

# A COMPARISON OF CONTROL STRATEGIES FOR MAGNETORHEOLOGICAL VEHICLE SUSPENSION SYSTEMS

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**Abstract.** *The behavior of magnetorheological suspension systems is highly nonlinear and one of the main challenges in the application of these devices is the development of appropriate control strategies. This paper presents a comparison of semi-active control strategies for magnetorheological vehicle suspension systems. Two semi-active control strategies based on the Clipped Control approach are proposed. A semi-active control synthesized in order to diminish the power transmitted to the vehicle body is also considered. In order to assess the performance of the proposed semi-active suspensions, numerical analyses considering a quarter car model were carried out. Disturbances induced by both road unevenness and a impact bump were considered and the aim of the proposed suspensions is to favour ride comfort on road holding. A mathematical model of the magnetorheological damper that accurately describes its inherent nonlinear dynamic behavior is considered in the analyses. The performances of the proposed suspensions relative to the ones of a standard passive suspension system are provided. A passive suspension obtained from a magnetorheological damper under a constant input voltage level is also considered for comparison purposes.*

**Keywords:** *Magnetorheological Suspension, Variable Structure Control, Quarter-Car Model.*

## 1. Introduction

Conventional vehicle suspension systems comprises passive elements such as springs and dampers. However, conflicting trade-offs between different performance measures, such as ride comfort and road holding are extremely difficult to achieve with passive suspensions (Sharp and Hassan, 1986). These conflicting trade-offs are specially difficult when one considers the very different dynamic modes that influence the ride comfort and road holding, namely, the bounce, pitch and roll modes. The use of active (Hrovat, 1997) and semi-active suspensions (Tseng and Hedrick, 1994) to overcome these conflicts between ride and handling is worthwhile.

Semi-active systems represent one of the most promising devices for practical applications in vibration isolation problems. This is due to its inherent stability and versatility, besides its relative simplicity and much lower power demand as compared with its active counterparts (Carlson and Spencer, 1996). In the magnetorheological (MR) dampers, for instance, the control force is passively generated, but its intensity can be actively controlled through an exposition of the damper MR fluid to a magnetic field. However, these systems present highly nonlinear dynamic behavior and one of the main challenges in the application of MR suspensions is the development of appropriate control algorithms. A comparison of control algorithms for seismic isolation using MR dampers can be found in (Jansen and Dyke, 1999).

Different control algorithms for semi-active vehicle suspension system have been proposed in the literature (Tseng and Hedrick, 1994) and (Guglielmino, 2001) to name but a few. Semi-active vehicle suspensions with MR dampers are found in (Yao et al., 2002) and (Yokoyama and Hedrick, 2001).

In the present work, a comparison of control strategies for MR vehicle suspension systems is presented. Two semi-active MR suspensions based on the Clipped Control approach (Dyke, 1996) are proposed. In this approach, the input voltage to the current driver is determined so that the MR damper force tracks a target force. Two control approaches are considered in the synthesis of the target control force: the Optimal Control and the Variable Structure Control (VSC). The Clipped Control approach does not require a model of the MR damper. In (Yokoyama and Hedrick, 2001), a variable structure control which uses an inverse model of the MR

damper is considered.

The VSC approach was chosen because of its robustness properties and for its simplicity in incorporating a reference model. The robustness of the variable structure control is required to deal with parameters uncertainties commonly present in the vehicle and a reference model is used to specify the desired performance of the suspension system. A MR suspension synthesized in order to diminish the total power transmitted to the vehicle body through the suspension system is also considered. This approach does not require the synthesis of a target control force and the input voltage is directly determined considering the passivity of the MR damper and the equation of the total power transmitted to the vehicle body.

In order to assess the performance of the proposed semi-active MR suspensions, numerical analyses considering a quarter car model were carried out. Disturbances induced by both road unevenness and an impact bump were considered and the aim of the proposed suspensions is to favour ride comfort on road holding. The performances of the proposed suspensions relative to the ones of a standard passive suspension system are provided.

## 2. Mathematical modelling

### 2.1. Quarter-car vehicle suspension

The present work considers the quarter-car model depicted in Fig.1 to assess the performance of semi-active control strategies for MR suspensions.

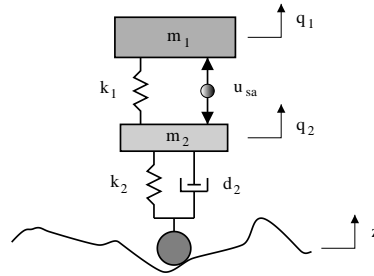


Figure 1: Quarter-car vehicle suspension model.

The dynamic behavior of the quarter car model is given by

$$\begin{cases} m_1 \ddot{q}_1 + k_1(q_1 - q_2) = u_{sa} \\ m_2 \ddot{q}_2 + d_2(\dot{q}_2 - \dot{z}) + k_2(q_2 - z) + k_1(q_2 - q_1) = -u_{sa} \end{cases} \quad (1)$$

where  $q_1$  is the absolute displacements of body mass  $m_1$ ,  $q_2$  is the absolute displacements of the effective mass  $m_2$  of the axle and wheel. The disturbance due to the movement of the vehicle over an unevenness road is given by  $z$ . The stiffness of the suspension and the tire are given by  $k_1$  and  $k_2$ , respectively,  $d_2$  is the damping coefficient of the tire and  $u_{sa}$  is the force applied by the MR damper. The equations in (1) may be written in matrix form as follows

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u_{sa} + \mathbf{B}_f f_z \quad (2)$$

where  $\mathbf{x}^T = [q_1 \quad q_2 \quad \dot{q}_1 \quad \dot{q}_2]$  is the state vector and  $f_z = d_2\dot{z} + k_2z$  is the disturbance vector. The state matrix  $\mathbf{A}$ , the control input matrix  $\mathbf{B}$  and the disturbance input matrix  $\mathbf{B}_f$  are given as follows

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -k_1/m_1 & k_1/m_1 & 0 & 0 \\ k_1/m_2 & -(k_1 + k_2)/m_2 & 0 & -d_2/m_2 \end{bmatrix}; \quad \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ 1/m_1 \\ -1/m_2 \end{bmatrix}; \quad \mathbf{B}_f = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1/m_2 \end{bmatrix} \quad (3)$$

### 2.2. Magnetorheological Damper

In order to model the nonlinear behavior of the MR damper, the modified Bouc-Wen model depicted in Fig.2 is adopted in this work. This model has been shown to accurately predict the nonlinear behavior of a prototype MR damper (Spencer et al., 1996) over a wide range of inputs in a set of experiments.

The MR damper model parameters were derived from the identified ones in (Lai and Liao, 2002). The idea is to scale the parameters so that the force is appropriate for the vehicle suspension problem, and the behavior in the force-displacement and force-velocity ranges is similar to that of the experimental device. The response

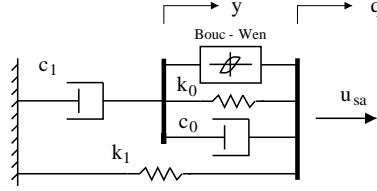


Figure 2: Modified Bouc-Wen model of the MR damper.

of the MR damper model due to a sinusoidal imposed displacement of frequency  $1Hz$  and amplitude of  $0.004m$  for different constant voltage levels is depicted in Fig.3.

In the modified Bouc-Wen model, the force generated by the MR damper is given by

$$u_{sa} = c_1 \dot{y} + k_1(q - q_0) \quad (4)$$

where the internal variable  $y$  and the hysteretic displacement  $h$  are given by the following equations

$$\dot{y} = \frac{1}{(c_0 + c_1)} [\alpha h + c_0 \dot{q} + k_0(q - y)] \quad (5)$$

$$\dot{h} = -\gamma |\dot{q} - \dot{y}| h |h|^{n-1} - \beta (\dot{q} - \dot{y}) |z|^n + \mathcal{A}(\dot{q} - \dot{y})$$

where the parameters  $\gamma$ ,  $\beta$ ,  $\mathcal{A}$  and  $n$  control the shape of the hysteresis loop. The model parameters are assumed to be dependent on the voltage  $V$  applied to the current driver as follows

$$\alpha(v) = \alpha_a + \alpha_b v, \quad c_1(v) = c_{1a} + c_{1b} v, \quad c_0(v) = c_{0a} + c_{0b} v \quad (6)$$

where  $v$  is the output of the first order filter used to model the dynamics involved in reaching the rheological equilibrium, viz.

$$\dot{v} = -\frac{1}{\tau}(v - V) \quad (7)$$

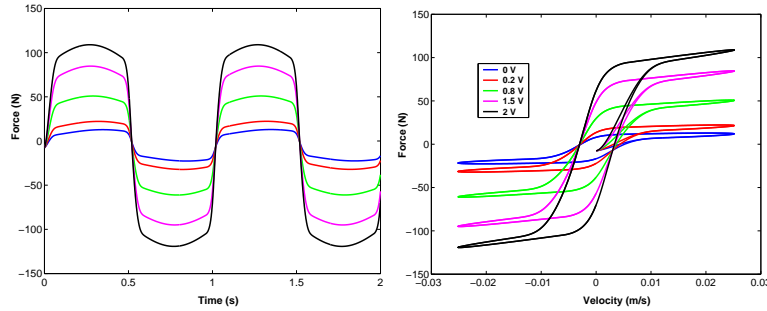


Figure 3: Response of the MR damper under different voltages to a sinusoidal excitation.

### 3. Clipped Control Approach

The Clipped Control approach has been shown to be effective in semi-active systems using MR dampers (Jansen and Dyke, 1999). This approach uses a force feedback loop to determine an input voltage to the current driver for the MR damper to approximately reproduce a target control force. In this work, the Optimal Control and the Variable Structure Control approaches will be considered in the synthesis of the target control force.

In the Clipped Control approach, the input voltage is selected as follows. When the MR damper is providing the target force  $u$ , the voltage  $V$  applied to the current driver should remain at the present level. If the magnitude of the force  $u_{sa}$  produced by the damper is smaller than the one of the target force and the two forces have the same sign, the voltage is increased to the maximum level, so as to increase the force produced by the damper to match the target force. Otherwise, the commanded voltage is set to zero. Mathematically, the input voltage is given as follows

$$V = \frac{V_{max}}{2} [1 + \text{sgn}((u - u_{sa})u)] \quad (8)$$

where  $V_{max}$  is the maximum voltage to the current driver and it is associated with the saturation of the magnetic field in the MR damper.

### 3.1. MR-Optimal vehicle suspension system

In the vehicle suspension named MR-Optimal suspension system, the target control force  $u$  is synthesized based on the Optimal Control approach so that it trades-off the ride comfort versus road holding, while maintaining constraints on the suspension rattle space and the control effort. Hence, the following performance index (Hać, 1985) is considered

$$J = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T [\rho_1(\dot{q}_1)^2 + \rho_2(q_1 - q_2)^2 + \rho_3(q_2 - z)^2 + \rho_4(u)^2] dt \quad (9)$$

The road disturbance  $z$  may be modeled as the output of a first order filter, viz.

$$\dot{z} + (a\mathcal{V})z = \xi \quad (10)$$

where  $\mathcal{V}$  is the vehicle velocity,  $a$  is a parameter depending on the type of the road surface,  $\xi$  is a white noise process with intensity  $W = 2\sigma^2 a\mathcal{V}$ , where  $\sigma^2$  is the variance of the road irregularities. The optimal control force is given by

$$u = -\mathbf{G}_a \mathbf{x}_a \quad (11)$$

where  $\mathbf{G}_a$  is the optimal control gain and  $\mathbf{x}_a^T = [\mathbf{x} \quad z]$  is the augmented state vector.

One should note, according to Eq.(11), that this control law requires the knowledge of the state vector  $\mathbf{x}$  describing the dynamics of the quarter car system as well as the road disturbance  $z$ . As the prime interest in this work is to assess the potentiality of MR suspension systems, the state vector  $\mathbf{x}$  is considered as known, i.e., the practical issues are not accounted for in the present moment. In the MR-Optimal suspension, the disturbance  $z$  is also considered as known.

### 3.2. MR-VSC vehicle suspension system

In the MR-VSC suspension system, the target control force  $u$ , which is required by the Clipped Control approach, is synthesized based on the VSC with sliding mode. A reference model is used to specify the desired dynamics of the vehicle body and a VSC force is synthesized in order to force the track error dynamic, between the displacement of the vehicle body and the one of the reference model, to attain the sliding mode (Yokoyama and Hedrick, 2001).

The dynamic equation of the vehicle body may be written as

$$m_1 \ddot{q}_1 + k_1 q_1 = u + k_1 q_2 \quad (12)$$

Defining the state vector

$$\mathbf{x}_1^T = [q_1 \quad \dot{q}_1] \quad (13)$$

the dynamic equation given in Eq.(12) may be written in the state-space form as

$$\dot{\mathbf{x}}_1 = \mathbf{A}_1 \mathbf{x}_1 + \mathbf{B}_1 [u + f_2 + f_p] \quad (14)$$

where  $f_2 = k_1 q_2$  is a known input signal because it is defined from the nominal parameter  $k_1$  and the unsprung displacement  $q_2$ , which is considered as known. The unknown disturbing signal  $f_p$  was added to account for parameter uncertainties and/or unmodeled dynamics. In the case of parameter uncertainties, the disturbance  $f_p$  is given by

$$f_p = -[\Delta m_1 \ddot{q}_1 + \Delta k_1 (q_1 - q_2)] \quad (15)$$

where  $\Delta m_1$  and  $\Delta k_1$  are the uncertainties of the vehicle body mass  $m_1$  and the suspension stiffness coefficient  $k_1$ , respectively.

#### 3.2.1. Synthesis of the Reference Model

In order to specify the desired dynamic of the vehicle body, the 1 *DOF* reference model depicted in Fig.4 is considered.

where  $m_m$  and  $k_m$  are the mass and stiffness coefficient of the reference model, respectively, and  $u_m$  is a control force that must be appropriately synthesized so that the reference model dynamic represents the desired one for the vehicle body. In (Yokoyama and Hedrick, 2001) an ideal semi-active skyhook control was considered in the synthesis of the reference model. One should note that the base dynamic of the 1 *DOF* reference model in

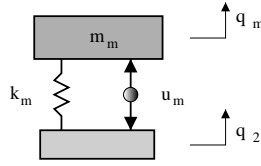


Figure 4: Single DOF reference model.

Fig.4 is governed by the actual dynamic of the unsprung mass  $m_2$  given by  $(q_2, \dot{q}_2)$  and that this model does not account directly for the road disturbance  $z$ .

The dynamic equation of the reference model may be written in matrix form as follows

$$\dot{\mathbf{x}}_m = \mathbf{A}_m \mathbf{x}_m + \mathbf{B}_m r \quad (16)$$

where  $\mathbf{x}_m^T = [q_m \quad \dot{q}_m]$  is the reference model state and  $r = k_m q_2 + u_m$  is the reference model excitation.

In this work, the control force  $u_m$  of the reference model was synthesized based on the Optimal Control approach and the following performance index was considered

$$J_m = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T [r_1 (\ddot{q}_m)^2 + r_2 (q_m - q_2)^2 + r_3 (u_m)^2] dt \quad (17)$$

It is worth noting that the performance index in Eq.(17) does not consider the tire deflection term  $(q_2 - z)$ , different from the performance index in Eq.(9). Therefore, the control force  $u_m$  and, consequently, the dynamic of the reference model, does not depend on the road disturbance  $z$ .

### 3.2.2. Synthesis of the target VSC force

Defining the tracking error vector  $\mathbf{e}$  between the response of the vehicle body  $\mathbf{x}_1$  and the response of the reference model  $\mathbf{x}_m$ , viz.

$$\mathbf{e} = \mathbf{x}_1 - \mathbf{x}_m \quad (18)$$

and considering Eqs.(14) and (16), the error dynamic is governed by the following equation

$$\dot{\mathbf{e}} = \mathbf{A}_m \mathbf{e} + \mathbf{B}_1 u + \mathbf{f}_k + \mathbf{f}_m \quad (19)$$

where the signals  $\mathbf{f}_k$  and  $\mathbf{f}_m$  are given by

$$\begin{aligned} \mathbf{f}_k &= (\mathbf{A}_1 - \mathbf{A}_m) \mathbf{x}_1 + \mathbf{B}_1 f_2 - \mathbf{B}_m r \\ \mathbf{f}_m &= \mathbf{B}_1 f_p \end{aligned} \quad (20)$$

The signal  $\mathbf{f}_k$  is a known one, since it is composed of nominal system parameters, reference model parameters and the known signals  $\mathbf{x}_1$ ,  $f_2$  and  $r$ . The signal  $\mathbf{f}_p$ , on the other hand, is associated with the parameters uncertainties and/or unmodeled dynamics and, therefore, it represents an unknown disturbance.

For the special case where the reference model parameters  $m_m$  and  $k_m$ , are, respectively, the nominal parameters  $m_1$  and  $k_1$  of the suspension model, the signal  $\mathbf{f}_k$  reduces to

$$\mathbf{f}_k = -\mathbf{B}_1 u_m \quad (21)$$

Consider the following sliding surface

$$\mathcal{S} = \{\mathbf{e} \in \mathbf{R}^n / s(\mathbf{e}) = 0\} \quad (22)$$

where  $s(\mathbf{e})$  is the switching function and it is defined as

$$s(\mathbf{e}) = \mathbf{S} \mathbf{e} \quad (23)$$

where  $\mathbf{S} = [S_1 \quad S_2]$ . Without loss of generality, from now on, one makes  $S_2 = 1$ .

If after a finite time  $t_s$ , a sliding mode takes place, one has

$$s = S_1 e_1 + e_2 = 0 ; \quad \forall t \geq t_s \quad (24)$$

where  $e_1 = (q_1 - q_m)$  and  $e_2 = \dot{e}_1$  are the components of the error vector  $\mathbf{e}$ . Therefore, the dynamic during sliding mode is governed by

$$e_1(t) = \exp(-S_1 t) e_1(t_s); \quad \forall t \geq t_s \quad (25)$$

According to Eq.(25), during the sliding mode, the vehicle body dynamic asymptotically tracks the dynamic of the reference model, despite the presence of the disturbance  $\mathbf{f}_m$ , i.e., despite the presence of parameter uncertainties and/or unmodeled dynamics.

The following proposed control law, which is comprised of two components, a linear  $u_l$  and a nonlinear  $u_n$  component, gives sufficient conditions to induce and maintain the sliding mode (Stutz and Rochinha, 2003) given by Eq.(25), viz.

$$u(t) = u_l(t) + u_n(t) \quad (26)$$

The linear component is given by

$$u_l(t) = -(\mathbf{SB}_1)^{-1} \mathbf{S} [\mathbf{A}_m \mathbf{e} + \mathbf{f}_k] + (\mathbf{SB}_1)^{-1} \phi s \quad (27)$$

where  $\phi$  is any negative constant and it is related with the transient of the dynamic before the trajectory reach the sliding surface.

The nonlinear component  $u_n$  is defined as

$$u_n(t) = -\rho(t, \mathbf{e}) (\mathbf{SB}_1)^{-1} \text{sgn}(s); \quad \text{para } s(t) \neq 0 \quad (28)$$

where  $\rho(t, \mathbf{e})$  is a modulation function, which is chosen to satisfy the following inequality

$$\rho(t, \mathbf{e}) \geq \frac{|f_p|}{m_1} + \eta \quad (29)$$

where  $\eta$  is any positive constant. The function  $\text{sgn}$  in Eq.(28) represents the signum function defined as

$$\text{sgn}(s) = \begin{cases} 1 & , \quad s > 0 \\ -1 & , \quad s < 0 \end{cases} \quad (30)$$

Therefore, if the modulation function is properly chosen according to Eq.(29), a sliding mode takes place in finite time and the tracking error dynamic is asymptotically stable with respect to the origin. The modulation function  $\rho$  may also be chosen as a constant value, however, the stability of the error dynamic can be only locally proven.

#### 4. MR suspension based on the minimization of the power transmitted to the vehicle body

This section presents a MR vehicle suspension system which is not based on the Clipped Control algorithm. The proposed suspension, which will be named MR-Power, is synthesized in order to diminish the total power transmitted to the vehicle body through the suspension system. In this approach, the input voltage to current driver is directly determined considering the passivity of the MR damper and the equation of the total power transmitted to the vehicle body.

Considering the quarter car model depicted in Fig.1, the total power transmitted through the suspension system to the vehicle body is given by

$$P_1 = [u_{sa} - k_1(q_1 - q_2)] \dot{q}_1 \quad (31)$$

As one of the aims of the suspension system is to isolate the vehicle body, the input voltage to the current driver is determined as follows. Whenever the MR damper force  $u_{sa}$  is such that power is dissipated from the vehicle body, the voltage to the damper is set to its maximum value, i.e.,  $V = V_{max}$ . Otherwise, the voltage is set to zero, i.e.,  $V = 0V$ . Mathematically one has

$$V = \begin{cases} V_{max} & , \quad u_{sa} \dot{q}_1 < 0 \\ 0 & , \quad u_{sa} \dot{q}_1 > 0 \end{cases} \quad (32)$$

It is worth noting that the MR damper, as a semi-active device, can only dissipate energy from the entire system it is attached, which means that

$$u_{sa}(\dot{q}_1 - \dot{q}_2) < 0 \quad (33)$$

However, when one considers only the vehicle body, the damper may inject into or dissipate energy from it.

According to Eq.(32), only the signs of the vehicle body velocity  $\dot{q}_1$  and the MR damper force  $u_{sa}$  are required for the application of this control system.

## 5. Numerical analyses

In order to assess the performance of the proposed semi-active MR suspension systems, numerical analyses considering the quarter car model were carried out. Disturbances induced by both road unevenness and an impact bump were considered and the aim of the proposed suspensions is to favour ride comfort on road holding. The disturbances induced by the road for a vehicle speed  $\mathcal{V} = 20m/s$  are depicted in Fig.5.

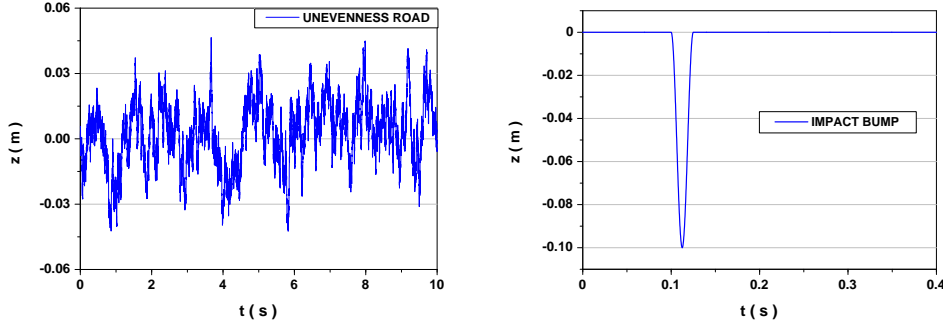


Figure 5: Road induced disturbances for a vehicle speed of  $\mathcal{V} = 20m/s$ .

The performances of the proposed MR suspensions are assessed relative to the ones of a standard passive suspension system, whose parameters are given in Table 1.

Table 1: Passive suspension parameters.

$m_1$	352.5 Kg	$m_2$	50 Kg
$k_1$	24 KN/m	$k_2$	250 KN/m
$d_1$	1560 Ns/m	$d_2$	0 Ns/m

In the following numerical analyses, the maximum input voltage to the current driver was adopted as  $V_{max}=2V$ . For comparison purposes, a passive suspension obtained from the MR damper, named MR-Passive, under a constant input voltage level of  $V = 0.75V$  was also considered. This value for the input voltage yielded lower *rms* acceleration values than at other constant voltage levels when the disturbance induced by the road unevenness was considered.

The MR-Optimal weighting coefficients were chosen as:  $\rho_1=1$ ,  $\rho_2=10^4$ ,  $\rho_3=2 \cdot 10^4$  and  $\rho_4=10^{-6}$ . This set of coefficients favours the ride comfort on the road holding.

The MR-VSC control parameters were chosen as follows. The nominal values of the body mass  $m_1$  and the stiffness coefficient  $k_1$  were adopted as the values of the corresponding parameters  $m_m$  and  $k_m$  of the reference model. The set of weighting coefficients adopted in order to obtain a desired dynamic for the vehicle body was:  $r_1=1$ ,  $r_2=3 \cdot 10^4$  and  $r_3=10^{-6}$ . The sliding surface  $\mathcal{S}$  was defined by making  $S_1=1$ . The parameter  $\phi$  of the linear control component given in Eq.(27) was adopted as zero. The modulation function of the nonlinear control component given in Eq.(28) was adopted as  $\rho=1$ .

The relative performance of the proposed MR vehicle suspension systems were assessed in terms of the *rms* and maximum values of the acceleration of the vehicle body, suspension and tire deflections and total power transmitted to the vehicle body. Along with these performance indices, the functional  $J$ , given in Eq.(9), was also considered and was named Global in what follows. Further studies of the MR vehicle suspension systems will be carried out considering the flexibility of the vehicle body, therefore, the power transmitted to the vehicle body through the suspension system is of great concern.

The relative performance of the proposed MR vehicle suspension systems in terms of the *rms* values is given in Table 2. These values were obtained by considering the disturbance induced by the unevenness road depicted in Fig.5 and a simulation time of 20 seconds.

As one can see from Table 2, all the proposed MR suspensions presented a significant improvement on ride comfort relative to the standard passive suspension system. Although the MR-Passive suspension also presented a relative improvement on the ride comfort, this was achieved at the expense of high *rms* levels of suspension and tire deflections. In the proposed semi-active MR suspensions a trade-off is achieved. The semi-active suspensions also presented lower *rms* values of the power transmitted to the vehicle body. Among the proposed semi-active suspensions, the MR-Power presented a greater relative *rms* value of the tire deflection. This is due to the

fact that this suspension system was synthesized without taking into account the tire deflection. However, the MR-Power suspension presented the most significant improvement on the *rms* value of the transmitted power.

Table 2: Relative performance of the proposed MR suspensions for an unevenness road.

Strategy	<i>rms</i> values				Global
	Accel.	Susp.	Tire	Power	
MR-Passive	0.80	1.48	1.62	1.06	1.21
MR-Optimal	0.76	1.06	1.31	0.62	0.83
MR-VSC	0.75	1.12	1.33	0.60	0.85
MR-Power	0.72	1.09	1.41	0.50	0.84

The corresponding relative performance of the proposed MR vehicle suspension systems in terms of the maximum values is given in Table 3. All the proposed semi-active MR suspensions presented a significant improvement with respect to the maximum value of the vehicle body acceleration, which can also be considered as a ride comfort index. Once again, the MR-Passive suspension presented a relative improvement on the ride comfort at the expense of high levels of suspension and tire deflections. The MR-Passive suspension also presented a high level of power transmitted to the vehicle body. The MR-Optimal suspension system presented a better performance when one considers the maximum values.

Table 3: Relative performance of the proposed MR suspensions for an unevenness road.

Strategy	maximum values				Global
	Accel.	Susp	Tyre	Power	
MR-Passive	0.74	1.54	1.71	1.68	1.10
MR-Optimal	0.69	1.11	1.33	0.74	0.64
MR-VSC	0.77	1.09	1.34	0.82	0.71
MR-Power	0.74	1.14	1.46	0.85	0.80

In order to assess the transient performance of the proposed MR vehicle suspension systems, the impact bump depicted in Fig.5 was also considered. The relative performance of the proposed MR vehicle suspension systems in terms of the *rms* values for a simulation time of 1 second is given in Table 4. As one can see, all MR suspensions presented a significant improvement on the ride comfort. The MR-Power suspension presented the most significant improvement with respect to the transmitted power, however, high *rms* values of suspension and tire deflection were obtained.

Table 4: Relative performance of the proposed MR suspensions for an impact bump.

Strategy	<i>rms</i> values				Global
	Accel.	Susp.	Tire	Power	
MR-Passive	0.70	1.88	1.61	0.35	1.14
MR-Optimal	0.81	1.25	1.19	0.59	0.88
MR-VSC	0.77	1.39	1.29	0.43	0.91
MR-Power	0.74	1.56	1.42	0.29	0.99

The corresponding relative performance of the suspension systems in terms of the maximum values is given in Table 5. As one can see, the suspensions presented a maximum tire deflection at the same level as that of the standard passive suspension system. Once again the MR-Power suspension presented the most significant improvement with respect to the transmitted power. The response of the semi-active suspension systems subjected to the impact bump is depicted in Fig.6. The MR-Passive suspension presented the better performance when one considers the maximum values for an impact bump.

## 6. Concluding remarks

The present work considered semi-active control strategies for MR vehicle suspension systems. Two semi-active control strategies based on the Clipped Control approach and a semi-active control synthesized in order to



Table 5: Relative performance of the proposed MR suspensions for an impact bump.

Strategy	maximum values				
	Accel.	Susp	Tyre	Power	Global
MR-Passive	0.44	1.26	0.99	0.26	0.47
MR-Optimal	0.65	1.13	1.00	0.52	0.59
MR-VSC	0.60	1.21	1.00	0.38	0.51
MR-Power	0.63	1.34	0.99	0.23	0.51

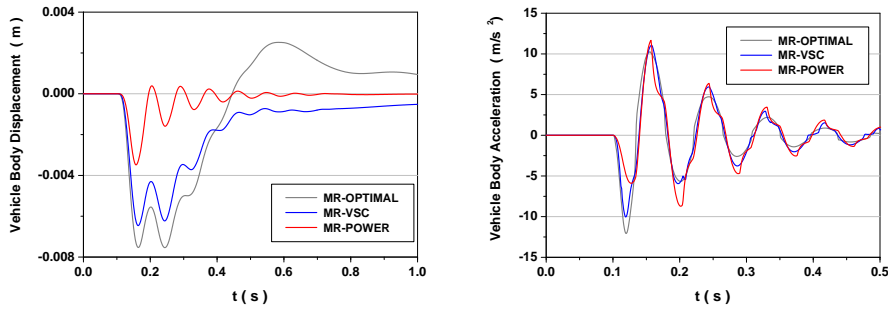


Figure 6: Response of the MR suspensions subjected to the impact bump.

diminish the power transmitted to the vehicle body were proposed. The proposed suspensions were synthesized in order to favour the ride comfort on the road holding and their performances were assessed through numerical analyses considering a quarter-car model subjected to disturbances induced by both road unevenness and an impact bump. The performances were assessed relative to the ones of a standard passive suspension system. A passive suspension obtained from a magnetorheological damper under a constant input voltage level was also considered for comparison purposes. For both disturbances considered, the proposed semi-active suspensions presented significant improvements with respect to ride comfort and power transmitted to the vehicle body. The MR-Passive suspension also presented an improvement on the ride comfort at the expense of high levels of tire and suspension deflections. The MR-Optimal suspension presented great performance for both disturbances considered, however, it is worth noting that in the optimal control approach considered, it was assumed the ideal situation where the road disturbance is measured. The MR-Power presented great improvements when considering the ride comfort and the total power transmitted to the vehicle body. For the impact bump, this suspension presented high *rms* levels of tire and suspension deflections. The control algorithm of the MR-Power suspension is relatively simple, but it presents a lack of flexibility in the control synthesis because the input voltage is direct derived from the equation of the total power transmitted to the vehicle body. The MR-VSC presented a great performance for both disturbances considered. The MR-VSC suspension presents a great flexibility in its synthesis due to the choice of the reference model along with the control parameters. From the analyses carried out in this study, the MR-VSC suspension was found to be the most promising one for the magnetorheological vehicle suspension system due to its performance along with its great flexibility. The performance in the presence of unmodeled dynamics and/or parameters uncertainties must also be considered. The present work presents an initial effort in the study of semi-active isolation systems, particularly the suspension systems. Parallel studies of semi-active MR suspensions in a half car model with flexible body are under consideration.

## 7. References

- J.D. Carlson and Spencer Jr., 1996, *Magneto-Rheological Fluid Dampers: Scalability and Design Issues for Application to Dynamic Hazard Mitigation*, Proc. 2nd Int. Workshop on Structural Control, Hong Kong, 99–109.
- S.J. Dyke, 1996, *Acceleration Feedback Control Strategies for Active and Semi-Active Systems: Modelling, Algorithm Development and Experimental Verification*, Ph.D. thesis, University of Notre Dame, Notre Dame.
- E. Guglielmino, 2001, *Robust Control of a Hydraulically Actuated Friction Damper for Vehicle Applications.*, Ph.D. thesis, Bath, UK.
- A. Hać, 1985, *Suspension Optimization of a 2-DOF Vehicle Model Using a Stochastic Optimal Control Technique*, Journal of Vibration and Control **100**, no. 3, 343–357.

- D. Hrovat, 1997, *Survey of Advanced Suspension Developments and Related Optimal Control Applications*, Automatica **33**, no. 10, 1781–1817.
- L.M. Jansen and S.J. Dyke, 1999, *Semi-Active Control Strategies for MR Dampers: A Comparative Study*, ASCE Journal of Engineering Mechanics **126**, no. 8, 795–803.
- B.F. Spencer Jr., S.J. Dyke, M.K. Sain, and J.D. Carlson, *Phenomenological Model of a Magneto-Rheological Damper*, J. Engrg. Mech. **123** (1996), no. 3, 230–238.
- C.Y. Lai and W.H. Liao, 2002, *Vibration Control of a Suspension System via a Magnetorheological Fluid Damper*, Journal of Vibration and Control **8**, 527–547.
- R.S. Sharp and S.A. Hassan, 1986, *An Evaluation of Passive Automotive Suspension Systems with Variable Stiffness and Damping Parameters*, Vehicle System Dynamics **15**, 335–350.
- L.T. Stutz and F.A. Rochinha, 2003, *Variable Structure Control with Model-Following Applied to a Quarter-Car Model Subjected to Random Excitation*, Proc. of 16th Brazilian Congress of Mechanical Engineering - COBEM, Brazil.
- H.E. Tseng and J.K. Hedrick, 1994, *Semi-Active Control Laws - Optimal and Sub-Optimal*, Vehicle System Dynamics **23**, 545–569.
- G.Z. Yao, F.F. Yap, G. Chen, W.H. Li, and S.H. Yeo, 2002, *MR Damper and its Application for Semi-Active Control of Vehicle Suspension System*, Mechatronics **12**, 963–973.
- M. Yokoyama, J.K. Hedrick, and S. Toyama, 2001, *A Model Following Sliding Mode Controller for Semi-Active Suspension Systems with MR Dampers*, Proceedings of the American Control Conference, Arlington, 2652–2657.

## 8. Responsibility notice

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