the material type and body dimensions on the estimated parameters, the experiments were run on three different specimens. The specimens' material and dimensions are summarized in table 1. Each specimen was instrumented with two type-K thermocouples, which were located near the body lateral surface and body center, respectively. The thermocouple locations are also presented in Tab. (1), by taking as reference the cylinder center, as illustrated in Fig. (1). The temperature readings were automatically recorded by using a data logger, with a frequency of 1 measurement per sensor per second. The three specimens are illustrated in Fig. (2).

Table 1. Specimens' characteristics

Specimen	Material	Thickness (mm)	Diameter (mm)	Location of thermocouple 1* (r,z)	Location of thermocouple 2* (r,z)
1	Teflon	20.6	52.1	(0,0)	(0,8.4)
2	Teflon	9.4	51.6	(5.3,0)	(3.6,2.6)
3	Aluminum	72.7	28.6	(0,0)	(13.5,0)

* Measurements taken as reference the cylinder center (origin)

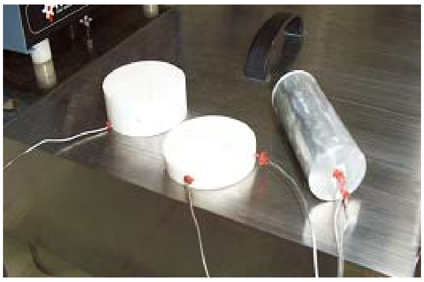

Figure 2. Specimen

In the experiments, the specimen, initially in equilibrium at room temperature, was fully immersed into the water. The time instant when the specimen was immersed was carefully recorded, in order to provide the time reference for the problem. The water temperature control in the bath was set to 50 ^{o}C, but during the experiments the water temperature was recorded. The experiment was run until the specimen was practically in thermal equilibrium with the water.

10. Results and Discussion

Before examining the estimation of the boundary heat flux by using actual measurements from the thermocouples located in accordance with Tab. (1), let's estimate the unknown function with simulated measurements. The simulated measurements were obtained from the solution of the direct problem (1.a-f) at the thermocouple locations, by using a function specified for $Q(\tau)$. The solution of the inverse problem obtained with simulated measurements is compared with the functional form used to generate the simulated measurements, in order to address the accuracy of the solution technique.

Figures (3) and (4) present comparisons between exact and estimated heat fluxes for specimen 3. The most difficult functions to be recovered by inverse analysis, involving discontinuities in the function and in its first derivative, were used to generate the simulated measurements in Figs. (3) and (4), respectively. These figures show that the present solution approach is capable of recovering such functional forms quite accurately. Similar results were obtained with simulated measurements for specimens 1 and 2.

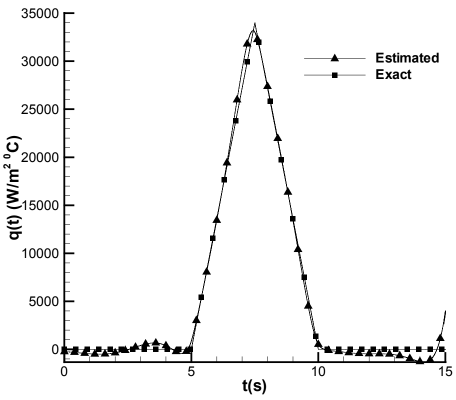

Figure 3. Estimation of a function with discontinuous first derivative by using simulated measurements for specimen 3

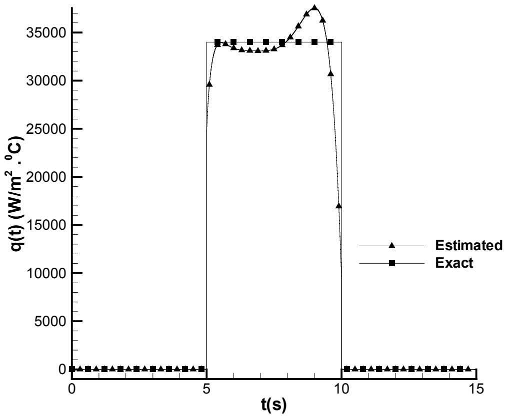

Figure 4. Estimation of a discontinuous function with simulated measurements for specimen 3

For the estimation of the boundary heat flux with actual measurements presented below, values for the thermophysical properties of the specimens were required. For the results obtained in this work, the values of thermal conductivity and volumetric heat capacity presented in Tab. (2) were used for the inverse analysis. Such values were obtained from the literature and from Part I of this paper.

Table 2. Values used for the thermophysical properties

Specimen	k (*W/mK*)	C (*J/m^{3}K*)
1	0.24	2.3×10^6
2	0.30	2.3×10^6
3	147.2	2.4×10^6

Figure 5 presents the estimated heat flux obtained with actual measurements from specimen 1. We also present in this figure the heat flux *calculated* with the heat transfer coefficient and the surface temperature obtained in Part I of this paper. As expected, the estimated heat flux is initially large in magnitude and gradually decreases to zero. Eventually, the heat flux becomes negative because of uncertainties in the temperature measurements. Although the calculated hat flux follows the same trend of the estimated heat flux, it reaches negative values with larger magnitudes. Such is the case because this heat flux is calculated with the difference between the surface and water temperatures, and it is thus more sensitive to the uncertainties on the water temperature measurements.

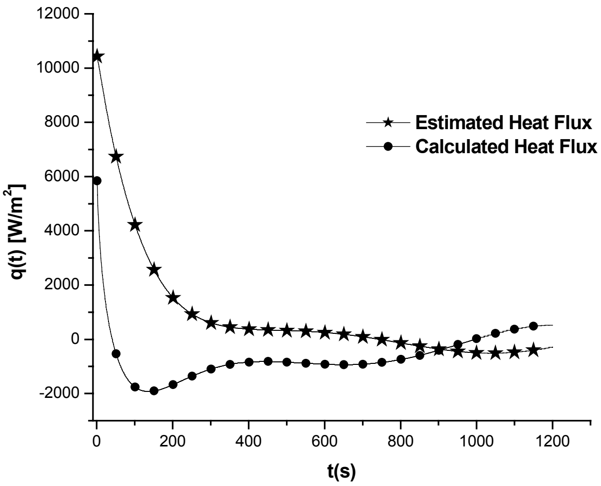

Figure 5. Heat flux for specimen 1

A comparison between the estimated heat flux, obtained with the function estimation approach, and that calculated in Part I of this paper for specimen 2, is presented in Fig. (6). This figure shows a very good agreement between the estimated and calculated heat fluxes for this case. Also, we notice that the heat flux does not become negative for specimen 2. Figure 7 presents a comparison between measured and estimated temperatures for thermocouple 2 in specimen 2. The agreement is very good.

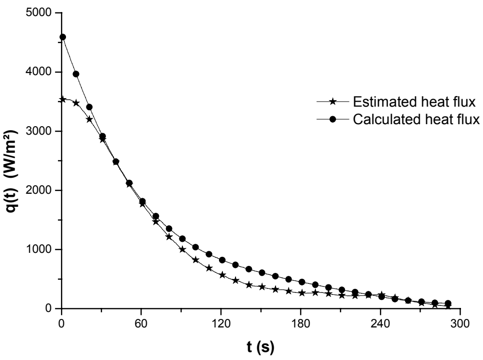

Figure 6. Heat flux for specimen 2

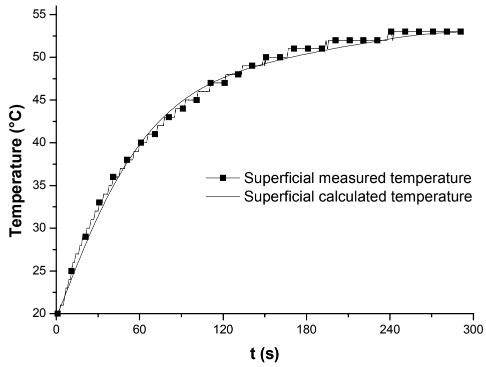

Figure 7. Comparison between measured and estimated temperatures for specimen 2

Figure 8 presents the estimated and the calculated heat fluxes for specimen 3. As for specimen 2, the agreement between the heat fluxes obtained with the parameter and estimated approaches is very good.

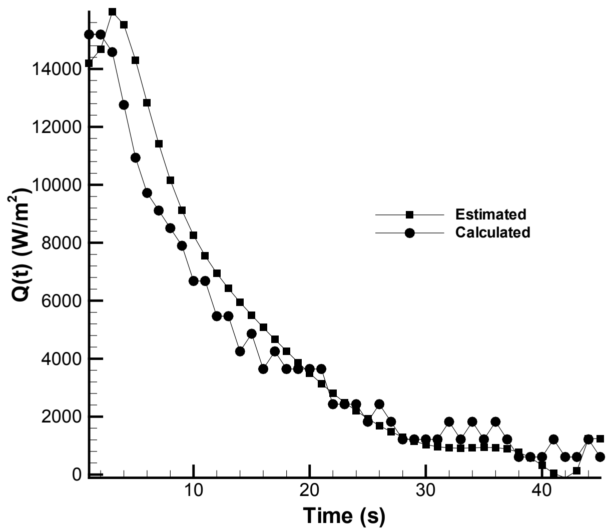

Figure 8. Heat flux for specimen 3

11. Conclusions

This paper presented the solution of the inverse function estimation problem involving the heating of cylindrical bodies in hot water. Experiments were run with three different cylinders, in order to examine the effects of size and material properties on the estimation results. The present inverse problem is concerned with the estimation of the time-dependent boundary heat flux, which is assumed to be uniform at the cylinder surface.

The use of simulated measurements revealed that the present function estimation approach is capable of recovering the boundary heat flux quite accurately, even for functional forms containing discontinuities and sharp corners. For the three specimens, the heat flux estimated with the present approach was in quite good agreement with that calculated with the heat transfer coefficient estimated in Part I of this paper. Therefore, the hypothesis of constant heat transfer coefficient used in Part I of this paper is reliable. This result reveals that forced convection is the dominant heat transfer mode between the cylinder and the water in the bath.

The continuation of this work, described in the two parts, involves the examination of the uncertainties on quantities assumed as exactly known for the inverse analysis. This represents a new research direction that involves Bayesian statistical analysis.

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