AN EFFICIENT MODEL FOR PREDICTING MIXING LENGTHS IN SERIAL PUMPING OF PETROLEUM PRODUCTS

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Abstract. *This paper presents a new model for estimating mixing volumes which arises in batching transfers in multiproduct pipelines. The novel features of the model are the incorporation of the flow rate variation with time and the use of a more precise effective dispersion coefficient, which is considered to depend on the concentration. The governing equation of the model forms a non-linear initial-value problem that is solved by using a predictor-corrector finite difference method. A comparison among the theoretical predictions of the proposed model, a field test and other classical procedures show that it exhibits the best estimate over the whole range of admissible concentrations investigated.*

Keywords: *Mixing volume, Batching transfer, Turbulent mass transfer*

1. INTRODUCTION

Multiproduct pipelines are large-diameter lines used to carry different petroleum products or different grades of the same product. Such a transport is performed by batching the products in continuous succession by either employing mechanical separators (usually pigs) between products or simply letting them mix at batch interfaces. Since

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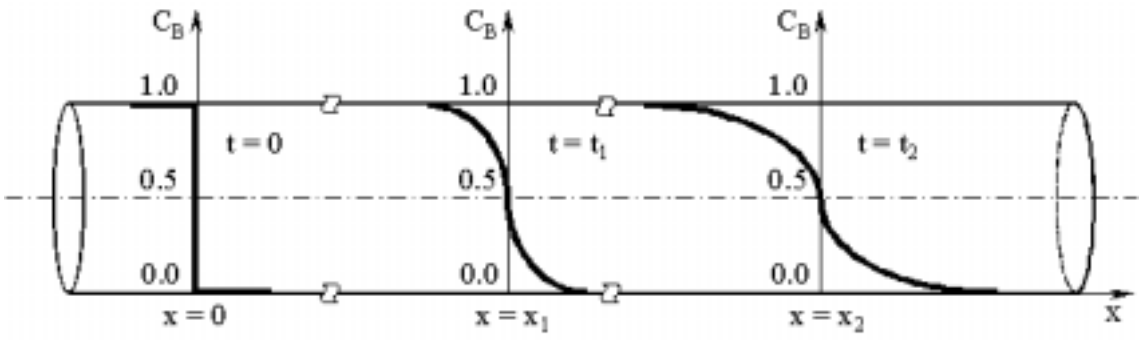

Figure 1: Evolution of the mixing region in a batching transfer.

the first approach requires a somewhat complex operation, specially when intermediate pump-station manifolds are present, the second one is in general preferred.

On the other hand, the absence of a medium to separate the products gives rise to a mixing zone at the batches' boundary, which increases in length as it travels along the pipeline towards the receiving point. The prediction of the volume of the mixing zone, for a certain degree of admissible concentrations is of great concern in practice, in order to ensure an efficient operation.

From the operational viewpoint, the occurrence of mixing zones implies in additional costs associated to shipping the mixture back to refinery for later reprocessing. In other words, the minimization of the mixing volumes should always be pursued.

To better characterize the problem, consider a batching transfer of two distinct products labeled as "A" and "B", being "B" the following fluid and "A" the leading fluid. Let t and x represent, respectively, the time and the axial coordinate of the line which begins at the discharge of the pump station, $x = 0$, and ends at the receiving point labeled as $x = L$. Let $C_\imath(x,t) \in [0,1]$, with $\imath \in \{A, B\}$, be the time-averaged mean concentration of fluid $\imath$ within the mixture at the cross-section of the pipeline at x and t. Under certain circumstances, at the beginning of a batching transfer, the concentration profile can be represented as a jump at $x = 0$. For instance, if we work with the concentration of fluid "B", then C_B jumps down as illustrated in Fig 1. During the transfer of the products through the pipeline, a mixing zone is formed at the boundary of the two adjacent products. This mixing region is constituted by a slug of contaminated material which increases in extension as the stream flows along the line. This phenomenon can be sketched in Fig 1 at subsequent time instants $t = t_1$ and $t = t_2$ at the spatial positions $x = x_1$ and $x = x_2$.

The spread of the mixing region is mainly due to turbulent dispersion of matter and is currently estimated by a number of models of semi-empirical nature (Aunicky, 1970; Austin and Palfrey, 1964; Levenspiel, 1958; Ovádi and Török, 1977; Sjenitzer, 1958; Smith and Schulze, 1948a). However, the majority of these models is based on several simplifying assumptions which in some cases seem to be inadequate. As a consequence, they do not give satisfactory results as would be expected.

As an attempt to revert this picture, it is proposed in this paper a simple model, which takes into account the variation of the flow rate and mass dispersion coefficients during the transfer. The governing equation forms a non-linear initial-value problem that is solved by a finite difference method. A comparison with experimental data shows the proposed technique is the most accurate in predicting mixing volumes among several other methods used in the literature: Sjenitzer (1958), Austin and Palfrey (1964), Levenspiel (1958), Ovadi and Török (1977), Smith-Schulze (1948a, 1948b), Aunicky (1970), Netchval

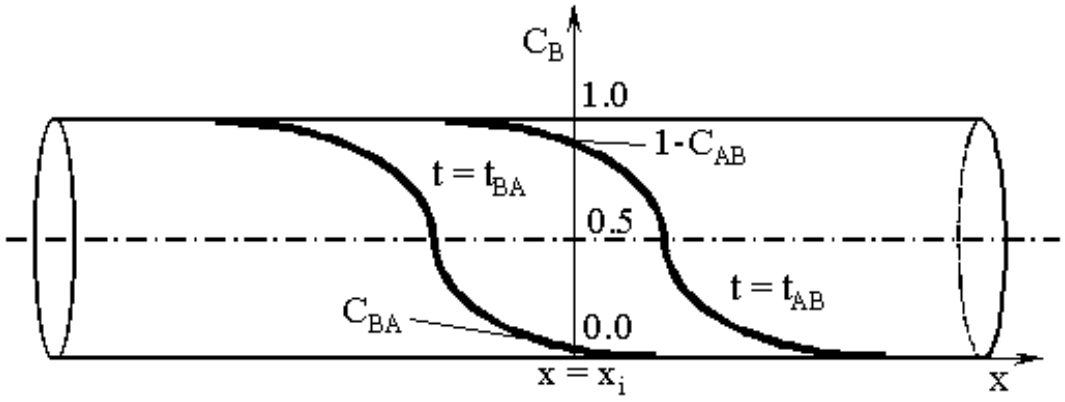

Figure 2: Characterization of the mixing volume at $x = x_i$ at different time instants, t_{BA} and t_{AB}.

et al. (1972).

2. MATHEMATICAL FORMULATION

From both practical and operational viewpoints, the mixing volume at a position $x = x_i$ in the pipeline can only be evaluated once that the admissible concentration of product "B" in "A" at the beginning of the mixing zone, C_{BA}, and the admissible concentration of product "A" in "B" at the end of the mixing zone, C_{AB}, are specified as shown in Fig 2. Once these values (which are not necessarily equal) have been chosen in such a way that the products technical specifications are not altered, the mixing volume is formally defined as:

$$V_c = \int_{t_{BA}}^{t_{AB}} Q(x = x_i, t)\, dt \tag{1}$$

in which $Q(t)$ denotes the volumetric flow rate at $x = x_i$, t_{BA} is the time instant associated to the beginning of the contaminated zone with concentration C_{BA} at $x = x_i$ and t_{BA} is the time instant associated to the end of the contaminated zone with concentration C_{AB} at $x = x_i$ (see Fig 2). In other words, t_{BA} and t_{AB} are such that $C_B(x = x_i, t = t_{BA}) = C_{BA}$ and $C_A(x = x_i, t = t_{AB}) = C_{AB}$, respectively.

With the knowledge of flow rate as a function of the time and the field $C_i(x, t)$, the mixing volume can be promptly determined by (1) at any point $x = x_i$ sufficiently away from $x = 0$.

To establish the mathematical formulation of the problem we consider as basic assumptions the following:

1. the fluids are supposed to be Newtonian and incompressible;
2. the pipeline diameter is constant and there are no intermediate pump-station manifolds;
3. the mixture process as well as the fluid mixture can be disregarded in the balance of linear momentum;
4. the specific weigths of the fluids do not significantly differ from each other;

5. minor losses can be neglected in the system.

Considering the above assumptions and designating by $x_m(t)$, with $x_m(t) \in [0, L]$, the position of the conventional half-length mixture (that is, the material coordinate in which $C_B(x = x_m(t), t) = C_A(x = x_m(t), t) = 0.5$) between the leading fluid "A" and the following fluid "B", the governing equations describing the mixing phenomenon can be written as:

$$\frac{f_B x_m + f_A(L - x_m)}{2Dg} u^2 = \frac{p_0}{\gamma_B} - \frac{p_L}{\gamma_A} + Z_0 - Z_L \tag{2}$$

$$\frac{dx_m}{dt} = u \tag{3}$$

$$\frac{\partial C_\imath}{\partial t} + u\frac{\partial C_\imath}{\partial x} = \frac{\partial}{\partial x}\Big[K^* \frac{\partial C_\imath}{\partial x}\Big] \tag{4}$$

for $(x, t) \in (0, +\infty) \times (0, T)$ in which $u = u(t)$ is the bulk time-average axial velocity in the tube, K^* is the effective dispersion coefficient, g is the gravitational acceleration, D stands for the pipeline diameter which is supposed to be constant along its length L. The specific weights of the products are denoted by $\gamma_\imath$, with $\imath \in \{A, B\}$, and the topographical heights at the beginning ($x = 0$) and at the receiving point ($x = L$) of the pipeline are designated by Z_0 and Z_L, respectively.

Equations (2) and (4) represent the balance of linear momentum for the fluids in the line and the concentration distribution which arises as a consequence of the dispersion phenomenon (Taylor, 1954). In equation (2) $f_\imath$, with $\imath \in \{A, B\}$, represent the Darcy-Weisbach friction factor related to the stretch of the pipeline where the flow of the fluid $\imath$ takes place and is given by

$$f_\imath = \left\{1.8 \log\left[\frac{6.9}{Re_\imath} + \left(\frac{\epsilon_r}{3.7}\right)^{1.11}\right]\right\}^{-2} \tag{5}$$

in which $Re_\imath = uD/\nu_\imath$ is the Reynods number associated to the flow of fluid $\imath$ and ϵ_r is the pipeline relative roughness.

The terms p_0 and p_L in equation (2) are the pressures at $x = 0$ and at $x = L$, respectively. They describe different operational equipments and so are generically represented as functions of the time and fluid velocity. If during the transfer the pressure at the receiving point is held constant and the pump at $x = 0$ is supposed to be centrifugal and to be running at constant speed then:

$$p_0 = (a - bQ^m)\,\gamma_B \tag{6}$$

in which a, b and m are constant parameters of the pump curve and Q is the flow rate through the pump. If only a single pump is used in the transfer, $Q = u\pi D^2/4$.

By considering that the products "A" and "B" are pumped sequentially in such way that "A" is followed by "B" and that following relationship must always hold,

$$C_A + C_B = 1 \tag{7}$$

the initial conditions for (4) are given by, according the case $\imath = A$ or $\imath = B$:

$$\imath = B, \quad \begin{cases} C_B(x,t) = C_B(x \leq 0, 0) = 1 \\ C_B(x,t) = C_B(x > 0, 0) = 0 \end{cases} \tag{8}$$

or

$$\imath = A, \quad \begin{cases} C_A(x,t) = C_A(x \leq 0, 0) = 0 \\ C_A(x,t) = C_A(x > 0, 0) = 1. \end{cases} \tag{9}$$

As a consequence of the assumptions made so far, the bulk velocity $u(t)$ can be calculated by solving (2) along with (3) and (6) independently of (4). With the function $u(t)$ on one hand and the introduction of the following change of variables on the other:

$$y = \frac{x}{D} - \tau, \tag{10}$$

$$\tau = \frac{1}{D}\int_0^t u(t')dt', \tag{11}$$

the problem described by (4) along with (8) or (9) can be stated in a more convenient and peculiar form:

$$\frac{\partial C_\imath}{\partial \tau} = \frac{\partial}{\partial y}\left[K\frac{\partial C_\imath}{\partial y}\right] \text{ for } \imath \in \{A, B\} \tag{12}$$

subjected to

$$\imath = B, \quad \begin{cases} C_B(y,\tau) = C_B(y \leq 0, 0) = 1 \\ C_B(y,\tau) = C_B(y > 0, 0) = 0 \end{cases} \tag{13}$$

or

$$\imath = A, \quad \begin{cases} C_A(y,\tau) = C_A(y \leq 0, 0) = 0 \\ C_A(y,\tau) = C_A(y > 0, 0) = 1. \end{cases} \tag{14}$$

in which $K = K^*/Du$ stands for the dimensionless effective axial dispersion coefficient.

The problem of contamination in pipelines described by (12) with (13) or (14), which was formulated for the first time by (Taylor, 1954), is the basis for a number of models (Aunicky, 1970; Austin and Palfrey, 1964; Levenspiel, 1958; Ovádi and Török, 1977; Sjenitzer, 1958; Smith and Schulze, 1948a) currently used by several pipeline companies around the world. However, all of them consider, as simplifying hypotheses, that the axial velocity as well as the dispersion coefficient are constants. Moreover, they make use of poor correlations for the dispersion coefficient which, in turn, results in inacurate values of mixing volumes, as it will be seen later.

Based on a bibliographical review, we have found that the correlation for the dispersion coefficient proposed by (Krantz and Wasan, 1974) is the most accurate among the usual ones. This assertion is based on a proper choice of the mean velocity and diffusivity distributions in the pipe wall region employed to compute K.

Even though the work of (Krantz and Wasan, 1974) is devoted to non-Newtonian fluids, they have presented for Newtonian fluids a graphical correlation for K as a function of Re (for different Schimdt numbers), as a particular case. For $Sc = 1000$, the following titted relationship between K and Re with $2 \times 10^3 \leq Re \leq 10^6$ is obtained (Krantz and

Wasan, 1974)

$$\log K = -0.0641(\log Re)^4 + 1.1274(\log Re)^3 - 6.9173(\log Re)^2 + 16.379 \log Re - 10.597. \quad (15)$$

To assign a more precise feature to the model, we consider Re in the above expression as the Reynolds number of the mixture $Re = uD/\nu$, being ν the kinematic viscosity of the mixture. This viscosity depends on the concentration of the products in the mixture and can be evaluated according to (Gambill, 1959)

$$\nu^{\frac{1}{3}} = C_A \nu_A^{\frac{1}{3}} + C_B \nu_B^{\frac{1}{3}}. \quad (16)$$

This approach has already been considered by (Netchval et al., 1972) who has shown that this feature is responsible for the asymmetric shape of the spatial concentration distribution.

Finally, once a numerical approximation for $C_\imath(y, \tau)$ is available the mixing volume is readly calculated.

3. NUMERICAL TECHNIQUE

To numerically solve the non-linear problem described by (12) with (13) or (14) we have used the predictor-corrector method proposed by (Ames, 1977).

Let $h > 0$ be the increment of y used to discretize the spatial computational domain of the problem in a set of points $\{y_i\}_{i=0}^n$ with $y_i = ih$, and $k > 0$ be the increment of τ used to discretize the time domain in a set of time instants $\{\tau_j\}_{j=0}^m$ with $\tau_j = jk$. By denoting the approximation of $C_\imath(y_i, \tau_j)$ by $C_{i,j}$, the expression for the predictor can be written as

$$\begin{aligned} C_{i-1,j+\frac{1}{2}} - \left[2 + \frac{4h^2}{kK(\tau_{j+\frac{1}{2}}, C_{i,j})}\right] C_{i,j+\frac{1}{2}} + C_{i+1,j+\frac{1}{2}} = \\ -C_{i-1,j} + \left[2 - \frac{4h^2}{kK(\tau_{j+\frac{1}{2}}, C_{i,j})}\right] C_{i,j} - C_{i+1,j} \\ -\frac{1}{8}\frac{\partial K}{\partial C}(\tau_{j+\frac{1}{2}}, C_{i,j}) \left[C_{i+1,j} - C_{i-1,j}\right]^2 \end{aligned} \quad (17)$$

and this expression for the corrector as

$$\begin{aligned} C_{i-1,j+1} - \left[2 + \frac{2h^2}{kK(\tau_{j+\frac{1}{2}}, C_{i,j+\frac{1}{2}})}\right] C_{i,j+1} + C_{i+1,j+1} = \\ -C_{i-1,j} + \left[2 - \frac{2h^2}{kK(\tau_{j+\frac{1}{2}}, C_{i,j+\frac{1}{2}})}\right] C_{i,j} - C_{i+1,j} \\ -\frac{1}{8}\frac{\partial K}{\partial C}(\tau_{j+\frac{1}{2}}, C_{i,j+\frac{1}{2}}) \left[C_{i+1,j+\frac{1}{2}} - C_{i-1,j+\frac{1}{2}}\right]^2 \end{aligned} \quad (18)$$

The solution of (18), $(C_{0,j+1}, C_{1,j+1}, \cdots, C_{n,j+1})$, at the time instant $\tau = (j+1)k$ is obtained through (17), $(C_{0,j+\frac{1}{2}}, C_{1,j+\frac{1}{2}}, \cdots, C_{n,j+\frac{1}{2}})$ with the values of C at time instante $\tau = jk$. This procedure is carried out by advancing in time a sufficient number of times

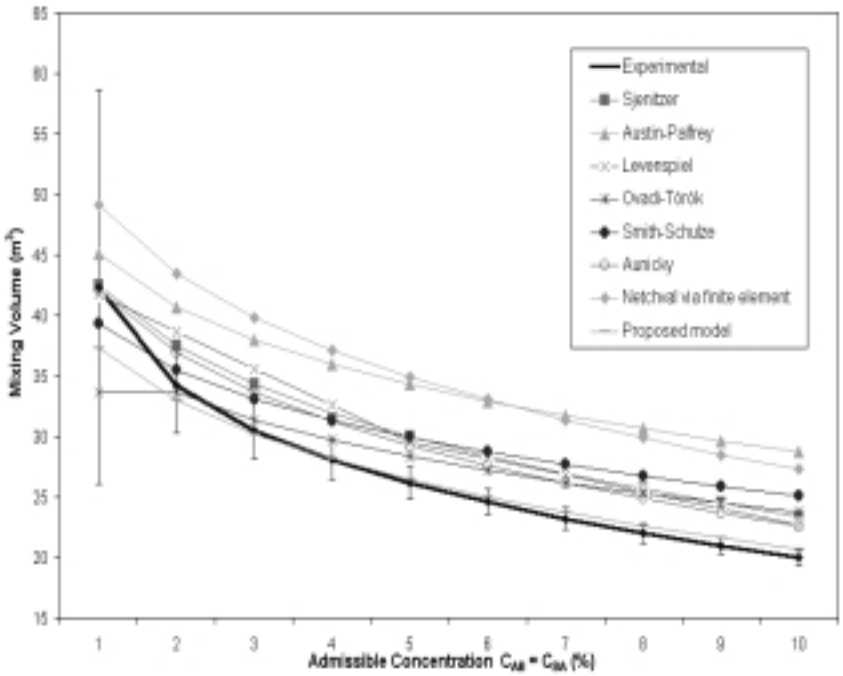

Figure 3: Predicted and measured mixing volumes as a function of admissible concentrations.

to ensure that the end of the mixing zone has crossed the spatial point $x = x_i$.

4. RESULTS AND DISCUSSION

To illustrate the performance of the model described in the past sections, a comparison between its predictions and experimental data are presented next for different values of admissible concentrations $C_{AB} = C_{BA} = 1, 2, \ldots, 9, 10\%$. The experimental data refers to a transfer of gasoline/diesel, being the gasoline the leading fluid ($\gamma_A = 734 kgf/m^3$, $\nu_A = 0.9 cSt$) and the diesel the following product ($\gamma_B = 833 kgf/m^3$, $\nu_B = 7.6 cSt$) which took place in a pipeline (10" of diameter and $199.8 km$ long) operated by Petrobras. The topographic difference level between the receiving point and the pump station is $Z_L - Z_0 = -895 m$.

The experimental mixing volume was evaluated by continuous monitoring of the sonic velocity of the mixture at the position $x = 135.9 km$. To do so, it was used a clamp-on transit-time ultrasonic flowmeter with an aquisition frequency of $0.2 Hz$ (Couto, 1998; Freitas Rachid et al., 1999a). The volumetric flow rate at the beginning of the transfer was $245 m^3/h$ and the pressure at the receiving point was $p_L = 9.21 kgf/cm^2$. The parameters a, b and m of equation (6) were determined from the pump curve and are equal to $a = 378.8 m$, $b = 5099.1 s^{1.75}/m^{4.25}$ and $m = 1.75$.

To better characterize the model performance, Fig 3 also shows the predictions of seven traditional methods currently used to predict mixing volumes: Sjenitzer (1958), Austin and Palfrey (1964), Levenspiel (1958), Ovadi and Török (1977), Smith-Schulze (1948a, 1948b), Aunicky (1970), Netchval et al. (1972). It should be noticed that the original method of Netchval is not capable to deal with kinematic viscosity ratios ν_A/ν_B or ν_B/ν_A greater than 8. The prediction of this method shown in Fig 3 was obtained by using a finite element technique proposed in (Freitas Rachid et al., 1999b).

The curves of mixing volume as a function of admissible concentrations $C_{AB} = C_{BA} = 1, 2, \ldots, 9, 10\%$ depicted in Fig 3 show that the proposed model is the one that presents the best agreement with experimental data over the whole range of concentration. The greatest discrepancies between the proposed model and experimental data take place in the range of concentrations $1\% \leq C_{AB} = C_{BA} \leq 2\%$. However, since the uncertainty associated to the mixing volume varies from $\pm 24.17 m^3$ (for $C_{AB} = C_{BA} = 1\%$) to $\pm 4.13 m^3$ (for $C_{AB} = C_{BA} = 2\%$), as seen in Fig 3, no comparisons among the methods can be effectively done for $C_{AB} = C_{BA} < 2\%$ inasmuch as their predictions are within the experimental uncertainty.

5. FINAL REMARKS

A model has been proposed in this paper to evaluate mixing volumes in pipeline batch transfers. The novel features of this model are the incorporation of the flow rate variation with time and the use of a more precise effective dispersion coefficient, which is considered to depend on the concentration. A comparison with experimental data and with other traditional methods available in the literature shows that the proposed model is the most accurate.

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