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# Nonlinear Dynamics and Chaos in a Shape Memory Alloy Oscillator

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*Abstract: The remarkable properties of Shape Memory Alloys (SMAs) are attracting much technological interest in several science and engineering fields, varying from medical to aerospace applications. Hysteretic response of these systems is one of their essential characteristics being related to the martensitic phase transformation. The dynamical response of systems with SMA actuators presents a rich behavior due to their intrinsic nonlinear characteristic. Since experimental results show that SMAs present an asymmetric behavior when subjected to tensile or compressive loads, it is important to evaluate the influence of this kind of behavior in the nonlinear dynamics of mechanical systems with SMA devices. This article discusses the nonlinear dynamics of shape memory systems, considering the influence of tensile-compressive asymmetry in the thermomechanical behavior of SMAs. An iterative numerical procedure based on the operator split technique, the orthogonal projection algorithm and the fourth order Runge-Kutta method is developed to deal with nonlinearities in the formulation. A numerical investigation is carried out showing some qualitative results such as chaotic-like response and multi-stability behavior for a single degree of freedom SMA oscillator.*

***Keywords: Nonlinear dynamics, shape memory alloys, hysteretic behavior.***

## INTRODUCTION

The remarkable properties of Shape Memory Alloys (SMAs) are attracting much technological interest in several science and engineering fields, varying from medical to aerospace applications. Machado and Savi (2002, 2003) make a review on the most relevant SMA applications within orthodontics and biomedical areas. Engineering applications are also extensive. They are ideally suited to be used as self-actuating fasteners, thermally actuator switches, seals, connectors and clamps (van Humbeeck, 1999). Moreover, aerospace technology is also exploiting SMA properties in order to build self-erectable structures, stabilizing mechanisms, solar batteries, non-explosive release devices and other possibilities (Denoyer *et al.*, 2000; Pacheco and Savi, 1997). Micromanipulators and robotics actuators have been conceived employing SMAs properties to mimic the smooth motions of human muscles (Garner *et al.*, 2001; Webb *et al.*, 2000; Rogers, 1995; Kibirkstis *et al.*, 1997). Furthermore, SMAs are being used as actuators for vibration and buckling control of flexible structures (Birman, 1997; Rogers, 1995).

Hysteretic response of shape memory alloys (SMAs) is one of their essential characteristics being related to martensitic phase transformation. Basically, hysteresis loop may be observed either in stress-strain or in strain-temperature curves. The major (or external) hysteresis loop can be defined as the envelope of all minor (or internal) hysteresis loops, usually denoted as subloops. Macroscopic description of the SMA hysteresis loops, together with their subloops due to incomplete phase transformations, is an important aspect in the phenomenological description of the thermomechanical behavior of SMAs, being of great interest in technological applications (Savi and Paiva, 2005).

The dynamical response of systems with SMA actuators presents a rich behavior due to their intrinsic nonlinear characteristic, being previously addressed in different references (Seelecke, 2002; Ghandi and Chapuis, 2002; Collet *et al.*, 2001; Salichs *et al.*, 2001; Saadat *et al.*, 2001, 2002; Schmidt and Lammering, 2004; Williams *et al.*, 2002; Feng and Li, 1996; Mosley and Mavroidis, 2001; Lagoudas *et al.*, 2004; Han *et al.*, 2005; Savi *et al.*, 2002a). Various applications are exploiting SMAs' dynamical response. SMAs' nonlinear response is associated with both adaptive dissipation related to their hysteretic behavior and huge changes in their properties caused by phase transformations. Concerning the dissipation effect, SMAs' high damping capacity may be exploited in adaptive passive control employed in bridges and civil structures subjected to earthquakes, for example (Han *et al.*, 2005; Williams *et al.*, 2002; Salichs *et al.*, 2001; Saadat *et al.*, 2002, Oberaigner *et al.*, 2002). SMAs' property changes due to phase transformations, on the other hand, may exploit either forces or displacements generated by this phenomenon as well as natural frequencies and stiffness variations (Williams *et al.*, 2002; Pietrzakowski, 2000). Chaotic behavior is also a possibility of SMA dynamical response discussed in different references (Savi and Braga, 1993a,b; Machado *et al.*, 2004; Savi and Pacheco, 2002; Machado *et al.*, 2003; Lacarbonara and Vestroni, 2003; Lacarbonara *et al.*, 2004; Bernardini and Rega, 2005). Recently, some experimental analyses confirm the presence of chaos in shape memory systems (Mosley and Mavroidis, 2001).

Regarding the dynamical behavior of SMA oscillators, Savi and Braga (1993a) discuss the chaotic behavior of shape memory helical springs. Machado *et al.* (2004) discuss bifurcation and crises in a shape memory oscillator. Savi and Pacheco (2002) study some characteristics of shape memory oscillators with one and two-degree of freedom, showing the existence of chaos and hyperchaos in these systems. Machado *et al.* (2003) revisited the analysis of coupled shape memory oscillators, considering two-degree of freedom oscillators. All these articles employ a polynomial constitutive model to describe the thermomechanical behavior of SMAs. Savi and Braga (1993b) also study shape memory oscillators employing another constitutive model to describe the restitution force provided by a shape memory helical spring.

This article deals with the nonlinear dynamics of shape memory systems where the restitution force is described by a constitutive model with internal constraints (Paiva *et al.*, 2005). This constitutive model presents close agreement with experimental data and therefore, can represent more accurately the qualitative behavior previously analyzed in the cited references, which use a simpler constitutive model. The accurate representation of the SMA hysteresis is critical to the nonlinear dynamics analysis and allows more realistic description of important characteristics as the adaptive dissipation influence in the system dynamics (Bernardini & Rega, 2005). Since experimental results show that SMAs present an asymmetric behavior when subjected to tensile or compressive loads, it is important to evaluate the influence of this kind of behavior in the nonlinear dynamics of mechanical systems with SMA devices. The constitutive model employed in this article allows one to capture this important issue. An iterative numerical procedure based on the operator split technique (Ortiz *et al.*, 1983), the orthogonal projection algorithm (Savi *et al.*, 2002b) and the fourth order Runge-Kutta method is developed to deal with nonlinearities in the formulation. Numerical investigation is carried out showing some characteristics of SMA dynamical response.

## CONSTITUTIVE MODEL

There are different ways to describe the thermomechanical behavior of SMAs. Here, a constitutive model that is built upon the Fremond's model and previously presented in different references (Savi *et al.*, 2002b, Baêta-Neves *et al.*, 2004, Paiva *et al.*, 2005) is employed. This model considers different material properties and four macroscopic phases for the description of the SMA behavior. The model also considers plastic strain and plastic-phase transformation coupling, which allows the two-way shape memory effect description. Moreover, tension-compression asymmetry is taken into account.

Besides strain ($\varepsilon$) and temperature ($T$), the model considers four more state variables associated with the volumetric fraction of each phase: $\beta_1$ is associated with tensile detwinned martensite, $\beta_2$ is related to compressive detwinned martensite, $\beta_3$ represents austenite and $\beta_4$ corresponds to twinned martensite. A free energy potential is proposed concerning each isolated phase. After this definition, a free energy of the mixture can be written weighting each energy function with its volumetric fraction. With this assumption, it is possible to obtain a complete set of constitutive equations that describes the thermomechanical behavior of SMAs as presented bellow:

$$\sigma = E\varepsilon + \left(\alpha^C + E\alpha_h^{\ C}\right)\beta_2 - \left(\alpha^T + E\alpha_{h\ 1}^{\ T}\right)\beta_1 - \Omega\left(T - T_0\right) \tag{1}$$

$$\begin{aligned}\dot{\beta}_1 = \frac{1}{\eta_1}\Big\{ & \alpha^T \varepsilon + \Lambda_1 + \beta_2\left(\alpha_h^{\ C}\alpha^T + \alpha_h^{\ T}\alpha^C + E\alpha_h^{\ T}\alpha_h^{\ C}\right) - \beta_1\left(2\alpha_h^{\ T}\alpha^T + E\alpha_h^{\ T^2}\right) + \\ & + \alpha_h^{\ T}\left[E\,\varepsilon - \Omega\left(T - T_0\right)\right] - \partial_1 J_\pi \Big\} + \partial_1 J_\chi\end{aligned} \tag{2}$$

$$\begin{aligned}\dot{\beta}_2 = \frac{1}{\eta_2}\Big\{ & -\alpha^C \varepsilon + \Lambda_2 + \beta_1\left(\alpha_h^{\ T}\alpha^C + \alpha_h^{\ C}\alpha^T + E\alpha_h^{\ C}\alpha_h^{\ T}\right) - \\ & - \beta_2\left(2\alpha_h^{\ C}\alpha^C + E\alpha_h^{\ C^2}\right) - \alpha_h^{\ C}\left[E\,\varepsilon - \Omega\left(T - T_0\right)\right] - \partial_2 J_\pi \Big\} + \partial_2 J_\chi\end{aligned} \tag{3}$$

$$\dot{\beta}_3 = \frac{1}{\eta_3}\left\{ -\frac{1}{2}\left(E_A - E_M\right)\left(\varepsilon + \alpha_h^{\ C}\beta_2 - \alpha_h^{\ T}\beta_1\right)^2 + \Lambda_3 + \right.$$
$$\left. + \left(\Omega_A - \Omega_M\right)\left(T - T_0\right)\left(\varepsilon + \alpha_h^{\ C}\beta_2 - \alpha_h^{\ T}\beta_1\right) - \partial_3 J_\pi \right\} + \partial_3 J_\chi \tag{4}$$

where $E = E_M + \beta_3\left(E_A - E_M\right)$ is the elastic modulus while $\Omega = \Omega_M + \beta_3\left(\Omega_A - \Omega_M\right)$ is related to the thermal expansion coefficient. Notice that subscript "*A*" refers to austenitic phase, while "*M*" refers to martensite. Besides, different properties are assumed to consider tension-compression asymmetry, where the superscript "*T*" refers to tensile while "*C*" is related to compressive properties. Moreover, parameters $\Lambda_1 = \Lambda_1(T)$, $\Lambda_2 = \Lambda_2(T)$ and $\Lambda_3 = \Lambda_3(T)$ are associated with phase transformations stress levels. Parameter $\alpha_h$ is introduced in order to define the horizontal width of the stress-strain hysteresis loop, while $\alpha$ helps vertical hysteresis loop control on stress-strain diagrams.

The terms $\partial_n J_\pi$ ($n$ = 1,2,3) are sub-differentials of the indicator function $J_\pi$ with respect to $\beta_n$ (Rockafellar, 1970). The indicator function $J_\pi\left(\beta_1, \beta_2, \beta_3\right)$ is related to a convex set $\pi$, which provides the internal constraints related to the phases' coexistence. With respect to evolution equations of volumetric fractions, $\eta_1$, $\eta_2$ and $\eta_3$ represent the internal dissipation related to phase transformations. Moreover $\partial_n J_\chi$ ($n$ = 1,2,3) are sub-differentials of the indicator function $J_\chi$ with respect to $\dot{\beta}_n$ (Rockafellar, 1970). This indicator function is associated with the convex set $\chi$, which establishes conditions for the correct description of internal subloops due to incomplete phase transformations and also avoids phase transformations $M+ \Rightarrow M$ or $M- \Rightarrow M$.

Concerning the parameters definition, linear temperature dependent relations are adopted for $\Lambda_1$, $\Lambda_2$ and $\Lambda_3$ as follows:

$$\Lambda_1 = -L_0^T + \frac{L^T}{T_M}(T - T_M) \qquad \Lambda_2 = -L_0^C + \frac{L^C}{T_M}(T - T_M) \qquad \Lambda_3 = -L_0^A + \frac{L^A}{T_M}(T - T_M) \tag{5}$$

Here, $T_M$ is the temperature below which the martensitic phase becomes stable. Besides, $L_0^T$, $L^T$, $L_0^C$, $L^C$, $L_0^A$ and $L^A$ are parameters related to critical stress for phase transformation, remembering that the indexes "*T*" refers to tensile, "*C*" to compression and "*A*" to austenite.

In order to contemplate different characteristics of the kinetics of phase transformation for loading and unloading processes, it is possible to consider different values to the parameter $\eta_n$ ($n$ = 1,2,3), which is related to internal dissipation: $\eta_n^L$ and $\eta_n^U$ during loading and unloading process, respectively. For more details about the constitutive model, see Paiva *et al.* (2005).

## SHAPE MEMORY OSCILLATOR

Consider a single-degree of freedom oscillator, which consists of a mass $m$ attached to a shape memory element of length $L$ and cross-section area $A$. A linear viscous damper, associated with a parameter $c$, is also considered (Figure 1). The system is harmonically excited by a force $F=F_0 \sin(\omega t)$.

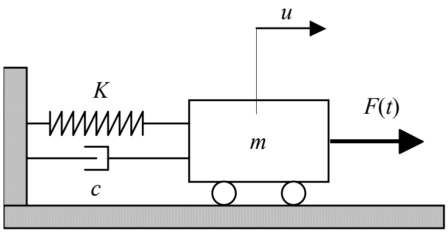


**Figure 1. Shape Memory Oscillator.**

With these assumptions, equation of motion may be formulated by considering the balance of linear momentum, assuming that the restitution force is provided by a SMA element described by the constitutive equation presented in the previous section. Therefore, the following equation of motion is obtained,

$$m\ddot{u} + c\dot{u} + K = F_0 \sin(\omega t) \tag{6}$$

Notice that the restitution force may be expressed as $K = \sigma A$. Using the constitutive equation for SMAs, one writes,

$$m\ddot{u} + c\dot{u} + EA\varepsilon + (A\alpha^C + EA\alpha_h^C)\beta_2 - (A\alpha^T + EA\alpha_h^T)\beta_1 - \Omega A(T - T_0) = F_0 \sin(\omega t) \tag{7}$$

In order to obtain a dimensionless equation of motion, system's parameters are defined as follows,

$$\omega_0^2 = \frac{E_R A}{mL}; \quad \xi = \frac{c}{m\omega_0} \quad ; \quad \overline{\alpha}^{C,T} = \frac{\alpha^{C,T} A}{mL\omega_0^2} = \frac{\alpha^{C,T}}{E_R}; \quad \overline{\alpha}_h^{C,T} = \frac{\alpha_h^{C,T} E_R A}{mL\omega_0^2} = \alpha_h^{C,T} \quad ; \tag{8}$$

$$\delta = \frac{F_0}{mL\omega_0^2} = \frac{F_0}{E_R A}; \quad \overline{\Omega} = \frac{\Omega_R A T_R}{mL\omega_0^2} = \frac{\Omega_R T_R}{E_R} \quad ; \quad \mu_E = \frac{E}{E_R}; \quad \mu_\Omega = \frac{\Omega}{\Omega_R}; \quad \varpi = \frac{\omega}{\omega_0}$$

These definitions allow one to define the following dimensionless variables, respectively related to mass displacement ($U$) and time ($\tau$).

$$U = \frac{u}{L}; \quad \theta = \frac{T}{T_R}; \quad \tau = \omega_0 t \tag{9}$$

Notice that dimensionless variables are defined considering some reference values for temperature dependent parameters. This is done assuming a reference temperature, $T_R$, where these parameters are evaluated. Therefore, parameters with subscript $R$ (specifically, $E_R$ and $\Omega_R$) are evaluated in this reference temperature. Moreover, it is assumed that strain $\varepsilon$ is represented by the dimensionless displacement $U$. The dimensionless equation of motion has the form:

$$U'' + \xi U' + \mu_E U + (\overline{\alpha}^C + \mu_E \overline{\alpha}_h^C)\beta_2 - (\overline{\alpha}^T + \mu_E \overline{\alpha}_h^T)\beta_1 - \mu_\Omega \overline{\Omega}(\theta - \theta_0) = \delta \sin(\varpi\tau) \tag{10}$$

where derivatives with respect to dimensionless time are represented by $(\ )' = d(\ )/d\tau$. This equation of motion can be written in terms of a system of first order differential equations as follows,

$$\begin{aligned} x' &= y \\ y' &= \delta \sin(\varpi\tau) - \xi y - \mu_E x - (\overline{\alpha}^C + \mu_E \overline{\alpha}_h^C)\beta_2 + (\overline{\alpha}^T + \mu_E \overline{\alpha}_h^T)\beta_1 + \mu_\Omega \overline{\Omega}(\theta - \theta_0) \end{aligned} \tag{11}$$

In order to deal with non-linearities of these equations of motion, an iterative procedure based on the operator split technique (Ortiz *et al.*, 1983) is employed. With this assumption, the fourth order Runge-Kutta method is used together with the projection algorithm proposed in Savi *et al.* (2002b) to solve the constitutive equations. The solution of the constitutive equations also employs the operator split technique together an *implicit Euler* method. For $\beta_n$ ($n$ = 1,2,3) calculation, the evolution equations are solved in a decoupled way. At first, the equations (except for the sub-differentials) are solved using an iterative *implicit Euler* method. If the estimated results obtained for $\beta_n$ does not fit the imposed constraints, an *orthogonal projection algorithm* pulls their value to the nearest point on the domain's surface (Paiva *et al.*, 2005).

## NUMERICAL SIMULATIONS

This section presents some numerical simulations developed in order to show the qualitative behavior of SMA dynamical responses. In all simulations, it is considered parameters presented in Table 1 that is related to typical NiTi alloys (Paiva *et al.*, 2005). It is also assumed a SMA element with $A = 1.96\times10^{-5}$ m and $L = 50\times10^{-3}$ m, and also a unitary mass. Figure 2 presents a quasi-static stress-strain curve obtained with the adjusted parameters for a high temperature ($T$ = 373K, where austenite is stable for a stress-free state). It is noticeable the tensile-compressive asymmetry, which represents a characteristic of SMA thermomechanical behavior. In order to analyze the effect of this characteristic, it is also considered a situation with tensile-compressive symmetry, assuming that tensile properties are applied to compressive behavior.

**Table 1. SMA parameters.**

| | | | | | |
|---|---|---|---|---|---|
| $E_A$ (GPa) | $E_M$ (GPa) | $\alpha^T$ (MPa) | $\alpha^C$ (MPa) | $\varepsilon_R^T$ | $\varepsilon_R^C$ |
| 54 | 42 | 150 | 165 | 0.0555 | −0.035 |
| $L_0^T$ (MPa) | $L^T$ (MPa) | $L_0^C$ (MPa) | $L^C$ (MPa) | $L_0^A$ (MPa) | $L^A$ (MPa) |
| 0.15 | 41.5 | 0.17 | 96.2 | 0.63 | 185 |
| | $\Omega_A$ (MPa/K) | $\Omega_M$ (MPa/K) | $T_M$ (K) | $T_A$ (K) | |
| | 0.74 | 0.17 | 291.4 | 307.5 | |
| $\eta_1^L$ (MPa.s) | $\eta_1^U$ (MPa.s) | $\eta_2^L$ (MPa.s) | $\eta_2^U$ (MPa.s) | $\eta_3^L$ (MPa.s) | $\eta_3^U$ (MPa.s) |
| 10 | 27 | 10 | 27 | 10 | 27 |

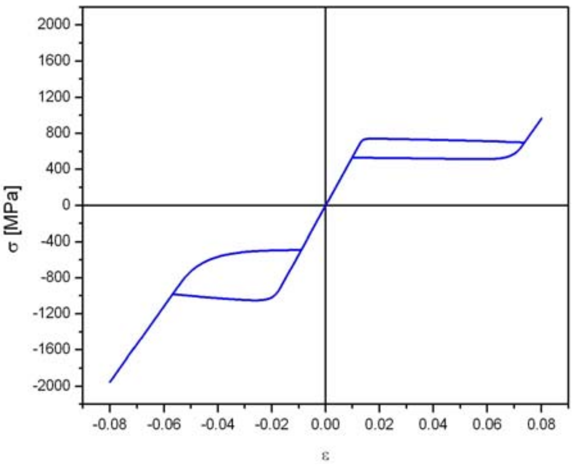


**Figure 2. Stress-strain curve for a high temperature (T = 373K).**

## Free Vibration

At first, free vibration is focused on, by letting $\delta$ vanish in the dimensionless equations of motion. It is assumed that reference parameters ($E_R$ , $\Omega_R$) are evaluated in the reference temperature $T_R = T_M$, that is, $E_R = E_M$ , $\Omega_R = \Omega_M$. The system has different equilibrium points depending on temperature. The oscillator free response is illustrated analyzing a system without viscous damping ($\xi = 0$). Results from simulations are presented in the form of phase portraits. In order to establish a comparison between the dynamical response of symmetric and asymmetric systems, tensile-compressive symmetry are considered assuming tensile parameters listed in Table 1 to both tensile and compression behaviors. Figure 3 presents the free response of a system with tensile-compressive symmetry, at different temperatures: $\theta = 1.28$, representing a high temperature where austenite is stable for a stress-free state; and $\theta = 0.99$, a low temperature where martensite is stable for a stress-free state. Between these two temperatures, martensite and austenite may coexist and it represents a transition region between the two cited situations (Savi & Pacheco, 2002; Machado *et al.*, 2003, 2004). For high temperatures, there is only a single equilibrium point. The system response presents dissipation for high amplitudes, converging to an elastic orbit near the equilibrium point, where phase transformations do not take place anymore. This behavior is due to hysteresis loop and initial conditions in the linear-elastic region do not present energy dissipation. For low temperatures, the dissipation characteristics are similar to the high temperature behavior but there is an increase in the number of equilibrium points. By observing the phase portrait, it is noticeable three stable equilibrium points (a stable point has a positive displacement, which is denoted as a positive equilibrium point, while a stable point that has a negative displacement is denoted as a negative equilibrium point), and it is possible to infer about the existence of unstable points among the stable ones.

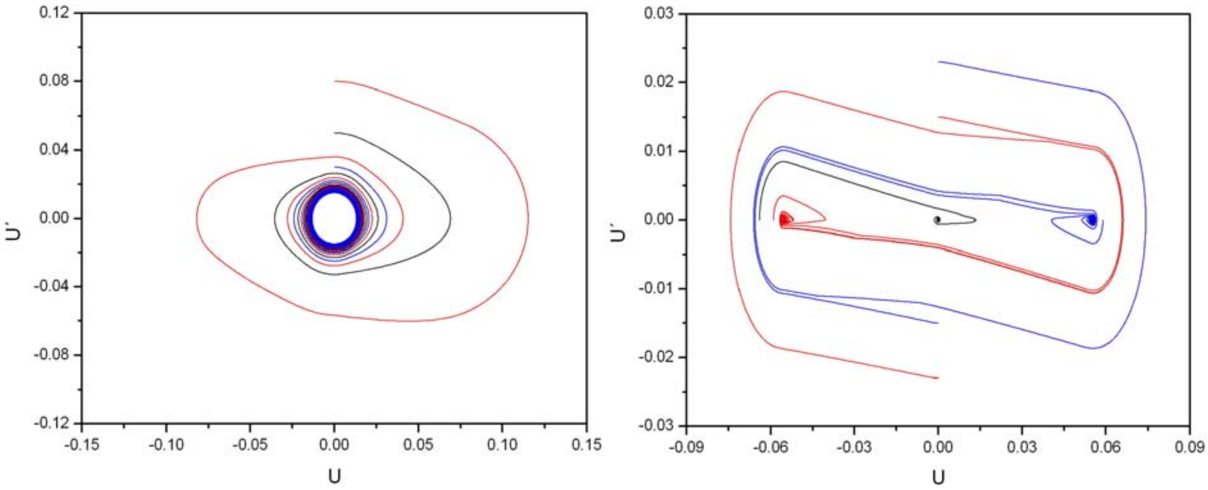


**Figure 3. Phase portrait for symmetric problem and different temperatures. (*a*) $\theta = 1.28$ and (*b*) $\theta = 0.99$.**

By considering the tensile-compressive asymmetry, phase portraits are deformed (Figure 4). For low temperature case, it is noticeable that the position of the negative equilibrium point is closer to the origin (when compared to the symmetric case), which causes differences in the dynamical response. Therefore, tensile-compressive asymmetry is an important characteristic to be verified in the design of SMA dynamical systems.

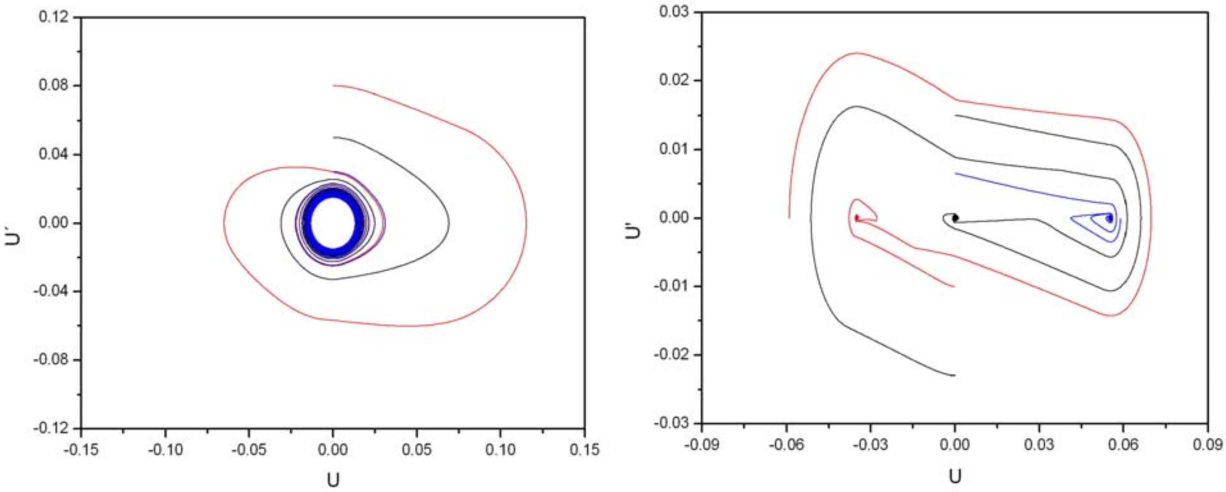


**Figure 4. Phase portrait for asymmetric problem and different temperatures. (*a*) $\theta = 1.28$ and (*b*) $\theta = 0.99$.**

## Forced Vibration

The behavior of the forced system is far more complex. At first, high temperature behavior is discussed exploiting the idea of the intelligent dissipation due to hysteresis loop. A paradigmatic way to visualize this kind of behavior is considering the system response under resonant conditions. Hence, an asymmetric condition is employed together with parameters $\xi = 0$, $\varpi = 1$, and $\theta = 1.28$. Moreover, austenitic properties are used as reference values ($T_R = T_A$, $E_R = E_A$ , $\Omega_R = \Omega_A$). As it is well-known, a non-dissipative linear system tends to increase the response amplitude indefinitely under this condition (Figure 5, left hand). Shape memory system, on the other hand, tends to dissipate higher energy levels as the response amplitude grows. This is due to phase transformation related to hysteresis loop and therefore, the amplitude tends to stabilize in lower values, as shown in Figure 5 (right hand side). This behavior is interesting to be exploited as a vibration passive control.

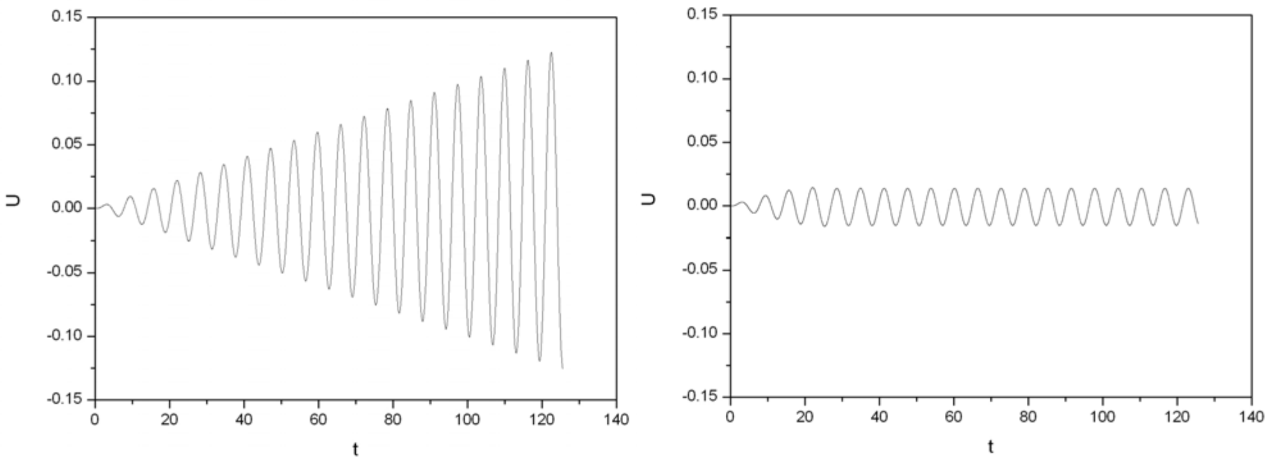


**Figure 5. Passive control exploiting hysteresis dissipation.**

At this point, low temperature behavior (where martensite is stable for a stress-free state) is focused on. Therefore, it is assumed that reference parameters ($E_R$ , $\Omega_R$) are evaluated in the reference temperature $T_R = T_M$, that is, $E_R = E_M$ , $\Omega_R = \Omega_M$. Moreover, it is assumed that $\xi = 5\times10^{-6}$, $\varpi = 1$, and $\theta = 0.99$. In order to perform a global analysis, bifurcation diagrams are constructed, sampling the position against the slow quasi-static variation of the forcing amplitude parameter. Figure 6 shows bifurcation diagrams obtained using two different initial conditions for each parameter value, showing the attractor coexistence. Another possibility to obtain other attractors related to this parameter range is to consider stabilized values of state variables as initial conditions for the next parameter value, which is not capable to capture the coexistence of different attractors. By considering tensile-compressive symmetry, the bifurcation diagrams tend to be more symmetric for the positive and negative initial conditions.

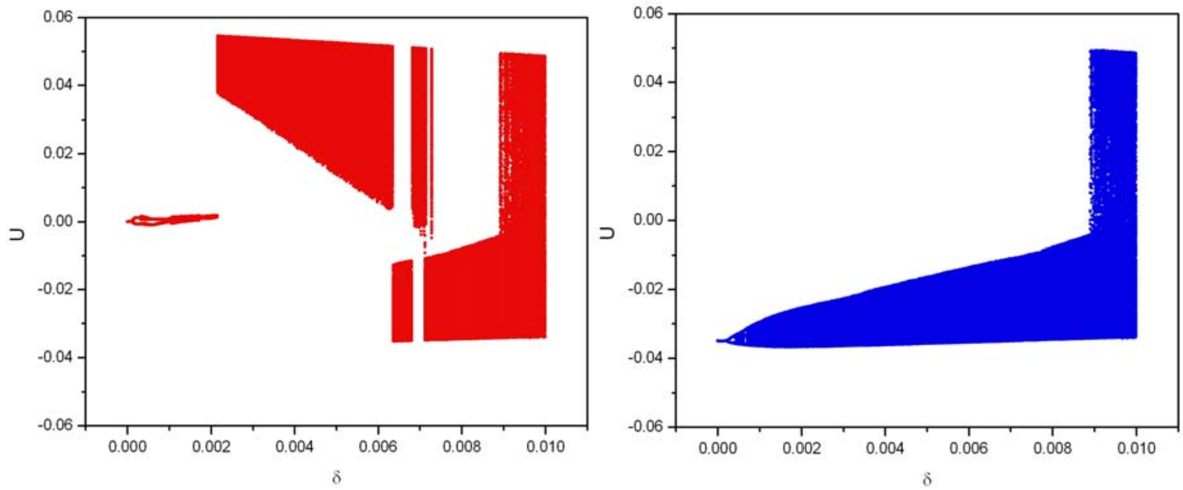


**Figure 6. Bifurcation diagrams for $\xi = 5\times10^{-3}$, $\varpi = 1$ and $\theta = 0.99$.**

Transient responses and multi-stability are interesting characteristics related to shape memory oscillators. Bifurcation diagrams presented in Figure 6 shows different cloud of points that appears depending on initial conditions. The forthcoming analysis exploits this coexisting attractors multi-stability by changing initial conditions and assuming $\delta = 4\times10^{-3}$, $\varpi = 1$ and $\xi = 6.4\times10^{-2}$. By assuming initial conditions within the cloud of points presented in Figure 6a, a chaotic-like response occurs, being related to oscillations in the positive part of phase space, and therefore, it is called positive chaotic-like response (Figure 7). On the other hand, by assuming initial conditions within the symmetric cloud (Figure 6b), the system presents a negative chaotic-like response that occurs in the negative part of phase space (Figure 8).

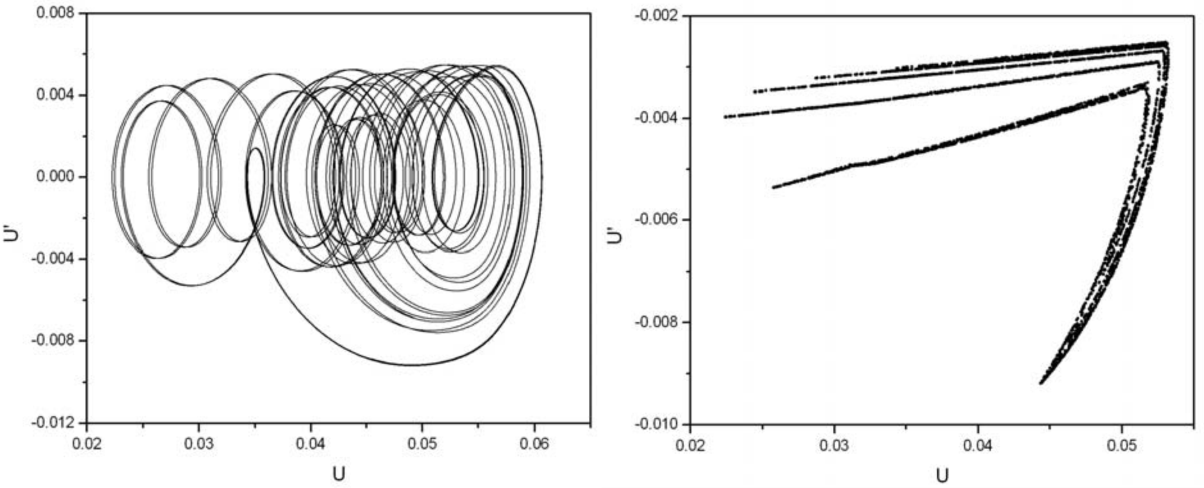


**Figure 7. Multi-stability: Positive chaotic-like response.**

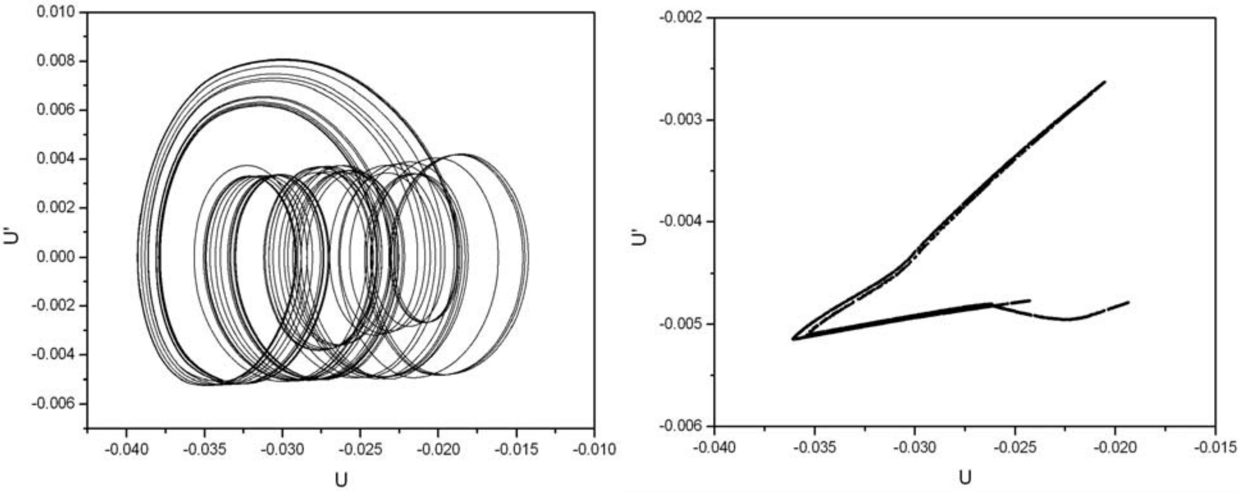


**Figure 8. Multi-stability: Negative chaotic-like response.**

At this point, a chaotic-like response related to the bifurcation diagram region with the same kind of response for both investigated situations is considered by changing the forcing characteristics. Figure 9 shows a chaotic-like response when $\delta = 9.5\times10^{-3}$, which is associated with all state space (including positive and negative parts).

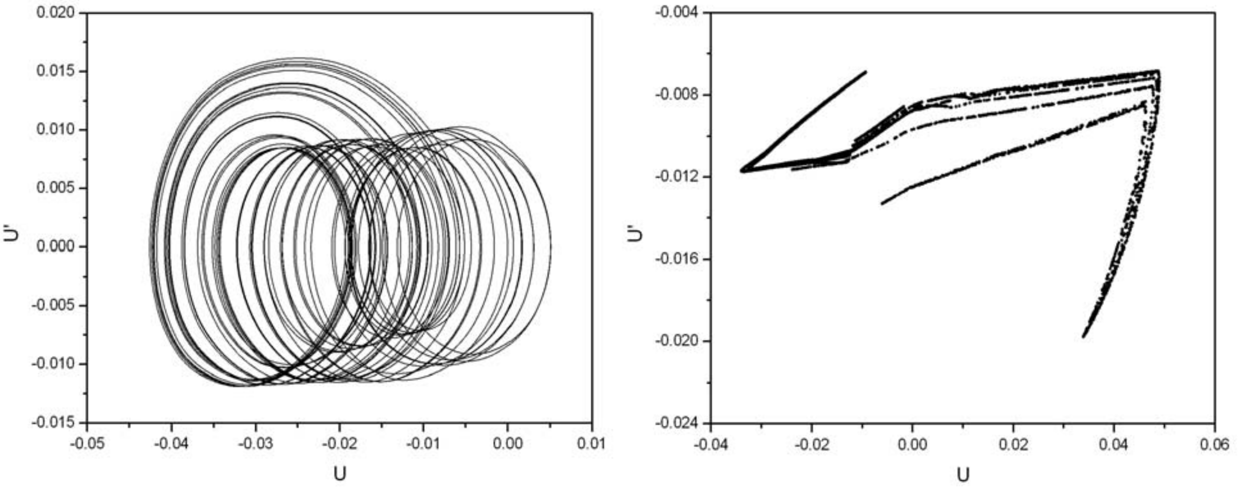


**Figure 9. Chaotic-like response.**

## CONCLUSIONS

This contribution analyzes the dynamical response of a single-degree of freedom shape memory alloy mechanical oscillator where the restitution force is described through a constitutive model with internal constraints. This model captures the general thermomechanical behavior of SMAs, allowing the description of various aspects of the dynamical system. An iterative numerical procedure is developed based on the operator split technique. Under this assumption, coupled governing equations are solved from uncoupled problems, where classical numerical methods can be employed. The fourth order Runge-Kutta method is employed together with the orthogonal projection algorithm, used to solve the constitutive equations. Results of numerical simulations indicate that this system has a rich behavior with different kinds of responses. An important characteristic of these systems is the equilibrium point temperature dependence, which means that the number and the characteristic of equilibrium points changes with the temperature. This behavior allows one to imagine changes of system position with temperature variation. Other interesting characteristic of SMA oscillator is the adaptive dissipation due to the hysteresis loop. Finally, it should be pointed out the possibility of SMA system to perform many types of behaviors, which can be exploited in the sense of giving flexibility to the system. Among various kinds of response, SMA oscillator may present chaotic-like response and also attractors multi-stability. Therefore, the response of SMA devices subjected to dynamic loadings can be very complex being of special interest to be accurately investigated.

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