SIMULTANEOUS ESTIMATION OF TWO BOUNDARY HEAT FLUXES IN PARALLEL PLATE CHANNELS

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Abstract. *This paper deals with the use of the conjugate gradient method of function estimation for the identification of two unknown boundary heat fluxes in a parallel plate channel. The fluid flow is assumed to be laminar and hydrodynamically developed. The boundary heat fluxes are supposed to vary in time and along the channel. Temperature measurements taken inside the channel are used in the inverse analysis. The accuracy of the present solution approach is examined by using simulated measurements containing random errors, for strict cases involving functional forms with discontinuities or sharp-corners for the boundary heat fluxes.*

Keywords: *Conjugate gradient method, function estimation, forced convection, laminar flow, parallel plate channel*

1. INTRODUCTION

The interest on the solution of inverse heat convection problems is more recent than on the solution of inverse heat conduction problems. To the best of the authors' knowledge, the first article dealing with an inverse heat convection problem is the one due to Moutsoglou (1989).Afterwards, several other articles involving inverse convection appeared in the literature (Moutsouglou, 1990; Huang and Özisik, 1992; Raghunath, 1993; Bokar and Özisik, 1995; Liu and Özisik, 1996a; Liu and Özisik, 1996b; Li et al, 1996; Machado and Orlande, 1996; Machado and Orlande, 1997; Szczygiel, 1997; Moaveni, 1997; Machado and Orlande, 1998; Aparecido and Ozisik, 1999; Colaço and Orlande, 2000). However, in all those works, a single unknown function was considered in the analysis.

In this paper, we present the solution of the inverse problem of estimating simultaneously the boundary heat fluxes at the two walls of a parallel plate channel. As the solution technique, we apply the conjugate gradient method of function estimation (Alifanov, 1974; Alifanov, 1994; Huang and Özisik, 1992; Bokar and Özisik, 1995; Liu and Özisik, 1996a; Liu and Özisik, 1996b; Machado and Orlande, 1996; Machado and Orlande, 1997; Machado and Orlande, 1998; Colaço and Orlande, 2000, Özisik and Orlande, 2000), by assuming that no information is available regarding the time and spatial variations of the unknown function. Simulated temperature measurements taken inside the channel are used in the inverse analysis, in order to address the accuracy of the present solution technique. Basic steps of the conjugate gradient method of function estimation include: (i) The Direct Problem, (ii) The Inverse Problem, (iii) The Sensitivity Problem, (iv) The Adjoint Problem, (v) The Gradient Equation, (vi) The Iterative Procedure, (vii) The Stopping Criterion and (viii) The Computational Algorithm. Details of such steps, as applied to the present inverse problem, are described next.

2. DIRECT PROBLEM

The physical problem considered here involves the laminar hydrodynamically–developed forced convection of a Newtonian fluid in a parallel plate channel. The fluid is initially at the temperature T_0, which is also the inlet temperature. The channel walls, separated by a distance h, are subject to different heat fluxes varying in time and also along the channel. In this work, we use the formulation of such physical problem in terms of generalized spatial coordinates (ξ,η) in a boundary-fitted coordinate system. The mathematical formulation is then given by:

$$\frac{\partial(\rho J T)}{\partial t}+\frac{\partial(\tilde{U}\rho T)}{\partial \xi}+\frac{\partial(\tilde{V}\rho T)}{\partial \eta}= \frac{\partial}{\partial \xi}\left[D_{11}\frac{\partial T}{\partial \xi}+D_{12}\frac{\partial T}{\partial \eta}\right]+\frac{\partial}{\partial \eta}\left[D_{21}\frac{\partial T}{\partial \xi}+D_{22}\frac{\partial T}{\partial \eta}\right] \tag{1.a}$$

in $1 < \eta < N$, $1 < \xi < M$; for $t > 0$

$$T = T_0 \qquad \text{t } \xi=1,\ 1<\eta<N;\ \text{for } t>0 \tag{1.b}$$

$$D_{11}\frac{\partial T}{\partial \xi}+D_{12}\frac{\partial T}{\partial \eta}=0 \qquad \text{t } \xi=M,\ 1<\eta<N;\ \text{for } t>0 \tag{1.c}$$

$$D_{21}\frac{\partial T}{\partial \xi}+D_{22}\frac{\partial T}{\partial \eta}=-\frac{q_1(\xi,t)\sqrt{\alpha_{22}}}{C_p} \qquad \text{t } \eta=1,\ 1<\xi<M;\ \text{for } t>0 \tag{1.d}$$

$$D_{21}\frac{\partial T}{\partial \xi}+D_{22}\frac{\partial T}{\partial \eta}=\frac{q_2(\xi,t)\sqrt{\alpha_{22}}}{C_p} \qquad \text{t } \eta=N,\ 1<\xi<M;\ \text{for } t>0 \tag{1.e}$$

$$T = T_0 \qquad \text{or } t=0;\ \text{n } 1<\eta<N,\ 1<\xi<M \tag{1.f}$$

where the boundaries η=1 and η=N correspond to the channel walls at y=0 and y=h, respectively, while the boundaries ξ=1 and ξ=M, correspond to the channel inlet and outlet, respectively. Different parameters appearing above are defined as

$$J = x_\xi y_\eta - x_\eta y_\xi; \quad \alpha_{22} = J^2\left(\eta_x^2+\eta_y^2\right) \tag{2.a,b}$$

$$D_{11} = \frac{J k}{C_p}\left(\xi_x^2+\xi_y^2\right); \quad D_{22} = \frac{J k}{C_p}\left(\eta_x^2+\eta_y^2\right) \tag{2.c,d}$$

$$D_{12} = D_{21} = \frac{J k}{C_p}\left(\xi_x \eta_x + \xi_y \eta_y\right) \tag{2.e}$$

$$\tilde{U} = J\left(u\,\xi_x + v\,\xi_y\right); \quad \tilde{V} = J\left(u\,\eta_x + v\,\eta_y\right) \tag{2.f,g}$$

where the subscripts ξ, η, x and y denote derivatives and the velocity profile is given by:

$$u(y) = \frac{6\bar{u}}{h^2}\left(y^2 - hy\right) \tag{2.p}$$

where $\bar{u}$ is the mean velocity of the fluid.

The *direct problem* is concerned with the determination of the temperature distribution inside the channel, from the knowledge of the velocity profile (2.p), initial and inlet temperature T_0, thermophysical properties and boundary heat fluxes $q_1(\xi,t)$ and $q_2(\xi,t)$.

3. INVERSE PROBLEM

The *inverse problem* under picture in this paper is concerned with the simultaneous estimation of the time and spatial variations of the boundary heat fluxes $q_1(\xi,t)$ and $q_2(\xi,t)$, by using temperature measurements taken inside the channel. The inverse problem is reformulated as a minimization problem involving the following objective functional:

$$S\left[q_1(\xi,t), q_2(\xi,t)\right] = \frac{1}{2}\int_{\tau=0}^{\tau_f} \sum_{s=1}^{S} \left\{T\left(\xi_s, \eta_s, t; q_1, q_2\right) - \mu_s(t)\right\}^2 dt \tag{3}$$

where S is the number of sensors used in the analysis, $\mu_s(t)$ are the measured temperatures at the position (ξ_s,η_s), while $T[\xi_s,\eta_s,t;\ q]$ are the estimated temperatures at the measurement positions.

The minimization of the objective functional (3) is obtained through the conjugate gradient method (Alifanov, 1974; Alifanov, 1995; Huang and Özisik, 1992; Raghunath, 1993; Machado and Orlande, 1996; Machado and Orlande, 1997; Machado and Orlande; 1998; Bokar and Özisik, 1995; Liu and Özisik, 1996a; Liu and Özisik, 1996b; Colaço and Orlande, 2000; Özisik and Orlande, 2000). Auxiliary problems, known as the *sensitivity and adjoint problems*, are required for the implementation of the iterative procedure of such a method, as described next.

4. SENSITIVITY PROBLEM

The sensitivity problem is used to determine the temperature variation due to changes in the unknown quantity. Since the present work deals with the estimation of two unknown functions, two sensitivity problems are required in the analysis. They are derived by considering perturbations of the boundary heat fluxes each at a time, as described next.

Let us consider that the temperature $T_1(\xi,\eta,t)$ undergoes a variation $\varepsilon\Delta T_1(\xi,\eta,t)$, when the boundary heat flux $q_1(\xi,t)$ is perturbed by $\varepsilon\Delta q_1(\xi,t)$, where ε is a small real number. In order to derive the sensitivity problem for $\Delta T_1(\xi,\eta,t)$, we write the direct problem in operator form and apply the following limiting process (Alifanov, 1994; Özisik and Orlande, 2000):

$$D_{\Delta q_1} T(\xi,\eta,t) = \frac{L_\varepsilon(q_{1\varepsilon}) - L(q_1)}{\varepsilon} = 0 \tag{4}$$

where $L_\varepsilon(q_{1\varepsilon})$ and $L(q_1)$ are the operator forms of the direct problem written for the perturbed $[q_1(\xi,t) + \varepsilon\Delta q_1(\xi,t)]$ and unperturbed $q_1(\xi,t)$ heat flux at the boundary $\eta=1$, respectively. A similar procedure is used for the derivation of the sensitivity problem for the function

$\Delta T_2(\xi,\eta,t)$, resultant from the perturbation of the heat flux $q_2(\xi,t)$ by $\varepsilon\Delta q_2(\xi,t)$, at the boundary η=N. We then obtain the sensitivity problems for the determination of the functions $\Delta T_j(\xi,\eta,t)$, j=1,2, respectively as:

$$\frac{\partial(\rho J \Delta T_j)}{\partial t}+\frac{\partial(\tilde{U}\rho\Delta T_j)}{\partial \xi}+\frac{\partial(\tilde{V}\rho\Delta T_j)}{\partial \eta}=$$

$$\frac{\partial}{\partial \xi}\left[D_{11}\frac{\partial \Delta T_j}{\partial \xi}+D_{12}\frac{\partial \Delta T_j}{\partial \eta}\right]+\frac{\partial}{\partial \eta}\left[D_{21}\frac{\partial \Delta T_j}{\partial \xi}+D_{22}\frac{\partial \Delta T_j}{\partial \eta}\right] \tag{5.a}$$

in $1<\eta<N$, $1<\xi<M$; for $t>0$

$$\Delta T_j = 0 \qquad \text{at } \xi=1,\ 1<\eta<N;\ \text{for } t>0 \tag{5.b}$$

$$D_{11}\frac{\partial \Delta T_j}{\partial \xi}+D_{12}\frac{\partial \Delta T_j}{\partial \eta}=0 \qquad \text{at } \xi=M,\ 1<\eta<N;\ \text{for } t>0 \tag{5.c}$$

$$D_{21}\frac{\partial \Delta T_j}{\partial \xi}+D_{22}\frac{\partial \Delta T_j}{\partial \eta}=-\delta_{1j}\frac{\Delta q_1\sqrt{\alpha_{22}}}{C_p} \qquad \text{at } \eta=1,\ 1<\xi<M;\ \text{for } t>0 \tag{5.d}$$

$$D_{21}\frac{\partial \Delta T_j}{\partial \xi}+D_{22}\frac{\partial \Delta T_j}{\partial \eta}=\delta_{2j}\frac{\Delta q_2\sqrt{\alpha_{22}}}{C_p} \qquad \text{at } \eta=N,\ 1<\xi<M;\ \text{for } t>0 \tag{5.e}$$

$$\Delta T_j = 0 \qquad \text{for } t=0;\ \text{in } 1<\eta<N,\ 1<\xi<M \tag{5.f}$$

where

$$\delta_{ij}=\begin{cases}0 & \text{, for } i\neq j\\ 1 & \text{, for } i=j\end{cases} \tag{6}$$

5. ADJOINT PROBLEM

The adjoint problem is obtained by multiplying equation (1.a) by the *Lagrange Multiplier* $\lambda(\xi,\eta,t)$, integrating the resulting expression over the time and space domains and adding the result to the functional given by equation (3). We obtain:

$$S[q(\xi,t)]=\frac{1}{2}\int_{\eta=1}^{N}\int_{\xi=1}^{M}\int_{t=0}^{t_f}\sum_{s=1}^{S}\{T(\xi_s,\eta_s,t;q)-\mu_s(t)\}^2\delta(\mathbf{r}-\mathbf{r}_s)\,dt\,d\xi\,d\eta+$$
$$\int_{\eta=1}^{N}\int_{\xi=1}^{M}\int_{t=0}^{t_f}\lambda(\xi,\eta,t)\left\{\frac{\partial(\rho J T)}{\partial t}+\frac{\partial(\tilde{U}\rho T)}{\partial \xi}+\frac{\partial(\tilde{V}\rho T)}{\partial \eta}-\right.$$
$$\left.\frac{\partial}{\partial \xi}\left[D_{11}\frac{\partial T}{\partial \xi}+D_{12}\frac{\partial T}{\partial \eta}\right]+\frac{\partial}{\partial \eta}\left[D_{21}\frac{\partial T}{\partial \xi}+D_{22}\frac{\partial T}{\partial \eta}\right]\right\}dt\,d\xi\,d\eta \tag{7}$$

where $\delta(\cdot)$ is the Dirac delta function.

We now perturb $q_1(\xi,t)$ by $\varepsilon\Delta q_1(\xi,t)$ and $T_1(\xi,\eta,t)$ by $\varepsilon\Delta T_1(\xi,\eta,t)$ in equation (7) and apply the following limiting process to obtain the directional derivative of the functional $S[q_1(\xi,t), q_2(\xi,t)]$ in the direction of the perturbation $\varepsilon\Delta q_1(\xi,t)$:

$$D_{\Delta q_1} S[q_1(\xi,t), q_2(\xi,t)] = \frac{S_\varepsilon(q_{1\varepsilon}) - S(q_1)}{\varepsilon} \tag{8}$$

where $S_\varepsilon(q_{1\varepsilon})$ and $S(q_1)$ denote the functional (7) written for the perturbed and unperturbed heat flux $q_1(\xi,t)$, respectively.

By employing integration by parts, utilizing the initial and boundary conditions of the sensitivity problem for $\Delta T_1(\xi,\eta,t)$ and also requiring that the coefficients of $\Delta T_1(\xi,\eta,t)$ in the resulting equation vanish, the following *adjoint problem* is obtained:

$$-\frac{\partial(\rho J^2 \lambda)}{\partial t} - \frac{\partial(\tilde{U}\rho\lambda J)}{\partial \xi} - \frac{\partial(\tilde{V}\rho\lambda J)}{\partial \eta} = \frac{\partial}{\partial \xi}\left[D_{11}\frac{\partial(\lambda J)}{\partial \xi} + D_{12}\frac{\partial(\lambda J)}{\partial \eta}\right] + \frac{\partial}{\partial \eta}\left[D_{21}\frac{\partial(\lambda J)}{\partial \xi} + D_{22}\frac{\partial(\lambda J)}{\partial \eta}\right] - \sum_{s=1}^{S}\{T(\xi_s,\eta_s,t;q) - \mu_s(t)\}^2 \delta(\mathbf{r}-\mathbf{r}_s) \tag{9.a}$$

in $1 < \eta < N$, $1 < \xi < M$; for $t > 0$

$$D_{11}\frac{\partial(\lambda J)}{\partial \xi} + D_{12}\frac{\partial(\lambda J)}{\partial \eta} = 0 \qquad \text{t } \xi=1,\ 1<\eta<N;\ \text{for } t>0 \tag{9.b}$$

$$\lambda = 0 \qquad \text{t } \xi=M,\ 1<\eta<N;\ \text{for } t>0 \tag{9.c}$$

$$D_{21}\frac{\partial(\lambda J)}{\partial \xi} + D_{22}\frac{\partial(\lambda J)}{\partial \eta} = 0 \qquad \text{t } \eta=1,\ 1<\xi<M;\ \text{for } t>0 \tag{9.d}$$

$$D_{21}\frac{\partial(\lambda J)}{\partial \xi} + D_{22}\frac{\partial(\lambda J)}{\partial \eta} = 0 \qquad \text{t } \eta=N,\ 1<\xi<M;\ \text{for } t>0 \tag{9.e}$$

$$\lambda = 0 \qquad \text{or } t = t_f;\ \text{in } 1<\eta<N,\ 1<\xi<M \tag{9.f}$$

A limiting process analogous to equation (8) is used in order to obtain the directional derivative of the functional $S[q_1(\xi,t),q_2(\xi,t)]$ in the direction of the perturbation $\varepsilon\Delta q_2(\xi,t)$. After performing similar manipulations, we obtain an adjoint problem resulting from the perturbation in $q_2(\xi,t)$ identical to that given by equations (9), resulting from the perturbation in $q_1(\xi,t)$. Therefore, one single adjoint problem needs to be solved at each iteration of the conjugate gradient method, despite the fact that two unknown functions are to be estimated.

6. GRADIENT EQUATION

In the process of obtaining the adjoint problem resulting from the perturbation in $q_1(\xi,t)$, the directional derivative of the functional in the direction $\varepsilon\Delta q_1(\xi,t)$ reduces to

$$D_{\Delta q_1} S[q(\xi,t)] = -\int_{\xi=1}^{M}\int_{t=0}^{t_f}\left[\left.\frac{\sqrt{\alpha_{22}}\,\Delta q_1 J \lambda}{C_p}\right|_{\eta=1}\right] dt\, d\xi$$

(10.a)

By assuming that the function $q_1(\xi,t)$ belongs to the space of square integrable functions in $0 < t < t_f$, $1 < \xi < M$, we can obtain the gradient equation of the functional for the estimation of the function $q_1(\xi,t)$ as:

$$\nabla S\left[q_1(\xi,t)\right] = -\frac{J\lambda}{C_p}\bigg|_{\eta=1} \tag{11.a}$$

An analogous procedure is used in order to obtain the gradient equation of the functional for the estimation of the function $q_2(\xi,t)$ as:

$$\nabla S\left[q_2(\xi,t)\right] = -\frac{J\lambda}{C_p}\bigg|_{\eta=N} \tag{11.b}$$

7. ITERATIVE PROCEDURE

The iterative procedure of the conjugate gradient method, as applied to the simultaneous estimation of $q_j(\xi,t)$, j=1,2, is given by:

$$q_j^{k+1}(\xi,t) = q_j^k(\xi,t) - \beta_j^k\, d_j^k(\xi,t) \tag{12.a}$$

where k is the number of iterations. The directions of descent $d_j^k(\xi,t)$, j=1,2, are obtained from

$$d_j^k(\xi,t) = \nabla S\left[q_j^k(\xi,t)\right] + \gamma_j^k\, d_j^{k-1}(\xi,t) \tag{12.b}$$

The conjugation coefficients γ_j^k , j=1,2, can be obtained from the Fletcher-Reeves expression as

$$\gamma_j^k = \frac{\displaystyle\int_{\xi=1}^{M}\int_{t=0}^{t_f} \left\{\nabla S\left[q_j^k(\xi,t)\right]\right\}^2 dt\, d\xi}{\displaystyle\int_{\xi=1}^{M}\int_{t=0}^{t_f} \left\{\nabla S\left[q_j^{k-1}(\xi,t)\right]\right\}^2 dt\, d\xi} \qquad \text{for } k=1,2,\ldots \tag{12.c}$$

with $\gamma_j^0 = 0$ for $k = 0$

Expressions for the search step sizes β_j^k $j=1,2$, are obtained by minimizing $S[q_1^{k+1}(\xi,t), q_2^{k+1}(\xi,t)]$ with respect to β_1^k and β_2^k. We obtain:

$$\beta_1^k = \frac{C_1 C_2 - C_3 C_4}{C_5 C_2 - C_3^2}; \quad \beta_2^k = \frac{C_4 C_5 - C_3 C_1}{C_5 C_2 - C_3^2} \tag{13.a,b}$$

where:

$$C_1 = \int \sum_{s=1}^{S} [T-\mu]\Delta T_2 \, dt; \quad C_2 = \int \sum_{s=1}^{S} (\Delta T_1)^2 \, dt; \quad C_3 = \int \sum_{s=1}^{S} \Delta T_1 \Delta T_2 \, dt$$

$$C_4 = \int \sum_{s=1}^{S} [T-\mu]\Delta T_1 \, dt; \quad C_5 = \int \sum_{s=1}^{S} (\Delta T_2)^2 \, dt \tag{14.a-e}$$

8. STOPPING CRITERION

The use of the stopping criterion based on the *Discrepancy Principle* gives the conjugate gradient method an iterative regularization character (Alifanov, 1974; Alifanov, 1994). In this case, the stopping criterion is given by

$$S[q(\xi,t)] < \varepsilon \tag{14}$$

where $S[q(\xi,t)]$ is computed with equation (3). The tolerance ε is chosen so that smooth solutions are obtained with measurements containing random errors. It is assumed that the solution is sufficiently accurate when

$$\left| T(\xi_s, \eta_s, t; q) - \mu_s(t) \right| \approx \sigma \tag{15}$$

where σ is the constant standard deviation of the measurement errors.

Thus, ε is obtained from equation (3) as

$$\varepsilon = S \sigma^2 t_f \tag{16}$$

For cases involving errorless measurements, ε can be specified *a priori* as a sufficiently small number, if the sensors are appropriately located.

9. RESULTS AND DISCUSSIONS

We consider here a test-case involving the laminar hydrodynamically developed flow of water with Re=100, in a channel of height 0.05 m. Direct, sensitivity and adjoint problems were solved with finite-volumes, by using a discretization with 100 and 200 volumes in the ξ and η directions, respectively. The final time was taken as 1000 s and 1000 time-steps were used for the finite-volume solution. For the inverse analysis, one measurement was assumed available per sensor at each time step.

In order to illustrate the accuracy of the present function estimation approach, we used simulated measurements containing random errors, normally distributed, with zero mean and constant standard-deviation (σ). Such simulated measurements were obtained by adding a random noise to the solution of the direct problem for *a priori* established functional forms for the boundary heat fluxes $q_1(\xi,t)$ and $q_2(\xi,t)$. For the results presented below, the boundary heat fluxes were taken as $q_j(\xi,t)=q_0 F_{\xi j}(\xi) F_{tj}(t)$, for j=1,2. The value of q_0 was taken as 100 W/m^2. Functional forms containing discontinuities and sharp-corners were examined for $F_{\xi j}(\xi)$ and $F_{tj}(t)$, because they represent the most difficult functions to be recovered by inverse analysis. Let us consider a test-case with $F_{\xi j}(\xi)$ and $F_{tj}(t)$ taken, respectively, in the form:

$$F_{\xi 1}(\xi)=\begin{cases}1 & \text{for} \quad 1<\xi<34 \quad \text{and} \quad 68<\xi\le 101\\ 2 & \text{for} \quad 34\le\xi\le 68\end{cases} \tag{17.a}$$

$$F_{\xi 2}(\xi)=\begin{cases}1 & \text{for} \quad 1\le\xi<34 \quad \text{and} \quad 68<\xi\le 101\\ \frac{1}{18}\xi-\frac{15}{18} & \text{for} \quad 34\le\xi\le 51\\ -\frac{1}{18}\xi+\frac{87}{18} & \text{for} \quad 51<\xi\le 69\end{cases} \tag{17.b}$$

$$F_{t1}(t)=\begin{cases}1 & \text{for} \quad 1\le t<334 \quad \text{and} \quad 666<t\le 1000\\ 2 & \text{for} \quad 334\le t\le 666\end{cases} \tag{18.a}$$

$$F_{t2}(t)=\begin{cases}1 & \text{for} \quad 1\le t<334 \quad \text{and} \quad 666<t\le 1000\\ \frac{6}{999}t-1 & \text{for} \quad 334\le t\le 500\\ -\frac{6}{999}t+5 & \text{for} \quad 500<t\le 666\end{cases} \tag{18.b}$$

Figures 1.a-c present the results for the spatial variations of $q_1(\xi,t)$ and $q_2(\xi,t)$ at selected times, by using simulated measurements with standard deviation σ=0.05. Such results were obtained by using the measurements of sensors located uniformly along the channel at 2.625 mm far from the walls at $\eta=0$ and $\eta=N$, respectively. Thirty-four sensors were located near each of the boundaries. Initial guesses of $q_1^0(\xi,t)=q_2^0(\xi,t)=0$ were used for the iterative procedure of the conjugate gradient method. Figures 1.a-c show that very accurate estimates can be obtained for the spatial and time variations of $q_1(\xi,t)$ and $q_2(\xi,t)$, even for such initial guesses far from the exact functions and by considering measurements containing random errors.

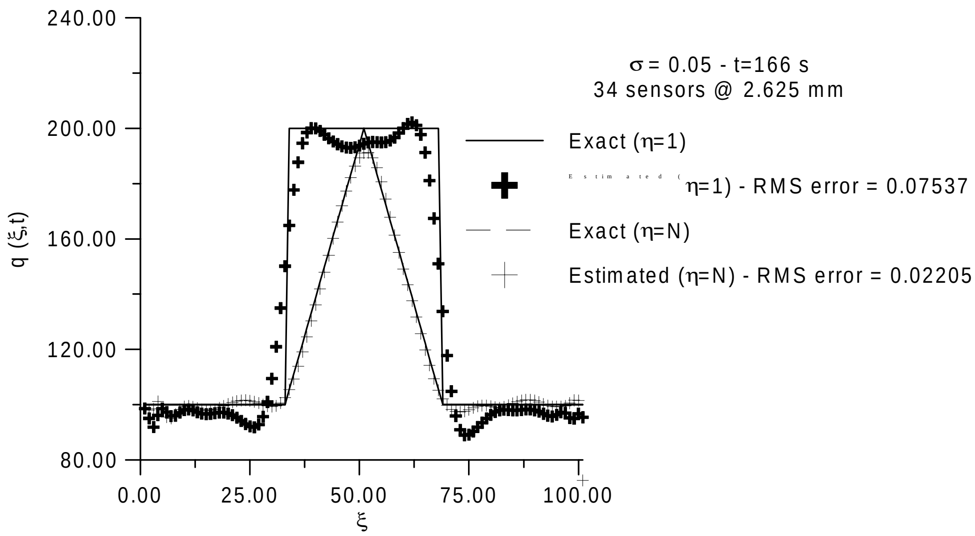

Figure 1.a. Estimation of $q_1(\xi,t)$ and $q_2(\xi,t)$ along the channel for t=116 s.

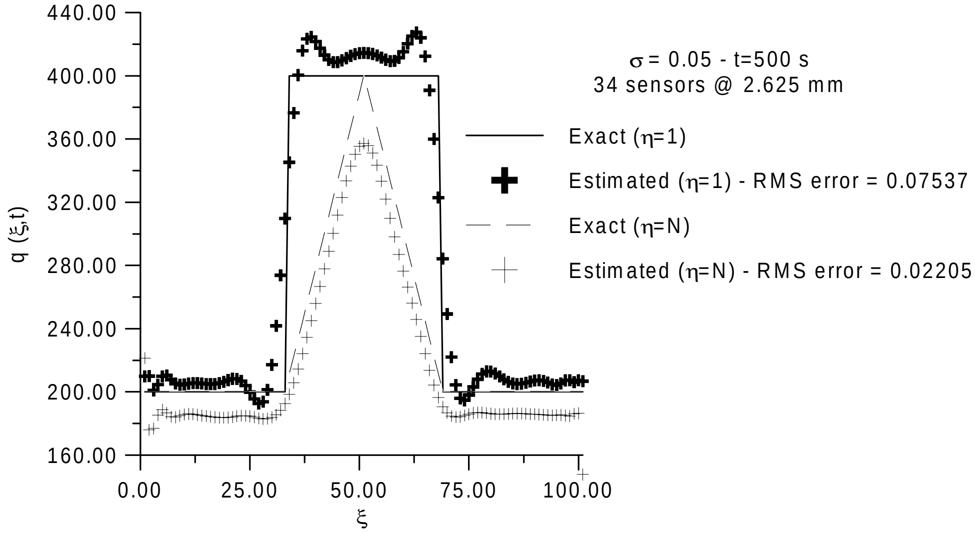

Figure 1.b. Estimation of $q_1(\xi,t)$ and $q_2(\xi,t)$ along the channel for t=500 s.

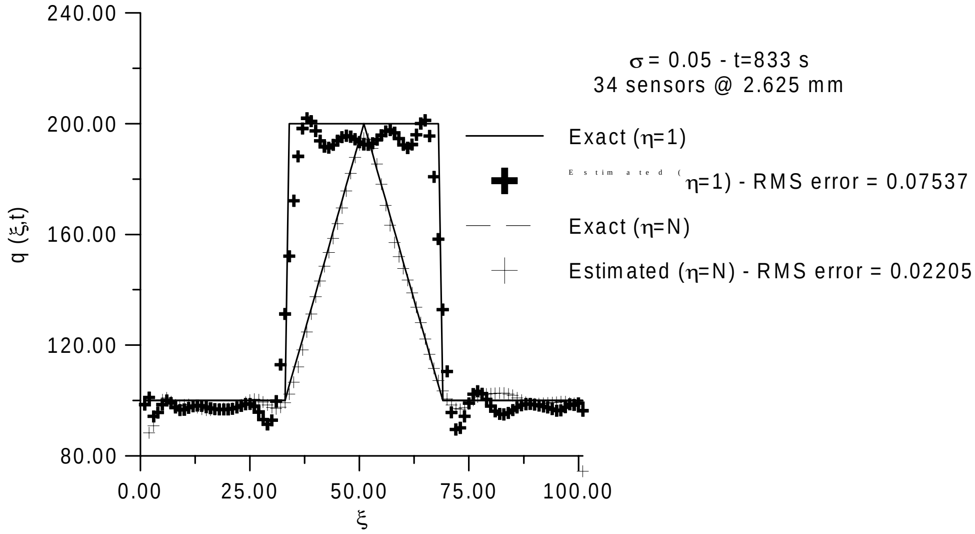

Figure 1.c. Estimation of $q_1(\xi,t)$ and $q_2(\xi,t)$ along the channel for t=833 s.

10. CONCLUSIONS

In this paper we applied the conjugate gradient method for the simultaneous identification of two boundary heat fluxes in a parallel plate channel. A function estimation approach was utilized, where no information was assumed available regarding the functional form of the unknown. The more difficult case of boundary heat fluxes varying in time and along the channel was examined in the paper. Results obtained with simulated temperature measurements reveal that quite accurate estimates can be obtained for the time and spatial variations of the unknown function.

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