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STABILITY ANALYSIS OF GROUND RESONANCE PHENOMENON IN HELICOPTERS: EXPERIMENTAL INVESTIGATIONS

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Abstract: *Helicopter ground resonance is an unstable dynamic phenomenon which can lead to the total destruction of the aircraft during take-off or landing phases. Coleman and Feingold were the earliest research in this domain around the 1960 decade. Their study leads to the prediction of instability by using classical procedure, when assuming all blades with the same mechanical properties (i.e.: isotropic rotor). Under this condition, the periodical equations of motion are simplified by introducing a change of variable, known as Coleman Variable Transformation. However, this assumption is no more valid for a helicopter under the aging effects. In fact, the mechanical properties of the blades may modify randomly with time and be different from each other (i.e.: anisotropic rotor). Recently, studies have been developed, by using the Floquet's Theory, in order to verify the influence of stiffness dissimilarities between blades. Stability chart highlights the appearance of new instability zones as function of the perturbation introduced on the lead-lag stiffness of one blade. In order to further understand the phenomenon and valid the theoretical results, this paper leads with the design and development of a new experimental setup. Different rotor configurations are investigated and the boundaries of instabilities are determined. Beyond the good correlation between both results, the new instability zones are found in asymmetric rotors. The temporal responses of the measured signals highlight the exponential divergence at the instability regions.*
Keywords: *Ground Resonance, Experimental Setup, Stability Analysis, Nonlinear Dynamics*

1. INTRODUCTION

Coleman and Feingold (1957) were the first to perform research into the problem of the ground resonance phenomenon in helicopters. Resulted from the coalescence between the fuselage and a cyclic rotor modes of vibrations, while the helicopter is on the ground over its landing skids, the phenomenon consists in a potential instability that may leads the aircraft to its total destruction.

Later, investigations carried out on the occurrences of the ground resonance for articulated, hingeless and bearingless rotors (Donham *et al.*, 1969; Hodges, 1979) conclude that the use of linearized periodic equations of motions provide accurate frequency prediction of critical zones. In all these investigations, only isotropic rotors configurations (all blades having the same mechanical properties) have been considered.

However, effects of aging or failure of mechanical elements in dynamical systems through time appear randomly by compromising its nominal operation, what may subject the user to dangerous conditions in some cases. Indeed, due to the aging effects, the stiffness properties of blades may vary randomly with time and be different from each other (i.e: anisotropic rotor).

The study of the influence of the aging effects on the ground resonance phenomenon, by assuming a slight lead-lag stiffness asymmetry of $\pm 5\%$ at one blade, verifies a shift on the boundary speeds of unstable regions, making them wider (Wang and Chopra, 1992).

Further analysis was performed by Sanches *et al.* (2011), in which takes into account high stiffness asymmetries between blades at different anisotropic rotors configurations. Stability charts, obtained by using Floquet's Theory, exhibit a complex evolution of the instability regions with respect to the asymmetries introduced and the appearance of new critical regions. The mechanical model adopted on the analysis assumes the hypothesis of no viscous damping in order to verify all the possible instabilities.

The present work aims to further comprehend the ground resonance in helicopters under the aging effects, by reproducing the phenomenon and validating the theoretical results experimentally. A new experimental setup, designed and conceived for this purpose, is described, as well as it is detailed its main components and their functioning. Isotropic and anisotropic rotor configurations are analysed. The boundaries of instability determined along an experimental test are compared with those predicted by using the Floquet's Theory.

2. GROUND RESONANCE: MECHANICAL MODEL & INSTABILITY PREDICTION

The mechanical model adopted for analysing the ground resonance phenomenon in helicopters with hinged blades and under the aging effects is based on that used by Sanches *et al.* (2011). However, it considers now the presence of viscous dampers on the longitudinal and lateral directions of the fuselage and on the blade lead-lag rotations, as shown in Fig. (1).

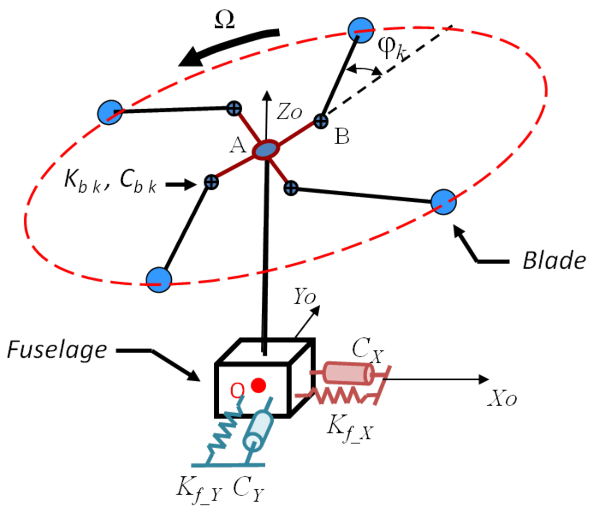


Figure 1: Schema of the mechanical system.

The fuselage movements are described as function of its longitudinal and lateral displacements, $x(t)$ and $y(t)$, respectively. Two springs, linked to the fuselage in both directions, represent the flexibility of the landing skids when the helicopter is on the ground. $K_{f\,X}$ and $K_{f\,Y}$ are the stiffness of the springs situated along *x* and *y* directions, respectively.

Similarly, the damping efforts, modelled as viscous damper, have coefficients equal to C_X and C_Y in the longitudinal and lateral directions of fuselage respectively. The helicopter at its equilibrium position has the center of mass of the fuselage (point *O*) coincident with the origin of inertial reference frame $(X_0,\, Y_0,\, Z_0)$.

The rotor head system is composed of one rigid rotor hub and an assembly of N_b blades. The blades have mass $m_{b\,k}$ and a moment of inertia $I_{zb\,k}$ around the *z* - axis located at its center of mass. The radius of gyration is defined by length *b*. Each *k*th blade owns an in-plane lead-lag motion defined by $\varphi_k(t)$ and an azimuth angle defined as $\zeta_k = 2\pi\,(k-1)\,/N_b$ with respect to the *x* - axis. Angular spring and viscous damper are considered on each blade hinge (point *B*) with spring stiffness and viscous damping coefficient been $K_{b\,k}$ and $C_{b\,k}$, respectively.

The origin of a rotational reference frame $(x,\, y,\, z)$ is parallel to the inertial one and is located at the geometric center of the rotor hub (coincident at point *O*). The rotor revolves at speed Ω.

Both, body and rotor head, are joined by a rigid shaft and aerodynamic forces on the blades are not taken into account. Such an assumption is quite realistic since the helicopter is on the ground. In the present work, the rotor is composed of N_b = 4 blades.

In order to obtain the equations of motions, LAGRANGE equation is applied on the energies and work expressions of the dynamical system. Only the linear terms are taken into account. The matrix equation of motion, with respect to **u**, is

$$\mathbf{M}\,\ddot{\mathbf{u}} + \mathbf{G}\,\dot{\mathbf{u}} + \mathbf{K}\,\mathbf{u} = \mathbf{F_{ext}} \tag{1}$$

where, **M**, **G** and **K** correspond to the mass, damping and stiffness matrix, respectively. They are non-symmetric and non-diagonal matrices due to the presence of periodic terms. $\mathbf{F_{ext}}$ is equal to zero once all blades possess the same inertial and geometrical properties.

The matrices then expressed in Eq. (1) are obtained from Sanches *et al.* (2011), excepted for the damping matrix **G**.

The diagonal elements correspond to the presence of viscous dampers in the helicopter, i.e.:

$$\mathbf{G}(t) = \begin{pmatrix} r_{c1} & 0 & -2\Omega r_{m1}\cos(\psi_1) & -2\Omega r_{m2}\cos(\psi_2) & -2\Omega r_{m3}\cos(\psi_3) & -2\Omega r_{m4}\cos(\psi_4) \\ 0 & r_{c2} & -2\Omega r_{m1}\sin(\psi_1) & -2\Omega r_{m2}\sin(\psi_2) & -2\Omega r_{m3}\sin(\psi_3) & -2\Omega r_{m4}\sin(\psi_4) \\ 0 & 0 & r_{c3} & 0 & 0 & 0 \\ 0 & 0 & 0 & r_{c4} & 0 & 0 \\ 0 & 0 & 0 & 0 & r_{c5} & 0 \\ 0 & 0 & 0 & 0 & 0 & r_{c6} \end{pmatrix} \quad (2)$$

where

$r_{c\,1..2} = \frac{C_{X..Y}}{m_f + \sum_{k=1}^{N_b} m_{b\,k}}$, $r_{c\,k+2} = \frac{C_{b\,k}}{b^2\, m_{b\,k} + I_{z_{b\,k}}}$ k= 1..4

The factors $r_{c\,1..2}$ are ratios between damping coefficient of the fuselage in *x* and *y* directions, respectively, and the total mass of helicopter, whereas $r_{c\,3..6}$ are ratios between damping coefficient and the total inertia of blade rotational motion.

The terms ω_x and ω_y represent the resonance frequencies of the fuselage in directions *x* and *y*, respectively. Moreover, $\omega_{b\,3..6}$ are the lead-lag resonance frequencies of blades 1 to 4.

The prediction of the boundaries of instability for the ground resonance phenomenon, represented by the periodic equations in Eq. (1), is obtained by applying Floquet's theory (Hammond, 1974; Skjoldan and Hansen, 2009; Sanches *et al.*, 2009, 2011).

Based on the procedure and development described in Sanches *et al.* (2011), the stability of the periodic dynamical system is given through the monodromy matrix **R** or the Floquet Transition Matrix (FTM).

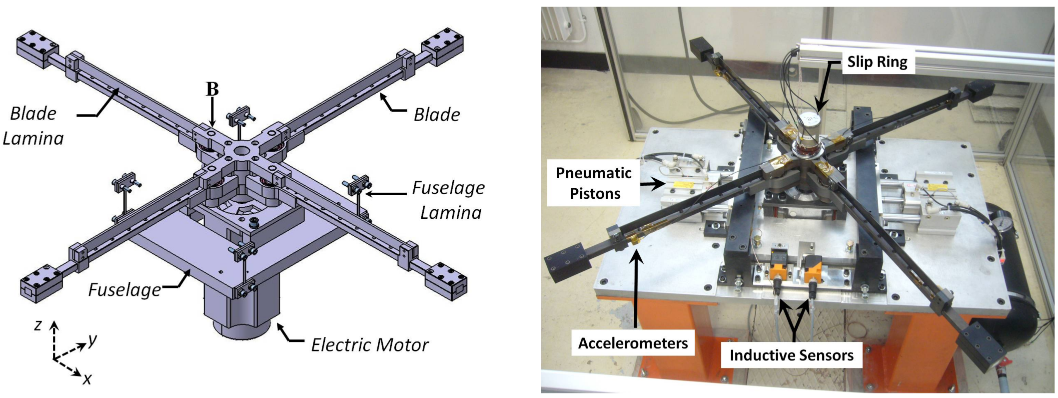
A special technique, called Picard interactions, is used and devoted to compute the monodromy matrix numerically (Dufour and Berlioz, 1998). It consists basically to approximate the periodic state-space system $S(t)$ by a series of p step functions along a period.

The dynamical system Eq. (1) is exponentially stable if **R** is Schur. This means that if all the norms of the eigenvalues of **R**, known as characteristic multipliers, are less than one.

3. EXPERIMENTAL SETUP

An experimental setup is designed and developed to reproduce the unstable motion of helicopters under the ground resonance phenomenon, contributing towards the physical understanding of the phenomenon and validating the theoretical results.

The final experimental helicopter conceived is represented in Fig. (2a). The conceptual design of the experimental aircraft is based on the same mechanisms and hypothesis considered on the mechanical system, in section 2.



(a) General Representation **(b)** Detailed View

Figure 2: The experimental helicopter: (a) general representation and (b) detailed view.

The rotor head system is composed of a rigid rotor hub and an assembly of four blades. Each blade features an in-plane lead-lag rotation along a vertical axis, passing through the point B. The angular spring stiffness of the blades are obtained through flexible laminas. Each lamina is located over the blades and links the rotor hub to the blade. At both joints (rotor head/lamina and lamina/blade), the kinematic coupling are of type clamped-clamped.

Regarding the fuselage, an electrical motor is the energy source necessary to drive the rotor system at a constant angular speed Ω. The motor is linked to the chassis and four vertical laminas are linked to it. The fuselage laminas, when later linked to inertial rigid beams (see Fig. (2b)), will support the whole weight of experimental helicopter.

The fuselage, suspended over an inertial table, can oscillate in the longitudinal and lateral directions (i.e.: x and y directions, respectively). The spring stiffness of the landing skids of the helicopter is given by the fuselage laminas. The kinematic couplings at each joints of the lamina are of type clamped-clamped.

The spring stiffness of the fuselage and blades are strongly dependent of the laminas geometry, once they are made of the same material. Intended to study different helicopter configurations (by altering the spring stiffness), three sets of laminas are used for the blades.

Table (1) shows the numerical data of the experimental helicopter (HE), as well as the main dimensions of the whole machine, including the inertial table. The pneumatic pistons and the inductive sensors are elements of the security system (see Fig. (2b)), beyond a safety enclosure cabine which is not represented in the figure. The maximum allowable rotor speed and the maximum longitudinal displacement of the fuselage are is 600 rpm and 2mm, respectively.

Table 1: Data of the experimental helicopter (HE).

System	Description	Units	Value
Fuselage	m_f	kg	45.2
Rotor	$m_{b\,1..4}$	kg	2.84
	a	m	0.1
	b	m	0.22
	I_{zb}	kg m^2	0.11
Machine	length	m	1.28
	width	m	1.28
	height	m	1.00

3.1 Measurement System

Since the ground resonance phenomenon is characterized by the instabilities, the measurement system aims to record the fuselage and blades oscillations. The analysis on the temporal responses will identify the critical rotor speeds, determining the boundaries of the unstable region.

Accelerometers are then used to measure the longitudinal acceleration of the fuselage and the angular acceleration of each blade. The revolving rotor speed is obtained through an optical encoder located in the motor. The slip ring element allows the signal transfer from a rotating to a fixed frame (see Fig. (2b)). A data-acquisition system records the measured signals along an experimental test.

A low-pass filter of type butter is applied (cutoff frequency = 20Hz and order = 5) on the measured signals in order to minimize the influence (i.e.: high frequencies) of the magnetic field provided by the electrical motor on the accelerometers.

3.2 Spring Stiffness Determination

By considering the fuselage and blade as uncoupled oscillators, their equivalent spring stiffness are determined dynamically through their natural frequencies of vibration. The natural frequencies are obtained from their free oscillations, when, independently, they are shifted from their equilibrium point.

In case of appearance of nonlinear oscillations, as encountered with the blades, the linear equivalent blade spring stiffness is determined by considering the natural blade frequency equal to that with the maximum power spectrum peak.

The structural damping factor is determined by analysing the exponential rate decay of the oscillations. An average structural damping factor of 2% is found for the blades and fuselage.

4. NUMERICAL VS. EXPERIMENTAL RESULTS

Numerical and experimental approaches are used on the study of ground resonance phenomenon for isotropic (called as RI) and anisotropic rotor (called as RA). At the RI configurations, all blades are mounted with the same set of blade lamina; while at RA configuration, one blade is mounted with a lamina from A different set with respect to the others.

Table (2) indicates the set of blade lamina used for each rotor configurations studied, as well as the blade natural frequency determined by following the proceedings described in section 3.2

Table 2: Blade natural frequency and the set of lamina of each blade for RI and RA rotor configurations.

Blade k		1	2	3	4
RI	Set of Lamina	2	2	2	2
	ω_{bk}[Hz]	3.19	3.14	3.47	3.07
RA	Set of Lamina	1	1	1	2
	ω_{bk}[Hz]	2.49	2.39	2.35	3.25

Table 3: The comparison between the boundary of instability predicted with FM and that determined experimentally for RI rotor configuration.

	With FM	Experimental	Deviation [%]
Lower Bound	7.33	7.82	6.68
Upper Bound	8.63	8.80	1.97

Concerning the equivalent spring stiffness of the fuselage, the natural frequencies identified are $\omega_x = 2.55$ Hz and $\omega_y = 15.8$ Hz in the longitudinal and lateral directions, respectively.

It is important to remark that the numerical analyses with Floquet's Theory are carried out by assuming $p = 64$.

4.1 Isotropic Rotor

Figure (3) shows the evolution of the characteristic multiplier (red line) with respect to the angular rate Ω, obtained through the Floquet's Theory. The data described previously about the experimental helicopter with RI rotor configuration is considered. The green square symbols represent the predicted boundary of instability whereas that determined experimentally is given in blue diamonds. Table (3) presents their numerical values.

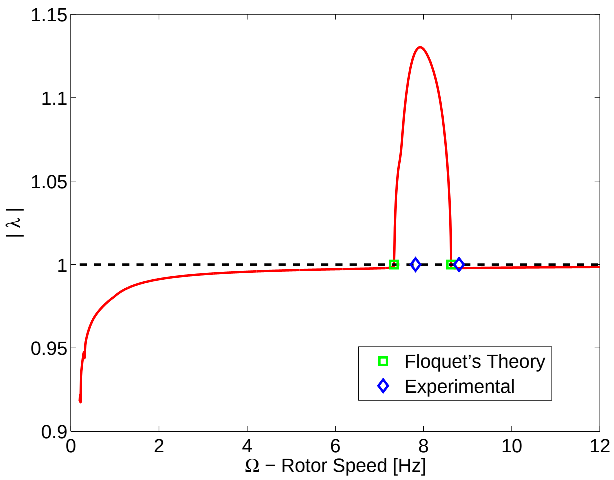


Figure 3: Comparison between the numerical vs experimental prediction of the instability zone for RI.

The analysis carried out with Floquet's Method is done until the maximum allowable rotor speed value in the experimental helicopter. This fact explains the observation of only one instability region, i.e.: region where the absolute value of the characteristic multiplier is bigger than the unity. Normally, an helicopter with isotropic rotor and two distinct fuselage natural frequencies have two instability regions.

The boundaries of instability verified experimentally, with respect to that predicted numerically, presents a very good correlation and a maximum deviation of 6.68%.

Regarding the fuselage and blades measured accelerations of the experimental helicopter along a critical region, it is possible to verify the exponential growth of the envelop of those signals (see Fig. (4)). It evidences then the presence of an instability.

4.2 Anisotropic Rotor

The study of anisotropic rotors are very interested from a practical point of view. Indeed, the effects of aging or failure of mechanical elements may alter the boundaries of instabilities of the ground resonance phenomenon. The stiffness asymmetries considered in this work are given in only one blade (see RA in Tab. (2)).

By applying the Floquet's Theory, Figure (5) shows the evolution of the characteristic multiplier (red line) with respect to the angular rate Ω. The predicted boundary of instability is represented by the green square symbols whereas that determined experimentally is given in blue diamonds. Table (4) presents their numerical values.

Three instability regions are predicted with Floquet in the case with one dissimilar blade. It is important to remark that the analysis is carried out until the maximum allowable rotor speed value of the experimental helicopter.

Two new instability zones (i.e.: zones 1 and 2) are predicted with respect to an isotropic rotor. The third zone corresponds the same critical region found with the RI configuration .

Experimentally, only the zone 1 and lower and upper limit of zones 2 and 3, respectively, were possible to be determined. The stable region between zone 2 and 3 is not found due to their proximity.

Regarding the fuselage and blades measured accelerations of the experimental helicopter along a critical region, it is

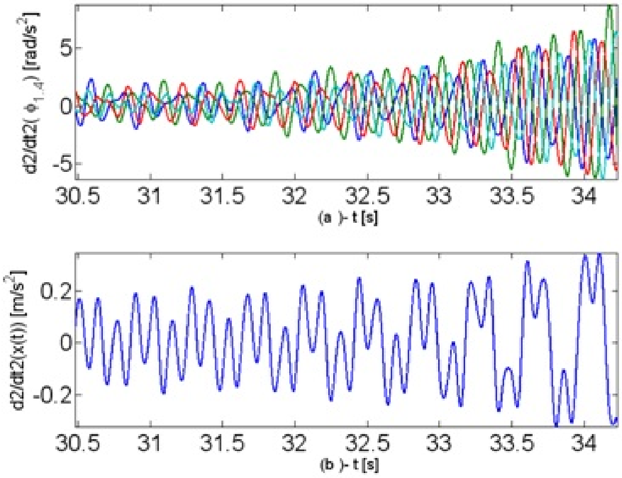


Figure 4: Time response of the experimental helicopter with RI rotor configuration at $\Omega \approx 7.5Hz$.

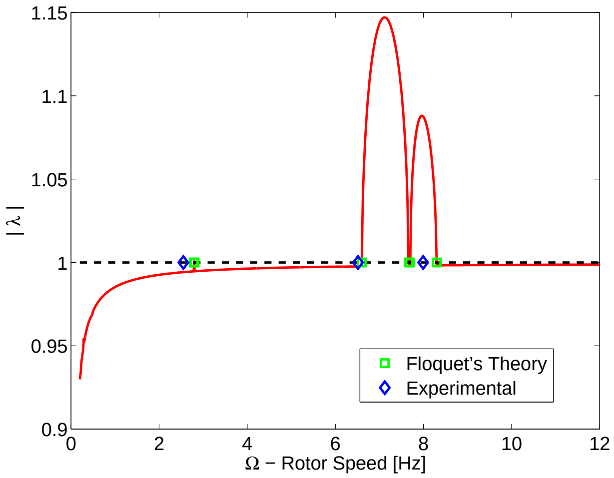


Figure 5: Comparison between the numerical vs experimental prediction of the instability Zone for RI configuration.

possible to verify the exponential growth of the envelop of those signals at zone 1 (see Fig. (6)). It evidences then the presence of new instability region once asymmetries between blades are introduced.

5. CONCLUSIONS

The dynamics of helicopters on the ground, but under the aging effects, have been recently studied. In fact, these effect may vary the spring stiffness of blades randomly, by compromising the nominal operation of the aircraft and leading the aircraft to the ground resonance phenomenon. The investigations predict the existence of new instabilities zones with high asymmetries between blade.

The present work is aimed to reproduce experimentally the ground resonance phenomenon for isotropic and anisotropic rotors, to further comprehend the phenomenon and to validate the theoretical results.

The new designed experimental helicopter, used for the analysis, is described and its main components are presented. The conceptual design of the apparatus was based on the same mechanisms and hypothesis considered on the simplified

Table 4: The comparison between the boundary of instability in Hz predicted with FM and that determined experimentally for the anisotropic rotor RA.

Zone		With FM	Experimental	Deviation [%]
1	–	2.81	2.55	9.20
2	Lower Bound	6.60	6.51	1.36
	Upper Bound	7.66	–	–
3	Lower Bound	7.69	–	–
	Upper Bound	8.30	7.99	3.73

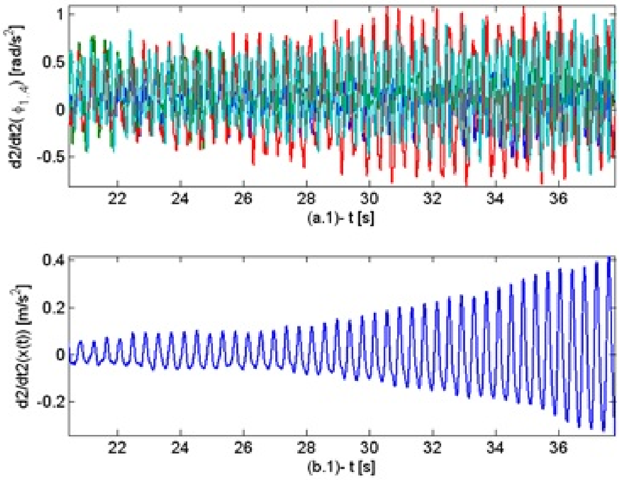


Figure 6: Temporal response of the experimental helicopter with RA configurations at $\Omega \approx 2.5Hz$.

helicopter with hinged blades, used for the theoretical analysis. Efforts have been made to get an easily adjustable parametric system (i.e.: blade mass, blade and fuselage spring stiffness). During an experimental test, the accelerations signals of the fuselage and the blade lead-lag rotations are recorded.

Between the both approaches, numerical (i.e.: Floquet's Theory) and experimental, the boundaries of instability found are quite similar for isotropic and anisotropic rotor configurations.

The appearance of new instability zones in anisotropic rotors with respect to the isotropic one is also verified experimentally. This fact is evidenced by the exponential divergence of the measured signals, which characterizes the unstable oscillations during the occurrence of the phenomenon.

6. ACKNOWLEDGEMENT

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8. DIREITOS AUTORAIS