EVALUATION OF BENDING CRITICAL SPEEDS OF HYDROGENERATOR SHAFT LINES BY USING THE TRANSFER MATRIX METHOD

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Abstract. In this work, the Transfer Matrix Method is used to evaluate critical speeds of hydrogenerator shaft lines. A computer program was developed, in MatLab™ language, for the calculation of bending critical speeds and this code was applied in some industrial hydrogenerators. The obtained results are compared with the data/guarantees from the equipments’ manufacturers. It is concluded that the present methodology is suitable for rotor dynamics analysis at early design phases, providing the engineers a reliable tool for quick assessments on shaft line arrangements.

Keywords: rotor dynamics, mechanical vibration, transfer matrix method, bending critical speed, hydrogenerator.

1. INTRODUCTION

In Brazil, thanks to its vast hydraulic resources, hydropower plants produce almost 85% of the annually consumed electrical energy in the country (EPE, 2010). Further, the Brazilian hydropower potential is around 260 GW, while the current hydropower installed capacity is near 80 GW, i.e., only 31% of the potential. These simple numbers confirm the historical importance of hydropower generation in Brazil and clearly indicate that this scenario shall be maintained for the next decades. Another evidence for this statement is the current decennial plan for energy expansion (PDE, 2011), in which the federal government foresees 24 new large hydropower stations to be erected up to 2020, increasing in more than 18,000 MW the installed capacity of the national interconnected electrical system.

Brazil is not the only country in which hydropower plays an important role in the energy matrix. Canada, United States of America, Russia, China, India, Venezuela, Equator, Argentina, Paraguay, Colombia, Switzerland, Norway, Portugal, among others, strongly depend on this type of power generation, with several ongoing projects (IEO 2011).

In a hydropower station, the hydraulic energy is converted into electricity in machines named hydrogenerators (or generating units), which are the electromechanical set formed by a hydro turbine coupled to an electrical generator. Figure 1 shows the lay-out of the hydrogenerators at Itaipu power station.

The good operational performance of a hydropower plant is closely linked to the perfect running of its generating units. For this, besides the rigorous compliance of the permissible operating ranges and maintenance routines, it is necessary that all the aspects of design, manufacture and assembly have been well studied and executed. In particular, the hydrogenerator design task involves multidisciplinary knowledge, like fluid mechanics, turbomachinery, electrical machines, strength of materials, vibrations, machine elements, science of materials, manufacturing processes, etc. Thereby, there are generally engineering teams to investigate the many technical solutions in order to reach an optimum and feasible design, under the several involved criteria.

One of the basic problems that arise during the mechanical design of a hydrogenerator – as well as in almost all types of rotating machines – is the evaluation of the shaft line mechanical vibrations, specially the calculation of natural frequencies and associated critical speeds. In fact, the rotor dynamics of a hydrogenerator is of paramount importance even at early design phases, because this study provides the engineers important data and constraints to be considered at the final executive design.

The analysis of the dynamic behavior of a hydrogenerator shaft line has the basic purpose of identifying fundamental vibratory phenomena that may occur during the machine operation, as well as the design changes to avoid or mitigate such problems. Special attention should be given to the critical speeds, which are angular velocities that may cause dangerous resonances, i.e., driving excitations very close to natural frequencies of the mechanical system. In this context, the evaluation of natural frequencies/critical speeds (torsion or bending ones) assumes priority already at early design phases. The present work aims to provide a useful calculation tool for assisting engineers in the correct dimensioning of shafts and positioning of guide bearings in hydrogenerator shaft lines.
For hydrogenerators, the bending vibrations of the shaft line are usually more relevant than the torsion vibrations. Actually, it is a common practice for hydrogenerators to respect the shaft shear stress requirements – due to torque – and the bending critical speeds and after only check if the torsion critical speeds are not reached too. In this way, the main concern of this work is to establish a calculation routine for the evaluation of bending critical speeds.

Hydrogenerators, being hydraulic machines, are sub-critical machines, i.e., they operate always below the first critical speed (Thomson, 1978). Other types of machines, like steam turbines, gas turbines and diesel engines, are super-critical machines, operating above the first critical speed. When a mechanical system (e.g., a shaft line) reaches a critical speed, its natural vibration mode is “energized”, which is dangerous for its structural integrity. Thus, in the case of super-critical machines, some special cares are taken to attenuate the passage through critical speeds.

Hydrogenerators, however, are equipments with very large weights in the shaft line, especially the units of large hydropower plants. Therefore, it is prohibited to a hydrogenerator reaching any critical speed. If this happens, the mechanical stresses due to resonance, even in fast transients, would cause damages to the shafts, bearings, bearing supports, embedded structures in the powerhouse, etc.

Therefore, it is crucial to ensure, already at early design phases, that the maximum speed of a hydrogenerator, in any condition of operation, mainly in runaway, should be always lower than the first critical speed, with a suitable safety margin that provides reliability and durability for the equipment. Putting in a different way, the engineer’s task is to design the shaft line in a way that the first critical speed is sufficiently higher than the maximum speed of the machine.

Among the available methods for rotor dynamics analysis, as the methods of Rayleigh and Dunkerley (Rao, 1996), and the finite element method (Vaz, 2010), it was chosen the Transfer Matrix Method (Almeida, 1990). The Transfer Matrix Method is of easy computational implementation and very suitable for matrix-based softwares, like MatLab™. Also, this method presents as advantage the fact that the order of the involved matrices does not depend on the number of the problem degrees of freedom. So, the same type of matrices can be used independently the level of discretization of the mechanical system. Moreover, a relatively small number of elements is generally enough for reaching results with good accuracy level. So these features make the Transfer Matrix Method a suitable approach for quick assessments of hydrogenerator shaft line arrangements (Quitzrau, 2002).

2. OBJECTIVE

Within this context, one perceives the importance/utility to dispose, at early design phases, of a simple and reliable calculation tool, being able to evaluate, with good accuracy, the bending critical speeds of shaft lines. With this tool, the mechanical engineer can quickly investigate suitable arrangements for a shaft line, specifying the shaft diameters and the guide bearing positions.

The main goal of this work is to develop such a calculation tool for hydrogenerators, based on the computational implementation of the Transfer Matrix Method. By using this tool, it will be calculated the first bending critical speed of some industrial hydrogenerators and the results will be compared with the design data provided by the equipments’ manufacturers.
3. TRANSFER MATRIX METHOD FOR BENDING VIBRATIONS IN SHAFT LINES

The Transfer Matrix Method is a suitable technique for solving mechanical vibration problems in beams and shafts (Almeida, 1990). Any section or station of the structure is described by a “state vector”, whose coordinates are the physical quantities that define the vibratory state of that section. In two-dimensional bending vibrations, the state vector, \( S \), is defined in terms of transversal displacement, \( Y \), axis slope, \( \theta \), bending moment, \( M \), and shear force, \( Q \), as shown in Eq. (1).

\[
S = \begin{bmatrix} Y \\ \theta \\ M \\ Q \end{bmatrix}
\]

The entire shaft line should be properly discretized in elements of shaft, elements of mass and elements of guide bearing, as illustrated in Fig. 2. The state vectors of the elements ends are related by proper matrices, named “transfer matrices”. The method consists in relating the ends of the shaft line by means of several transfer matrices. Then, the boundary conditions can be imposed and this leads to the natural frequencies (critical speeds) calculation.

![Figure 2. Shaft line discretization: (a) actual shaft line; (b) discrete shaft line. (adapted from Quitzrau (2002))](image)

The masses in the shaft line are considered as concentrated in points, as shown in Fig. 3. For generator rotors and turbine runners, this is a reasonable assumption, since the axial length of these rotors uses to be small in comparison with the shaft line total length. For the shafts, however, one should divide the masses in a convenient number of points. The experience with a particular problem will show appropriate choices.

![Figure 3. Element of mass in a shaft line. (adapted from Almeida (1990))](image)

By applying the Newton’s laws of motion and assuming harmonic vibrations, one can found (Almeida, 1990; Barbosa, 2011) that the state vectors of a mass element ends can be related by Eq. (2). Note: the gyroscopic effect is not taken in account in the present formulation.
In Eq. (2), $m$ is the concentrated mass and $\omega$ is the angular frequency of the transversal vibration. Thus, the “mass matrix” or “point matrix”, $T_p$, is defined by Eq. (3). This is the transfer matrix for each mass element in the discrete shaft line.

$$
T_p = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
\text{mass}^2 & 0 & 0 & 1 \\
\end{bmatrix}
$$

The shafts are treated as elastic segments without mass, as sketched in Fig. 4.

Figure 4. Element of shaft in a shaft line. (adapted from Almeida (1990))

Considering the equilibrium conditions and the beam elastic line theory for the slope/displacement calculations (Popov, 1978), one can found (Almeida, 1990; Barbosa, 2011) that the state vectors of a shaft element ends can be related by Eq. (4).

$$
\begin{bmatrix}
Y \\
\theta \\
M \\
Q
\end{bmatrix}_{i+1} = \begin{bmatrix}
1 & L & \frac{L^2}{2EI} & \frac{L^3}{6EI} \\
0 & 1 & \frac{L}{EI} & \frac{L^2}{2EI} \\
0 & 0 & 1 & L \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\begin{bmatrix}
Y \\
\theta \\
M \\
Q
\end{bmatrix}_i
$$

In Eq. (4), $L$ is the element length, $E$ is the Young modulus of the shaft material and $I$ is the moment of inertia of area of the shaft cross-section, for the bending ($I = \pi(D_e^4 - D_i^4)/64$; $D_e$ = shaft outer diameter; $D_i$ = shaft inner diameter). Thus, the “field matrix”, $T_e$, is defined by Eq. (5). This is the transfer matrix for each shaft element in the discrete shaft line.

$$
T_e = \begin{bmatrix}
1 & L & \frac{L^2}{2EI} & \frac{L^3}{6EI} \\
0 & 1 & \frac{L}{EI} & \frac{L^2}{2EI} \\
0 & 0 & 1 & L \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
$$

The guide bearings on the shaft line are treated as flexible supports, with a total rigidity coefficient $k$, as sketched in Fig. 5. The flexibility of the bearing is due to the flexibilities of the oil film between the shaft and the bearing, the bearing frame and the bearing support. This flexibility significantly affects the rotor dynamics of the shaft line and thus should be carefully investigated by the engineer.
Neglecting the mass of the bearing, one can found (Almeida, 1990; Barbosa, 2011) that the state vectors of a guide bearing element ends can be related by Eq. (6).

\[
\begin{bmatrix}
Y \\
\theta \\
M \\
Q
\end{bmatrix}_{n+1} =
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
-k & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
Y \\
\theta \\
M \\
Q
\end{bmatrix}_n
\]

(6)

Thus, the “bearing matrix” or “rigidity matrix”, \( T_m \), is defined by Eq. (7). This is the transfer matrix for each guide bearing element on the discrete shaft line.

\[
T_m =
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
-k & 0 & 0 & 1
\end{bmatrix}
\]

(7)

Table 1 summarizes the transfer matrices for each discrete element on the shaft line.

<table>
<thead>
<tr>
<th>Discrete element</th>
<th>Transfer matrix</th>
<th>Physical quantities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Element of concentrated mass (without gyroscopic effect)</td>
<td>( T_p = \begin{bmatrix} 1 &amp; 0 &amp; 0 &amp; 0 \ 0 &amp; 1 &amp; 0 &amp; 0 \ 0 &amp; 0 &amp; 1 &amp; 0 \ m\omega^2 &amp; 0 &amp; 0 &amp; 1 \end{bmatrix} )</td>
<td>( m = ) concentrated mass ( \omega = ) angular frequency of the bending vibration</td>
</tr>
<tr>
<td>Element of shaft (without mass)</td>
<td>( T_e = \begin{bmatrix} 1 &amp; L &amp; \frac{L^3}{2EI} &amp; \frac{L^3}{6EI} \ 0 &amp; 1 &amp; L &amp; \frac{L^2}{2EI} \ 0 &amp; 0 &amp; 1 &amp; L \ 0 &amp; 0 &amp; 0 &amp; 1 \end{bmatrix} )</td>
<td>( L = ) element length ( E = ) Young modulus of the shaft material ( I = ) moment of inertia of area of the shaft cross-section (for the bending)</td>
</tr>
<tr>
<td>Element of guide bearing (without mass)</td>
<td>( T_n = \begin{bmatrix} 1 &amp; 0 &amp; 0 &amp; 0 \ 0 &amp; 1 &amp; 0 &amp; 0 \ 0 &amp; 0 &amp; 1 &amp; 0 \ -k &amp; 0 &amp; 0 &amp; 1 \end{bmatrix} )</td>
<td>( k = ) rigidity coefficient</td>
</tr>
</tbody>
</table>

By covering the entire shaft line, from one end to another, one can relate the state vectors of the shaft line ends by the product of all elementary transfer matrices. For instance, considering the discrete system of Fig. 2, one can write \( S_{14} = T S_1 \), where \( T \) is the product of all elementary transfer matrices, from station 14 to station 1. \( T \) is named “total transfer
matrix”, and is of the same order of the elementary transfer matrices, i.e., 4 by 4 in the case of bending vibrations. Therefore, the total transfer matrix order does not depend on the number of degrees of freedom of the discrete system.

For hydrogenerator shaft lines, a common boundary condition is “free-ends”, in which the bending moment, \( M \), and the shear force, \( Q \), assume zero value in the state vectors at the shaft line extremes. Denoting by \( S_i \) the state vector at the first station and \( S_n \) the state vector at the last station of an arbitrary shaft line, with total transfer matrix \( T \), one can write:

\[
S_n = TS_0
\]  

or

\[
\begin{bmatrix}
Y \\
\theta \\
M \\
Q
\end{bmatrix}_n =
\begin{bmatrix}
T_{11} & T_{12} & T_{13} & T_{14} \\
T_{21} & T_{22} & T_{23} & T_{24} \\
T_{31} & T_{32} & T_{33} & T_{34} \\
T_{41} & T_{42} & T_{43} & T_{44}
\end{bmatrix}
\begin{bmatrix}
Y \\
\theta \\
M \\
Q
\end{bmatrix}_0
\]

where \( T_{ij}, i, j = 1, \ldots, 4 \), are the total transfer matrix elements.

By imposing the free-ends conditions, \( M_n = M_0 = 0 \) and \( Q_n = Q_0 = 0 \), Eq. (9) turns:

\[
\begin{bmatrix}
Y \\
\theta \\
M \\
Q
\end{bmatrix}_n =
\begin{bmatrix}
T_{11} & T_{12} & T_{13} & T_{14} \\
T_{21} & T_{22} & T_{23} & T_{24} \\
T_{31} & T_{32} & T_{33} & T_{34} \\
T_{41} & T_{42} & T_{43} & T_{44}
\end{bmatrix}
\begin{bmatrix}
Y \\
\theta \\
M \\
Q
\end{bmatrix}_0
\]

The 3rd and 4th lines of the previous matrix equation give a homogeneous subsystem:

\[
\begin{align*}
0 &= T_{31} Y_0 + T_{32} \theta_0 \\
0 &= T_{41} Y_0 + T_{42} \theta_0
\end{align*}
\]  

or

\[
\begin{bmatrix}
0 \\
0
\end{bmatrix} =
\begin{bmatrix}
T_{31} & T_{32} \\
T_{41} & T_{42}
\end{bmatrix}
\begin{bmatrix}
Y \\
\theta
\end{bmatrix}_0
\]

The condition for non-trivial solutions for Eq. (12) is the singularity of the sub-matrix in it. That is:

\[
T_{31} T_{42} - T_{32} T_{41} = 0
\]

One should remember that the total transfer matrix is obtained by the product of all elementary transfer matrices, naturally including the mass matrices of the discrete shaft line. Consequently, some of the \( T_{ij} \) are function of \( \omega \) – the angular frequency of the bending vibration, which appears in Eq. (3). Thus, the natural frequencies of the bending vibration are the values of \( \omega \) that satisfy Eq. (13). The first value of \( \omega \) that satisfies Eq. (13) corresponds, then, to the first bending critical speed.

One simple approach for solving this problem is to assume progressive values for \( \omega \) (in a convenient search range) and successively calculate the total transfer matrix and the associated sub-determinant of Eq. (13). Then, these sub-determinant values are plot against \( \omega \) and this plot – \( T_{31} T_{42} - T_{32} T_{41} = f(\omega) \) – will indicate the solutions of Eq. (13), i.e., the bending natural frequencies of the shaft line.

This routine was properly implemented in a computer program by using the MatLab™ language, which is particularly suitable for handling matrices. As a search range for the first critical speed, one may adopt from 0 to \( \sim 3 \) times the turbine maximum runaway speed, since in most cases with industrial hydrogenerators the first critical speed lies within this interval.

4. EXAMPLES OF APPLICATION

By using the presented methodology, the bending critical speeds of two industrial hydrogenerators will be evaluated in this section. The calculated natural frequencies will be compared with those ones informed by the equipments’ manufacturers. This comparison is a good way for checking the effectiveness of the developed calculation tool.
4.1. Hydrogenerator 1: low head Francis turbine with two guide bearings shaft line

This example refers to a large size, low head, Francis type hydrogenerator, with two guide bearings vertical shaft line. This generating unit, designed by a multinational manufacturer of power equipments, would be installed in a run-of-river hydropower station at Brazil’s south region. The turbine rated conditions are: net head = 43.0 m, speed = 144 rpm, power output = 59.6 MW. The turbine maximum runaway speed is 295 rpm. Figure 6 illustrates this machine. In this figure, one can identify the basic elements on the shaft line: rotors, shafts and bearings. Regarding the guide bearings, one is on the generator lower bracket (combined with the thrust bearing) and the other one is on the turbine head cover. Table 2 shows the hydrogenerator 1 design data supplied by its manufacturer.

![Figure 6. Francis type hydrogenerator, with two guide bearings shaft line (“umbrella” arrangement).](image)

Table 2. Design data of hydrogenerator 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass of generator rotor</td>
<td>$136 \times 10^3$ kg</td>
</tr>
<tr>
<td>Mass of turbine runner</td>
<td>$32.5 \times 10^3$ kg</td>
</tr>
<tr>
<td>Mass of turbine shaft</td>
<td>$16.0 \times 10^3$ kg</td>
</tr>
<tr>
<td>Mass of generator shaft</td>
<td>$14.2 \times 10^3$ kg</td>
</tr>
<tr>
<td>Young’s modulus of the shafts material (forged carbon-steel)</td>
<td>206 GPa</td>
</tr>
<tr>
<td>Generator guide bearing rigidity</td>
<td>$6.54 \times 10^8$ N/m</td>
</tr>
<tr>
<td>Turbine guide bearing rigidity</td>
<td>$1.40 \times 10^9$ N/m</td>
</tr>
<tr>
<td>Outer diameter of the shafts</td>
<td>800 mm</td>
</tr>
<tr>
<td>Inner diameter of the shafts</td>
<td>300 mm</td>
</tr>
</tbody>
</table>

For the hydrogenerator 1, three discrete models have been tried, as sketched in Fig. 7. The first model (Fig. 7a) does not consider the shaft weights, but only the rotor weights; the second one (Fig. 7b) takes into account the shaft weights as concentrated masses in two single points; the third model (Fig. 7c) distributes the weights a little better through the shaft line.

Considering the elements indicated in Fig. 7, the total transfer matrix $T$ of each shaft line model is given by:

$$
\begin{align*}
\text{Fig. 7a} \Rightarrow T &= T_{p,7,6} T_{c,6,5} T_{m,5,4} T_{c,4,3} T_{m,3,2} T_{c,2,1} T_{p,1,0} \\
\text{Fig. 7b} \Rightarrow T &= T_{p,11,10} T_{c,10,9} T_{m,9,8} T_{c,8,7} T_{p,7,6} T_{c,6,5} T_{p,5,4} T_{c,4,3} T_{m,3,2} T_{c,2,1} T_{p,1,0} \\
\text{Fig. 7c} \Rightarrow T &= T_{p,13,12} T_{c,12,11} T_{m,11,10} T_{c,10,9} T_{p,9,8} T_{c,8,7} T_{p,7,6} T_{c,6,5} T_{p,5,4} T_{c,4,3} T_{m,3,2} T_{c,2,1} T_{p,1,0} 
\end{align*}
$$

By running the developed computer code for each shaft line model, it is obtained the results in Fig. 8. A closer look, by zooming in the computer screen, reveals the first and second critical speeds. These values are in Tab. 3, where the first bending critical speed of each discretized shaft line is compared with the value given by the manufacturer.
Figure 7. Discrete models for the hydrogenerator 1 shaft line. (a) only the rotor weights; (b) shaft weights as concentrated masses in two single points; (c) shaft weights distributed a little better through the shaft line.

Figure 8. Results for hydrogenerator 1 shaft line. (a) first model; (b) second model; (c) third model.
Table 3. Bending critical speeds of hydrogenerator 1.

<table>
<thead>
<tr>
<th></th>
<th>1st critical speed (rpm)</th>
<th>2nd critical speed (rpm)</th>
<th>Difference of 1st critical speed regarding manufacturer’s guarantee</th>
<th>1st critical speed/runaway speed (runaway speed = 295 rpm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Manufacturer’s guarantee</td>
<td>471.7</td>
<td>n/a</td>
<td>-</td>
<td>1.60</td>
</tr>
<tr>
<td>Results with first model</td>
<td>494.2</td>
<td>1138</td>
<td>+ 4.8%</td>
<td>1.67</td>
</tr>
<tr>
<td>Results with second model</td>
<td>489.3</td>
<td>1128</td>
<td>+ 3.7%</td>
<td>1.66</td>
</tr>
<tr>
<td>Results with third model</td>
<td>483.4</td>
<td>1047</td>
<td>+ 2.5%</td>
<td>1.64</td>
</tr>
</tbody>
</table>

These results show that a relatively small number of elements can lead to good evaluations. As the refinement of the shaft line discretization is improved, the difference between the evaluated first critical speed and manufacturer’s guarantee ("target value") is reduced. The third model (Fig. 7c), which presents a better weight distribution, led to only 2.5% difference. The last column (1st critical speed/runaway speed) is a common “safety coefficient” for checking the dynamic stability of the shaft line. Usually, the owner’s engineering establishes a criterion for the minimum ratio (1st critical speed/runaway speed), like 1.15, 1.20 or 1.25. So, the hydrogenerator manufacturer should design the shaft line to fulfill this condition – and also many other criteria not mentioned in this work.

Further, one can perceive that the influence of the shafts weights is reduced in comparison with that of the rotors weights, which is ordinary for hydrogenerators. So, a very accurate discretization for the shafts weights is not mandatory for achieving reliable results. For instance, the first model (Fig. 7a), which simply does not account the shafts weights, gives only 4.8% difference regarding the manufacturer’s guarantee for the first critical speed.

4.2. Hydrogenerator 2: high head Pelton turbine with three guide bearings shaft line

This example refers to a medium size, high head, 5-jets, Pelton type hydrogenerator, with three guide bearings vertical shaft line. This generating unit, designed again by a multinational manufacturer of power equipments, would be installed in a run-of-river hydropower plant at the Peruvian Andes Mountain. The turbine rated conditions are: net head = 852m, speed = 720 rpm, power output = 57.1 MW. The turbine maximum runaway speed is 1270 rpm. Figure 9 illustrates the mechanical arrangement of the machine. In this figure, one can observe the basic elements on the shaft line. Regarding the guide bearings, one is on the generator upper bracket (combined with the thrust bearing), other one is on the generator lower bracket and the last one is on the turbine casing. Table 4 shows the hydrogenerator 2 design data supplied by its manufacturer.

![Figure 9. Pelton type hydrogenerator, with three guide bearings shaft line.](image)
Table 4. Design data of hydrogenerator 2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass of generator rotor</td>
<td>$53 \times 10^3$ kg</td>
</tr>
<tr>
<td>Mass of turbine runner</td>
<td>$3.2 \times 10^3$ kg</td>
</tr>
<tr>
<td>Mass of turbine shaft</td>
<td>$6.3 \times 10^3$ kg</td>
</tr>
<tr>
<td>Mass of generator shaft</td>
<td>$21 \times 10^3$ kg</td>
</tr>
<tr>
<td>Young’s modulus of the shafts material (forged carbon-steel)</td>
<td>206 GPa</td>
</tr>
<tr>
<td>Generator upper guide bearing rigidity</td>
<td>$1.23 \times 10^9$ N/m</td>
</tr>
<tr>
<td>Generator lower guide bearing rigidity</td>
<td>$1.64 \times 10^9$ N/m</td>
</tr>
<tr>
<td>Turbine guide bearing rigidity</td>
<td>$1.23 \times 10^9$ N/m</td>
</tr>
<tr>
<td>Outer diameters of the generator shaft</td>
<td>1000.550 mm</td>
</tr>
<tr>
<td>Outer diameter of the turbine shaft</td>
<td>550 mm</td>
</tr>
<tr>
<td>Inner diameter of the shafts</td>
<td>300 mm</td>
</tr>
</tbody>
</table>

For the hydrogenerator 2, only one discrete model has been tried, with a convenient weight division on the shaft line, as sketched in Fig. 10.

Figure 10. Discrete model for hydrogenerator 2 shaft line.

Considering the elements indicated in Fig. 10, the total transfer matrix $T$ of the shaft line model is given by:

$$ T = T_{m \_14 \_13} T_{r \_13 \_12} T_{r \_12 \_11} T_{p \_11 \_10} T_{r \_10 \_9} T_{m \_9 \_8} T_{r \_8 \_7} T_{p \_7 \_6} T_{r \_6 \_5} T_{p \_5 \_4} T_{r \_4 \_3} T_{m \_3 \_2} T_{r \_2 \_1} T_{p \_1 \_0} $$

(15)

By running the calculation tool, it is obtained the result in Fig. 11. A closer look, by zooming in the computer screen, reveals the first and second critical speeds. These values are in Tab. 5, where the evaluated first bending critical speed is compared with the value given by the manufacturer.

Figure 11. Result for hydrogenerator 2 shaft line.
Table 5. Bending critical speeds of hydrogenerator 2.

<table>
<thead>
<tr>
<th></th>
<th>1st critical speed (rpm)</th>
<th>2nd critical speed (rpm)</th>
<th>Difference of 1st critical speed regarding manufacturer’s guarantee</th>
<th>1st critical speed/runaway speed (runaway speed = 1270 rpm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Manufacturer’s guarantee</td>
<td>1588</td>
<td>n/a</td>
<td>-</td>
<td>1.25</td>
</tr>
<tr>
<td>Present results</td>
<td>1568</td>
<td>2340</td>
<td>– 1.3 %</td>
<td>1.23</td>
</tr>
</tbody>
</table>

This result shows that a suitable discretization for the shaft line, even with relatively few elements, can provide good accuracy evaluations. In this example, the calculated first bending critical speed is only 1.3% lower than the manufacturer’s guarantee. Thus, for quick assessments on the hydrogenerator shaft line arrangement, the developed calculation tool is useful, providing reliable results.

5. CONCLUDING REMARKS

The Transfer Matrix Method for bending vibration analysis has been implemented in a MatLab™ language computer code. This program is a simple, fast and reliable calculation tool for the evaluation of bending critical speeds of hydrogenerator shaft lines.

The obtained results for the first bending critical speed of some industrial hydrogenerators are in good agreement with the data provided by the equipments’ manufacturers, even when the shaft line discrete model presents relatively few elements.

Thus, the present methodology is capable to aid the engineers in quick investigations on suitable arrangements for hydrogenerator shaft lines.

6. ACKNOWLEDGEMENTS

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7. REFERENCES


8. RESPONSIBILITY NOTICE

The authors are the only responsible for the printed material included in this paper.