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CON10-2342 - MODELING GAS TRANSMISSION LINES TAKING INTO ACCOUNT RANDOM FAILS

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Abstract : *Gas transmission lines are known to be an efficient way of transporting gas through long distances, from production wells to consumer centers. As the consumer market begins to grow rapidly, new production wells come into operation. As a result, an expressive increase in the gas network is experienced. To accommodate this new scenario, complex operational management of the whole system is required in order to ensure gas delivery to all markets. To properly address this problem in suitable and simple way, it is presented in this paper a mathematical modeling which aims to forecast the capability of an existing gas network in attending the gas demand of whole market. The model is based on the mass balance equation and takes into account, in a suitable fashion, pressure and flow rate as well as physical restrictions of the gas lines. To give the model a more realistic behavior, random failure has been taken into account in components such as compressors, gas processing units and wells. Thus, by specifying the mean time between failures and the mean time to repair, the impact of the failures are properly accounted for in the simulation. Given the production capacity, the infrastructure of the network and the gas demand of each consumer market, the model predicts the behavior of the whole distribution system, verifying automatically all the associated constraints. Important features such as packing and unpacking phenomena in the gas network are considered, enabling the model as a promising tool in the task associated with operational management.*

Keywords: *gas networks; distribution systems; linear programming; random failure; failure rate*

1. INTRODUCTION

Gas transmission networks are systems that combine different ways of transporting gas through long distances, from production wells to consumer centers. As the consumer market begins to grow rapidly new production wells come into operation. As a result, an expressive increase in the gas network is experienced. To accommodate this new scenario, complex operational management of the whole system is required in order to ensure gas delivery to all markets.

In a complex branched gas network, decision making regarding to gas deliverability to a market is in general not an easy task, since the cost associated with the gas production depends on each production well (Mokhatab *et al*., 2006). Moreover, the cost of the gas to a particular market depends on the distance traveled and the specific route the gas has traveled in the network (McAllister, 2005).

As a first step to address this problem, it is presented in this paper a simple mathematical modeling which aims to forecasting the capability of an existing gas network in attending the gas demand of whole market. The model is based on the mass balance equation and takes into account, in a suitable fashion, pressure and flow rate as well as compressor stations restrictions of the gas lines. Given the production capacity, the infrastructure of the network and the gas demand of each consumer, the model predicts the behavior of the whole distribution system, verifying automatically all the associated constraints.

Among others, important features such as packing and unpacking phenomena in the gas network are accurately described enabling the model as a promising tool in the task associated with operational management.

2. PROBLEM DESCRIPTION

A gas transmission system is a complex network which aims to delivering gas from the production wells to the consumer markets. To achieve this task, different infrastructures can be used by employing a variety of equipments, routes and transport media as well as the random failures associated with each component. If these items have been chosen, then the infrastructure is established and operational restrictions have automatically been imposed in the network. As a result, once the gas production capacity in a scenario has been specified, question arises as to the capability of the given infrastructure in attending the prescribed gas demand of the consumer market. When the gas demand is not fully attended, it also becomes important to precisely identify the consumers, the deficit and also the periods for the gas shortage.

No matter how complex the network can effectively be, the components of the infrastructure can be grouped into four distinct classes: production/processing, stock, transport and consumption. Typical components from the production/processing class are the associated or non-associated gas wells and the units of gas processing. As components belonging to the consumption class, one may cite the industries, the thermo-electrical power plants and the cities as a whole. To sum up, given the infrastructure of a gas network, its gas production capacity and its gas demand and also the parameters associated with the random failures of the components, the problem in analysis consists in properly determine the temporal evolution of the effective production and consumption in the whole network.

3. FAILURE OF THE COMPONENTS

Failures of components usually cause production losses, which in turn affect the operating performance of the systems as a whole. In complex gas distribution networks the security of supply to the consumers (as thermo plants, industries and cities), may be seriously affected by component failures, depending on how frequent they occur and how long they last. Even shortfalls may imply on heavy fines for the gas suppliers.

In the proposed modeling, focus was kept on functional random failures. Functional failure is considered here as the inability of an equipment or component to meet a desired standard of performance. This concept covers complete and partial loss of function, resulting in losses of gas production or gas supply to the consumers. Random failure is characteristic of failures caused by random events, and is defined by the assumption that the rate λ at which the component fails is constant and independent of its age. The frequently used Bathtub Distribution Curve, also known as mortality curve (Smith, 1976), seeks to describe the variation of failure rate λ of components during their life. The middle portion of that curve is referred to as the useful life and it is assumed that failures exhibit a constant failure rate, that is to say they occur at random. The constant failure rate model for continuously operating systems, as is the case of gas networks, leads to an exponential distribution of the failure probability density function (pdf) (Lewis, 1994).

$$pdf = f(t) = \lambda\, e^{-\lambda t} \tag{1}$$

Similarly, the $F(t)$ function, also known as Cumulative Distribution Function (cdf), and $R(t)$ function (or reliability function) can also be expressed in terms of the failure rate λ, as follows:

$$cdf = F(t) = P\,(T \le t) \text{ (probability that failure takes place at a time } T \text{ less than or equal to } t\text{)} \tag{2}$$

The reliability $R(t)$, defined as the probability that a component will perform properly for a specified period of time (t) under a given set of operating conditions (Lewis, 1994), can be expressed as:

$$R(t) = P\,(T > t) \tag{3}$$

in which $R(t)$ expresses the probability that a component operates without failure for a length of time t (operating time or mission life), and T is the time to component failure, usually referred to as time to fail or TTF.

Assuming λ as a constant, we have (Lewis, 1994):

$$R(t) = e^{-\lambda t} \tag{4}$$

From equations (2), (3) and (4) one can write:

$$cdf = F(t) = 1 - R(t) = 1 - e^{-\lambda t} \tag{5}$$

To give the model a more realistic operating behavior, random failures have been taken into account for some selected critical facilities of the gas network, such as natural gas processing units (NGPU), production wells (PW) and compressor stations (CS). In the modeling, such facilities were considered as individual components.

Thus, by estimating the failure rate λ and the mean time to repair (MTTR) of those components, the impact of the failures is properly accounted for in the simulations. Estimates of λ and MTTR were based on collected data from similar facilities and international failure data (OREDA, 2002). For each component of the system the following estimate for λ was used:

$$\lambda = \frac{number\ of\ reported\ failures}{total\ time\ in\ service} \tag{6}$$

Once the component failure rate λ has been defined, the cumulative distribution function (cdf) of each component can be found by Eq. (5). TTFs can also be determined by taking the inverse of Eq. (5). To simulate the occurrence of unscheduled failures in time (TTFs), a random number between 0 and 1 is generated in a simulator and used to sample against the cdf curve generated for each component (see Figure 1). The occurrence of an event causes the system to sample the distribution once more to generate the next point in time for failure of the relevant component. In this manner, the successive times to failures (TTFs) of each component are generated. Progress of the simulation is in steps, from the occurrence of one event to the occurrence of the next until the simulated time exceeds the specified life time of the system being modeled.

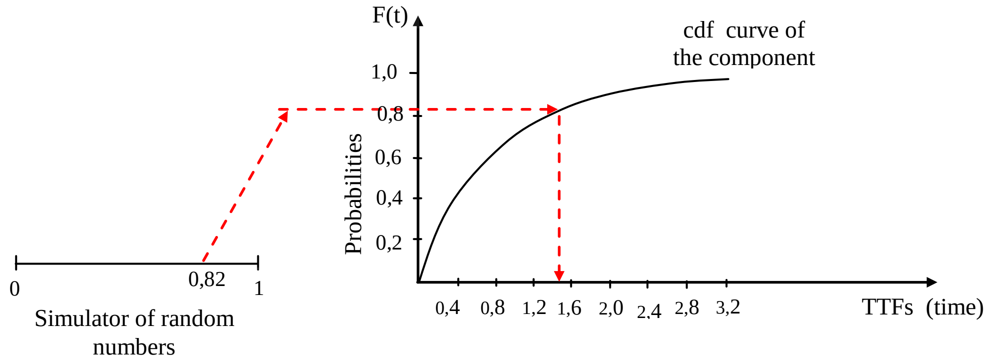

Figure 1. Determining of TTFs (time to fail) of each component - graphic representation

The overall duration of the individual failure events, referred to as time to repair (TTR), was estimated considering the total time the component was out of operation, which includes the required time to mobilize maintenance personnel and spares prior starting repair, added to the active repair time. For each class of component it was determined a mean time to repair (MTTR). Figure 2 ilustrates graphically the parameters TTF and MTTR, as well as the impact of failures events upon the gas flow rate passing through a component. The successive TTFs are designated by TTF_1, TTF_2, TTF_3. On a selected time scale the duration of the failure events of each component are represented by the intervals (TTF_1, TTF_1 + MTTR), (TTF_2, TTF_2 + MTTR) and ((TTF_3, TTF_3 + MTTR), respectively. The term CLF expresses the capacity loss at failure.

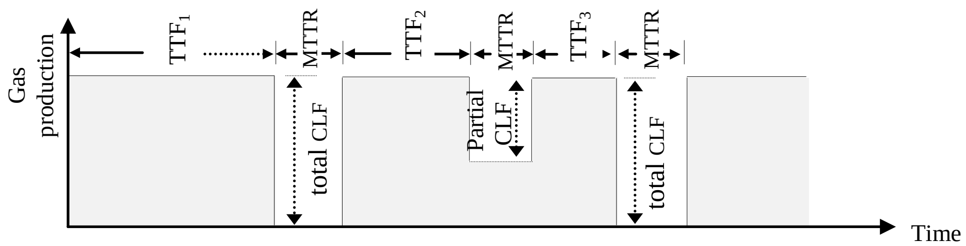

Figure 2. Impact of failures events upon the gas flow passing through a component.

4. MATHEMATICAL MODELING

The basic physical principle used to govern the gas motion through the distribution system is the mass conservation law, which must be stated not only globally (for the system as a whole) but also locally (for each component of the system). In fact, if the mass conservation principle is verified for each component of the system, then it is automatically satisfied for the system as a whole. As the reciprocal is not true, the starting point in the mathematical modeling process is to express the aforementioned principle for each component of the system. In the course of this process, the four classes of components (production/processing, stock, transport and consumption) mentioned in the past section are fully characterized.

4.1. Class Description

Whatever the class the component belongs, every component in the system may be physically characterized by only one general equation which expresses the mass conservation principle. Thus, for each component n of the system consisting of N components we can write, for every time instant t:

$$\frac{d}{dt}V^{(n)} + \sum_{j=1}^{J^{(n)}} S_j^{(n)} - \sum_{i=1}^{I^{(n)}} E_i^{(n)} = P^{(n)} - C^{(n)} \quad for \quad n = 1, \ldots, N \tag{7}$$

in which $V^{(n)}$ stands for the quantity of gas mass expressed in Nm^3, whose definition is presented ahead, inside the *n*-th component at the instant t, $S_j^{(n)}$ and $E_i^{(n)}$ denote the instantaneous mass flow rate which, respectively, leaves and comes into the *n*-th component through the *j*-th inlet and the *i*-th outlet and, finally, $C^{(n)}$ and $P^{(n)}$ represent, respectively, the instantaneous time rate of gas consumption and production taking place in the *n*-th component at the time instant t. All the variables in Eq. (7) are functions of t, the only one independent variable in the problem formulation.

Due to the form as Eq. (7) is written, it becomes evident that the following conditions set below must be satisfied:

$$V^n \geq 0, \quad S_j^{(n)} \geq 0, \quad E_i^{(n)} \geq 0, \quad P^{(n)} \geq 0, \quad C^{(n)} \geq 0 \quad for \quad n = 1, ..., N \tag{8}$$

It is not difficult to realize that not all the components will have all the terms in Eq. (7). As a matter of fact, it is the identification of the null terms in Eq. (7) that will characterize not only the class the component belongs but also its peculiarities within each class. For instance, if the component in consideration belongs to the class production / processing, then $E_i^{(n)} \equiv 0, \ \forall i = 1, \ldots, I^{(n)}$, and $C^{(n)} \equiv 0$. In a similar fashion, components which belong to the class consumption will be characterized by having the following null terms in Eq. (7): $S_j^{(n)} \equiv 0, \ \forall j = 1, \ldots, J^{(n)}$, and $P^{(n)} \equiv 0$.

On the other hand, if the component belongs to the class stock or transport, then one must necessarily have $P^{(n)} \equiv 0$ and $C^{(n)} \equiv 0$ or $P^{(n)} \equiv 0$, respectively. Based on this classification, it is explicitly assumed that the components of the transport class may also stock as well as consume gas. By excluding the components used with the purpose of connecting the other components such as the junctions and the derivations, it is admitted as a basic assumption that the only difference between the components of the stock and transport class is that the components from this last class possess only one inlet and only one outlet, that is, $I^{(n)} = 1$ and $J^{(n)} = 1$. The junction and the derivation, which constitutes the exception to the rule of the transport class, give flexibility in mounting the system by allowing to considering branches of pipelines with multiples inlets and outlets, pipelines of varying diameters and so forth. The derivation component is characterized by assumption as having only one inlet and two outlets, i. e., $V^{(n)} \equiv 0$, $C^{(n)} \equiv 0$, $P^{(n)} \equiv 0$ with $I^{(n)} = 1$ e $J^{(n)} = 2$. Similarly, the junction component has only one outlet and two inlets, i. e; $V^{(n)} \equiv 0$, $C^{(n)} \equiv 0$, $P^{(n)} \equiv 0$ with $I^{(n)} = 2$ e $J^{(n)} = 1$.

To ensure that the mass conservation principle is satisfied globally (for the system as a whole), it is imposed compatibility conditions between the mass flow rates, at the inlet and at the outlet, of the components of the transport class and the mass flow rates of the components connected to them. Without loosing generality, if we admit that there exists *M* components of the transport class (with $M < N$), then there will be 2*M* compatibility equations as follows:

$$\left. \begin{array}{l} E_i^{(m)} - S_j^{(o)} = 0, \ for \ i = 1,2 \ with \ j \in \left\{1, \ldots, J^{(o)}\right\} \\ S_j^{(m)} - E_i^{(o)} = 0, \ for \ j = 1,2 \ with \ i \in \left\{1, \ldots, I^{(o)}\right\} \end{array} \right\} for \ m = 1, \ldots, M \ with \ o \in \{1, \ldots, N - M\}, o \neq m \tag{9}$$

Every variable which appear in Eq. (7) are submitted, for every time instant t, to the additional conditions in the following form:

$$C^{(n)} \leq {}^{max}C^{(n)}\zeta_C^{(n)}, \ V^{(n)} \leq {}^{max}V^{(n)}, \ S_j^{(n)} \leq {}^{max}S_j^{(n)}, \ E_i^{(n)} \leq {}^{max}E_i^{(n)}\zeta_E^{(n)}, \ P^{(n)} \leq {}^{max}P^{(n)} \tag{10}$$

for $n = 1, ..., N$, in which ${}^{max}C^{(n)}$, ${}^{max}V^{(n)}$, ${}^{max}S_j^{(n)}$, ${}^{max}E_i^{(n)}$ and ${}^{max}P^{(n)}$ are, for a given time instant, known positive constants which represent the maximum admissible values for $C^{(n)}$, $V^{(n)}$, $S_j^{(n)}$, $E_i^{(n)}$ and $P^{(n)}$, respectively. The parameters $\zeta_C^{(n)}$ and $\zeta_E^{(n)}$ assume either the values 1 or 0 and are introduced to take the random failures described in section 3 into account. They are equal to 1, except during the time intervals $(TTF^{(n)}, TTF^{(n)} + TTR^{(n)})$ when they assume the value 0 in order to mimic the complete failure of the n component. From the physical view point, these upper bounds stand for operational restrictions associated with each component. For instance, ${}^{max}V^{(n)}$ represents the maximum stock capacity of a component, ${}^{max}P^{(n)}$ the maximum time rate production of gas, ${}^{max}C^{(n)}$ the maximum time rate consumption of gas (also named demand) and finally, ${}^{max}S_j^{(n)}$ and ${}^{max}E_i^{(n)}$ the maximum mass flow rates in the components of the transport class. If the component is a pipeline, then

${}^{max}S_1^{(n)} = {}^{max}E_1^{(n)} = {}^{max}Q^{(n)}$, in which ${}^{max}Q^{(n)}$ is the maximum mass flow rate in the pipeline. Besides the specification of the upper bounds involved in Eq. (7), initial conditions are required with respect to the initial quantities of gas (say at time $t = 0$) of each variable $V^{(n)}$,

$$V^{(n)} = V_0^{(n)} \quad at \quad t = 0 \quad for \quad n = 1, \dots, N \tag{11}$$

With exception of the pipeline component, the initial quantity of gas in all components of the system is assumed to be an input. The approach used to estimate the initial quantity of gas in a pipeline based on the process variables will be described in section 4.3. Once the basic features of each of the four classes have been defined, we are able to enumerate the set of fundamental assumptions which characterize the nature of the gas which is transported through the whole system. To take the random fails into account described in section 3,

4.2. Basic Assumptions

To fully describe the proposed model, some fundamental assumptions enumerated ahead are assumed with regard to the constitutive nature of the gas:

1) The gas density, with respect to the air at the normal conditions of pressure and temperature, is for every time instant (including the initial at $t = 0$) and in every component of 0.725, which is equivalent to admit that the molecular weight is constant and equal to 21.

2) The compressibility factor of the gas can be approximated by the following expression of the CNGA – California Natural Gasoline Association (McAllister, 2005):

$$Z = \hat{Z}\left(p, T\right) := \frac{1}{1 + \dfrac{517060 \times 10^{1,294125}\, p}{T^{3,825}}} \tag{12}$$

with T expressed in degrees Kelvin and p, the gauge pressure, expressed in kgf/cm^2.

3) Whatever the system is, the gas temperature is assumed to be uniform and constant, being equal to 20^{o}C.

4) For all effects, it is assumed that the gas obeys the following equation of sate, in which V represents the volume, n the number of moles and R the universal constant of the gases.

$$pV = nZRT \tag{13}$$

5) The unit of mass adopted for every component is the normal cubic meter, Nm^3. By definition, it is the mass of gas in a 1 m^3 at a reference temperature T_{ref} = 20^{o}C and at a reference absolute pressure p_{ref} =1 atm.

6) The superior caloric power of the gas, PC_S, is constant and equal to 9500 kcal/Nm^3. According to the classification of the Brazilian Agency of Petroleum ANP, the gas is considered a medium natural gas with PC_S within the interval 8800 to 10200 kcal/Nm^3. Moreover, the superior caloric power is assumed to be 90 % do PC_S.

Based upon the aforementioned assumptions, it can be verified that the compressibility factor in the reference state, $Z_{ref} = \hat{Z}(p = 0\text{kgf/cm}^2,\ T = 293.15\text{K})$, obtained from Eq. (12), is equal to 1.

With the assumptions set before one can estimate the initial quantity of gas inside the pipeline, i. e., that is its initial condition. In the next section we present a simple way to estimate the amount of gas inside the pipe according to its operational parameters.

4.3. Initial Condition for the Pipeline

The procedure used to estimate the initial quantity of gas inside the pipeline, as well as its minimum and maximum stock capacities, is presented next. The quantity of gas, which obeys the assumptions set before in the past section, that can be stored in a pipeline of internal diameter D and length L can be expressed according to:

$$V = \hat{V}\left(Z_M, p_M, T\right) := V_0 \frac{Z_{ref}}{Z_M} \frac{p_M}{p_{ref}} \frac{T_{ref}}{T} \tag{14}$$

in which $V_0 = \dfrac{\pi D^2 L}{4}$ and $Z_M = \hat{Z}_M\left(p_M, T_{ref}\right)$ is the compressibility factor for the mean pressure p_M within the pipeline segment.

If the pipeline is in the shut-in condition (null flow rate or, equivalently, in static condition), the pressure p_M is the absolute and uniform pressure inside the pipeline. On the other hand, if it is running (or operates under dynamic condition), then the mean pressure p_M inside the pipeline can be estimated by the following expression (Mokhatab, 2006). p_1 e p_2 represent the absolute pressures at the inlet and at the outlet of the pipeline segment, respectively.

$$p_M = \frac{2}{3}\left(p_1 + p_2 - \frac{p_1 p_2}{p_1 + p_2} \right) \tag{15}$$

By knowing the pipeline geometrical dimensions, the inlet pressure and also the mass flow rate *Q*, then the outlet pressure can be estimated based on the Weymouth formulae (Mokhatab et al., 2006):

$$Q = 453\frac{D^{8/3}}{L^{0.51}}\left[p_1^2 - p_2^2 \right]^{0.51} \tag{16}$$

in which *Q* is given in Nm^3, *D* stands for the inside diameter (in inches), *L* is the pipeline length expressed in km and p_1 and p_2 are the pressures in kgf/cm^2.

By admitting that *Q*, *D*, *L* and p_1 are taken as input data for the pipeline, then Eq. (16) can be used to estimate p_2. It is worthwhile noting that one must have $p_2 \le p_1$. If $p_2 < p_1$ then, the pipeline segment operates dynamically. If, on the other hand, $p_2 = p_1$ is in shut-in condition. With p_1 and p_2 we compute via Eq. (15) the pressure p_M and, in the sequence, $Z_M = \hat{Z}\left(p_M, T_{ref}\right)$ through Eq. (12). Finally, with p_M and Z_M we compute the initial quantity of gas in the pipeline segment $V = \hat{V}\left(Z_M, p_M, T_{ref}\right)$ by using Eq. (14).

The minimum and maximum quantities of gas that can be stored in the pipeline segment can be estimated based upon the minimum p_{min} and maximum p_{max} operational pressures. These data are also assumed to be input data for each pipeline segment. The minimum gas quantity in the pipeline is $V^{min} = \hat{V}\left(Z_{min}, p_M = p_{min}, T_{ref}\right)$ in which $Z_{min} = \hat{Z}\left(p_{min}, T_{ref}\right)$. Analogously, the maximum quantity of gas in the pipeline segment is $V^{max} = \hat{V}\left(Z_{max}, p_M = p_{max}, T_{ref}\right)$ in which $Z_{max} = \hat{Z}\left(p_{max}, T_{ref}\right)$.

4.4. Mathematical formulation

The fundamental background required by the proposed model to describe gas transmission networks have been presented in the past sections. However, the use of the mass conservation principle itself is not sufficient to describe the gas motion throughout the network. From the mechanical viewpoint, it would be necessary to consider additionally the momentum conservation principle for each component. However, the use of this principle not only requires the knowledge of a large amount of operational data, in general not easily available, but also is itself not sufficient to fully describe the operandi modus of such networks. To overcome such difficulty, we proposed an alternative and simple way to describe the network operation, without appealing to the momentum conservation principle. As we shall see next, the motion of gas throughout the network is emulated by maximizing a linear functional suitably postulated, subjected to the restrictions imposed by the mass conservation principle Eq.(7-8), by the compatibility equations Eq.(9), by the effective capacities given by Eq.(10) and also the initial conditions expressed by Eq.(11).

Formally, by considering that ${}^{max}V^{(n)}$, ${}^{max}S_j^{(n)}$, ${}^{max}E_i^{(n)}$ are known quantities and that ${}^{max}C^{(n)}$ and ${}^{max}P^{(n)}$ are prescribed for each time instant, for $n = 1,\dots,N$, $i = 1,\dots,I^{(n)}$ and $j = 1,\dots,J^{(n)}$ (when pertinent), then the mathematical problem which describes the gas motion in the system in consideration consists to find $V^{(n)}$, $S_j^{(n)}$, $E_i^{(n)}$, $C^{(n)}$ and $P^{(n)}$ for $n = 1,\dots,N$, $i = 1,\dots,I^{(n)}$ and $j = 1,\dots,J^{(n)}$ (when pertinent), subjected to Eq.(7-11). By inspecting the system of equations formed by Eq. (7) and Eq. (9), along with Eq. (11), we can see that it is undetermined since there are more unknowns than equations. To allow that this initial value problem has a solution which consistently represents an actual operation of the gas network, it becomes necessary to impose additional condition(s). This is done by choosing a suitable linear functional of some variables of the problem, which has to be maximized. Thus, the mathematical problem can now be formally formulated as follows:

Given ${}^{max}C^{(n)}$ e ${}^{max}P^{(n)}$ for all time instant t, find $V^{(n)}$, $S_j^{(n)}$, $E_i^{(n)}$, $C^{(n)}$ e $P^{(n)}$ for $n = 1,\dots,N$, $i = 1,\dots,I^{(n)}$ and $j = 1,\dots,J^{(n)}$ (when pertinent) which satisfy Eqs. (1) to (5) and that maximize the linear function $f\left(V^{(n)}, S_j^{(n)}, E_i^{(n)}, C^{(n)}, P^{(n)}\right)$.

Naturally, the choice of f is crucial in order to ensure uniqueness of solution (if it exists!) and, at the same time, to properly describe the actual operation of a gas network. To achieve these goals, we proposed based, on some the practical information available in (Nayyar, 2000), that the linear functional have the following form:

$$f\left(V^{(n)},S_j^{(n)},E_i^{(n)},C^{(n)},P^{(n)}\right):=\sum_{n=1}^{N}\left[p_c^{(n)}C^{(n)}+p_p^{(n)}P^{(n)}+V_2^{(n)}\right] \tag{17}$$

in which $p_c^{(n)}$ and $p_p^{(n)}$ are prescribed functions of the time, with image within the real interval [1,10], and represent the priorities associated with the variables ${}^{max}C^{(n)}$ and ${}^{max}P^{(n)}$ of the component n of the consumption and production/processing classes, respectively.

To assign the model a more realistic behavior, we decompose the variable which represents the quantity of gas $V^{(n)}$ inside the pipeline into two distinct additive parcels, that is $V^{(n)}=V_1^{(n)}+V_2^{(n)}$, with $0\le V_1^{(n)}\le 0.2\ {}^{max}V^{(n)}$ and $0<{}^{min}V\le V_2^{(n)}\le 0.8\ {}^{max}V^{(n)}$, being ${}^{min}V^{(n)}$ and ${}^{max}V^{(n)}$ the minimum and maximum capacities of storing gas inside the pipeline. To emulate the packing and unpacking effects in the pipeline the variable $V_2^{(n)}$ is included in the linear functional given by Eq. (17). Besides the packing/unpacking behavior, this strategy implicitly imposes a mean operational pressure in steady-state around 80% of the maximum allowable operational pressure in the pipeline.

Finally, it is possible to prove that if the totally implicit Euler method is employed to approximate the time derivative in Eq.(7), then the mathematical formulation presented in this section forms a typical problem of linear programming (Luenberger, 1973). To numerically solve this problem we use the well-known SIMPLEX method (Luenberger, 1973).

5. NUMERICAL EXAMPLE

To illustrate the capability of the proposed model in properly describe the gas transmission network, taking random failures into account, a representative numerical example is shown in this section. The example presented ahead aims to illustrate the following features:

a) operational situations in which the effective production becomes inferior to its maximum capacity;

b) gas shortage; a situation in which at least one component of the consumption class is forced to present an effective consume that is inferior to its demand;

c) time variation of the gas stock in the network when the gas production supplants gas demand or is less than it;

d) the packing/unpacking effect in the gas pipeline, highlighting its ability to operate as a regulator element of the network gas supply;

e) and, finally, the net effect of random failures of the components on the system response.

The network simulated in this example has nineteen components and is illustrated in Figure 3. The gas from the wells GW 1 and GW 2 are transported through the pipelines Pipe 1and Pipe 2, which merge before the inlet of Pipe 3. After it has been processed in the nature gas process unit (NGPU), the gas is conveyed through the pipelines Pipes 4, 5, 6 and 7 to the consumer market, which is composed by an industry and a city (see Fig. 3).

To displace the gas throughout the whole network, five major compression stations, named CS 1, CS 2, CS 3, CS 4 and CS 5 are strategically positioned along the route. Since in this scenario there are two components of the class production (GW 1 and GW 2) and two of the class consumption (Industry and City), priorities are attributed to them in order to define the component which should have its production (or consume) reduced when the overall production becomes greater than the overall consumption (or the overall consumption becomes greater than the overall production capacity).

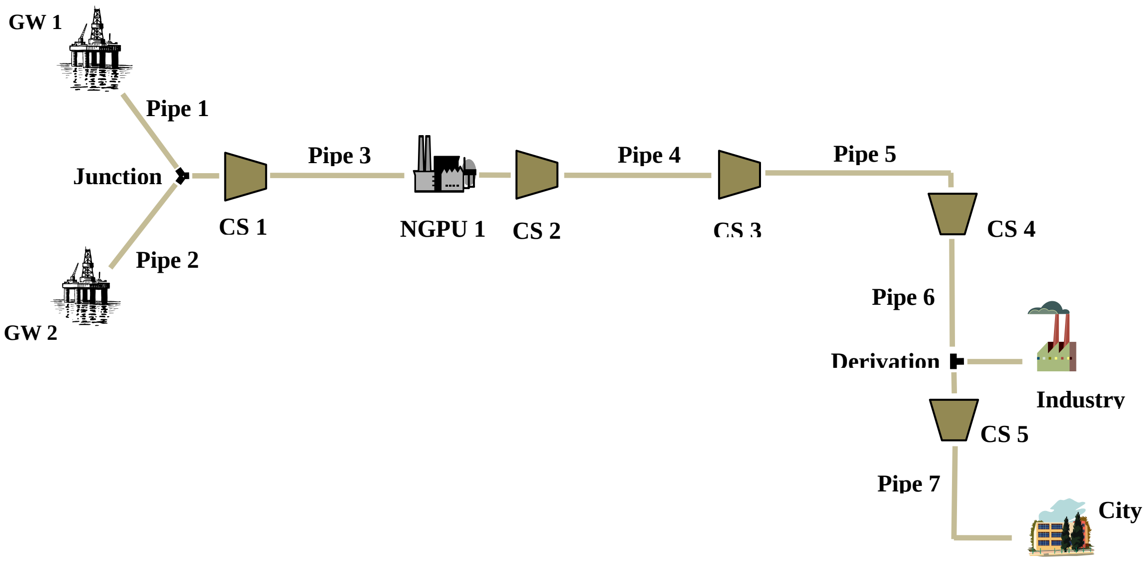

Figure 3. The network and its components.

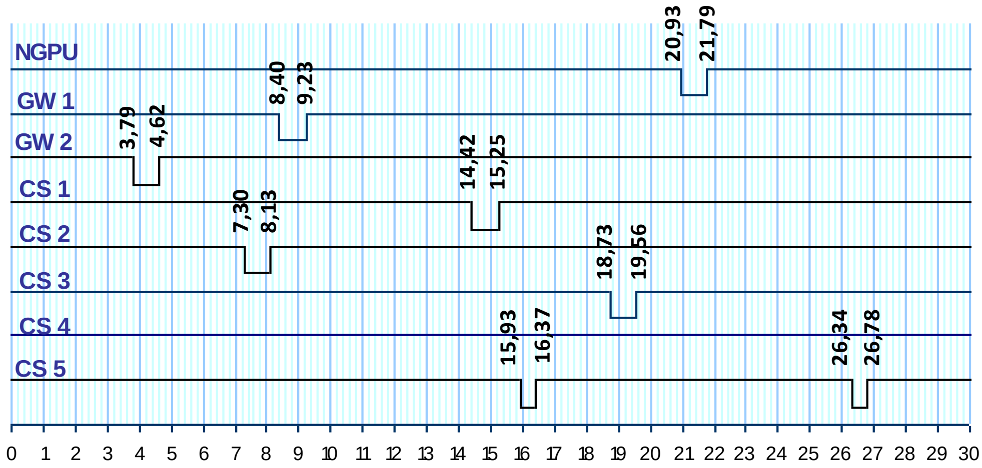

Figure 4. Time table of the failures of components in the network (see Fig.1) for a 30-day period.

In the present simulation, priorities $p_p = 1$ and $p_p = 2$ are attributed to the wells GW1 and GW2, whereas priorities $p_c = 1$ and $p_c = 2$ are attributed to the industry and city, respectively. The gas wells, the compression stations and the natural gas processing unit are susceptible to random failures. The time-table of failures with the time intervals (TTF, TTF+TTR) for each of these components computed as described in section 3 are illustrated in Fig. 4 for the 30-day period of simulation. Note that only the CS4 has not failed during this period and that the CS5 is the unique component to fail twice.

The maximum production capacity of the wells GW1 and GW2 are kept constant and equal to 3.0 MNm^3/day and 2.0 MNm^3/day respectively, during the 30-day period of simulation, as shown in the left graph of Fig. 5. On the other hand, the gas demand of the industry as well as the city, varies during the 30-day period of simulation. The gas demanded by the city is equal to 3.5 MNm^3/day until the 5^{th} day. From that time instant to the 10^{th} day the gas demanded by the city increases linearly to 4.0 MNm^3/day, and soon after, to 4.5 MNm^3/day at the end of the 12^{th} day. From the 12^{th} day until the end of the 17^{th} day it remains constant and equal to 4.5 MNm^3/day. After this abnormal period of gas consume on the city, its demand is linearly reduced to 4.0 MNm^3/day, and then, kept constant until the end of the 30-day period of simulation, as illustrated in the right graph of Fig.5. The complete set of input data for the components of the gas network shown in Fig. 3 are presented in Table 1 at the end of the paper.

Before showing the results of the simulation when failures are taken into account, it is convenient to carry out a simulation in which none of the components is allowed to fail. The response of the network system with no failure is presented in Fig. 5. The effective productions of the wells GW1 and GW2 are shown in the left graph, and the gas demand along with the effective consumptions of the industry and city are depicted in the right graph.

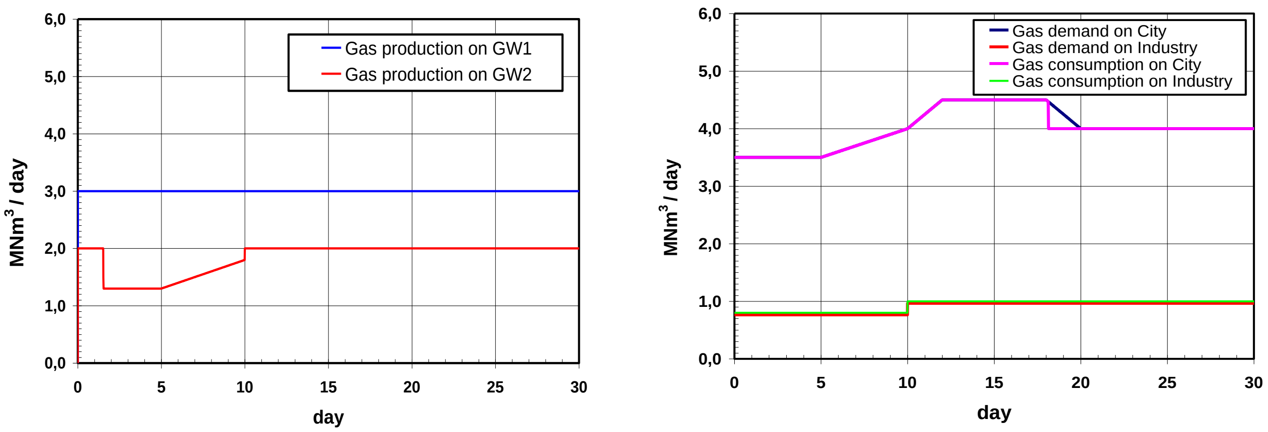

Figure 5. Gas production on the wells GW1 and GW2 (left) and gas demand along with gas consumption on the City and the Industry (right), without taking into account random failures.

Although the whole consumption in the industry and in the city is only 4.3 MNm^3/day in the first two days, the overall well production totalizes 5.0 MNm^3/day during this period. The excess of 0.7 MNm^3/day is used for packing all the pipelines in the network until the end of the 2^{nd} day, when the maximum operating pressures of all the pipelines have been reached and no additional gas is stored within them. From this time instant until the end of the 10^{th} day the overall production becomes equal to the overall demand even in the period between the 5^{th} and the 10^{th} day, when the city gas demand increases steadily to 4.0 MNm^3/day, as it can be seen in the right graph of Fig. 5 (right). Since the

GW2 has a secondary priority with respect to GW1, the effective production of GW2 becomes less than its maximum capacity (2.0 MNm^3/day) from the 2nd to the 10th day. From the 10th to the 18th day, the overall gas demand becomes greater than the overall production (5.0 MNm^3/day), but no gas shortage is noticed in the consumer market until the end of the 18th . During this period the pipelines are unpacking and the gas stored within them are used in order to attend the overall demand. However, from the end of the 18th to the end of the 20th day, being the overall production inferior to the overall consumption, the gas stored in the pipelines is not enough to attend the market. As a result, a gas shortage is observed during this period in the city (the consumption becomes inferior to the gas demand) which totalizes 1.0 MNm^3. Note that it has not been observed gas shortage in the industry since a higher priority had been established to it.

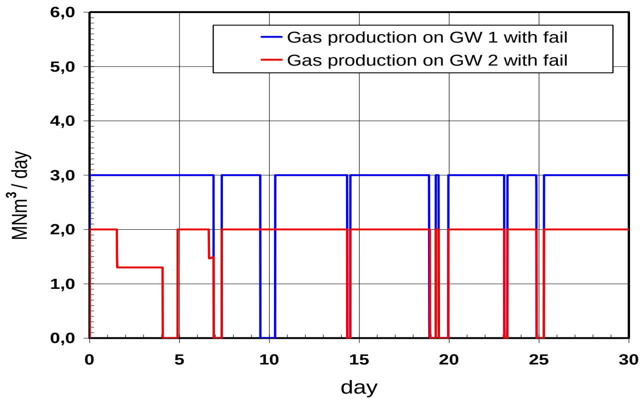

Figure 6. Gas production on the wells GW1 and GW2 taking into account random failures.

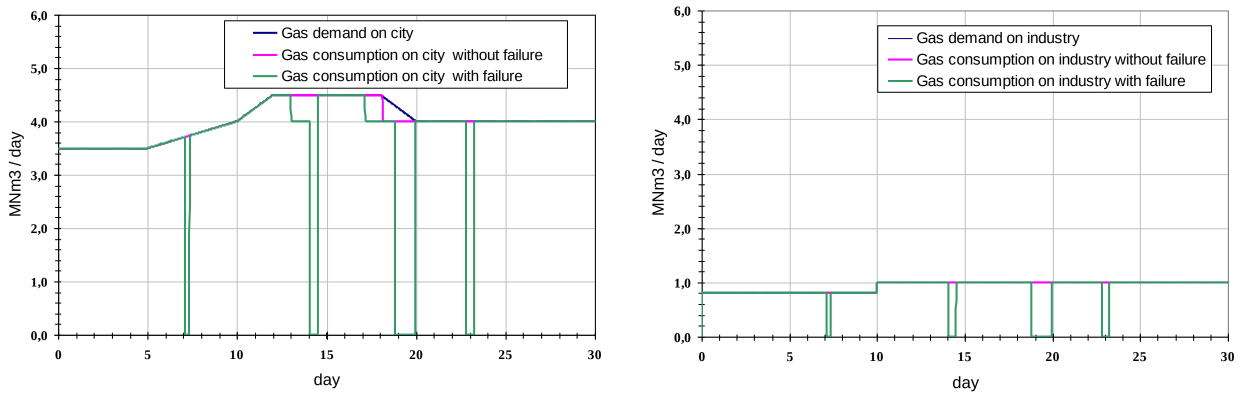

Figure 7. Left graph: gas demand along with gas consumption on City (without and with failures). Right graph: gas demand along with gas consumption on Industry (without and with failures).

To highlight the effects that component failures may introduce into the consumer and production market of a gas network, the past simulation has been carried out with the same input data by considering the time-table of failures shown in Fig. 4. The response of the network system with failure is presented in Fig. 6 and Fig. 7. The effective productions of the wells GW1 and GW2 are shown in Fig. 6 and the effective consumptions of the industry and city are depicted in Fig. 7. Despite of the failures on gas well GW1 and GW2, it can be observed in the graph of Fig. 6 that the failures of the other components, which are located ahead in the network, propagate backward causing the wells to stop producing. This condition is attenuated whenever the gas stock in the pipelines decreases, that is, the average pressure in the pipelines becomes far below its maximum average pressure. On the other hand, it can be noticed in the graphs of Fig. 7 that a failure in the production not necessarily implies in a gas shortage in the consumer market. Note that the failure in the gas well GW2 during the elapsed time interval [3.79, 4.62] has virtually no impact on the consumer market.

In contrast to what seems to be evident, the impact of the failures on the consumer market can not be predicted straightforwardly. Only four of a total of eight failures during the 30-day period have effectively caused shortages in the city an industry, as it can be seen in the right graph of Fig. 7. These shortages are mainly due to the failures in the compression stations CS 1, CS 2 and CS 3. Moreover, the combined effect of failures may worsen even further the shortage in scenarios susceptible to its occurrence. This is what happens in the present example. The shortage of 1.0 MNm^3 observed during the 18th and 20th days in the simulation without failures has increased to nearly 10 MNm^3 during the period delimited by the time interval [17.15, 20] days when the failures are accounted for. In the context of the simulation, this amount is significant showing therefore that random failure of components should be properly taken into account when accurate predictions are sought.

Table 1 - Component input data

Gas wells (GW1, GW2)	GW1 $^{max}P(t \geq 0$ day)	3.0 MNm^3/day , $p_p = 1$
	GW2 $^{max}P(t \geq 0$ day)	2.0 MNm^3/day, $p_p = 2$
Natural Gas Processing Unit (NGPU)	^{max}P	50000 kNm^3/day
	^{max}C	0 kNm^3
	^{max}V	0 kNm^3
	$^{max}S_i$	20000 kNm^3/day
Compression Stations (CS1, CS2, CS3, CS4,CS5)	^{max}C	0 MNm^3
City ($p_c = 2$)	^{max}V	0 MNm^3
	$^{max}C(t \leq 5$ day)	3.5 MNm^3/day
	$^{max}C(t = 10$ day)	4.0 MNm^3/day
	$^{max}C(t = 12$ day)	4.5 MNm^3/day
	$^{max}C(t = 17$ day)	4.5 MNm^3/day
	$^{max}C(t = 20$ day)	4.0 MNm^3/day
	$^{max}C(t = 30$ day)	4.0 MNm^3/day
Industry ($p_c = 1$)	^{max}V	0 MNm^3
	$^{max}C(t < 10$ day)	0.8 MNm^3/day
	$^{max}C(t \geq 10$ day)	1.0 MNm^3/day
Pipelines (Pipe 1, Pipe 2)	L	70 km
	D	8"
	p_1	80 kgf/cm^2
	Q	2.0 MNm^3/day
	$p_{\min}$	20 kgf/cm^2
	$p_{\max}$	100 kgf/cm^2
	^{max}S	4.0 MNm^3/day
Pipeline (Pipe 3)	L	100 km
	D	12"
	p_1	80 kgf/cm^2
	Q	3.5 MNm^3/day
	$p_{\min}$	20 kgf/cm^2
	$p_{\max}$	100 kgf/cm^2
	^{max}S	6.0 MNm3/day
Pipelines (Pipe 4, Pipe 5,Pipe 6)	L	80 km
	D	12"
	p_1	80 kgf/cm^2
	Q	3.5 MNm^3/day
	$p_{\min}$	20 kgf/cm^2
	$p_{\max}$	100 kgf/cm^2
	^{max}S	6.0 MNm^3/day
Pipeline (Pipe 7)	L	50 km
	D	10"
	p_1	70 kgf/cm^2
	Q	2.0 MNm^3/day
	$p_{\min}$	20 kgf/cm^2
	$p_{\max}$	100 kgf/cm^2
	^{max}S	5.0 MNm^3/day

6. CONCLUDING REMARKS

A mathematical model taking into account random failures has been presented in this paper to analyze gas networks. By specifying the infrastructure, the failure probability density function of its components as well as the production capacities and the consumer demands, the model is able not only to predict the occurrence of shortages but also to quantify them. Instead of solving the momentum balance equations to describe the transport of the gas throughout the network, a suitable mathematical linear programming problem is formulated based on the mass conservation principle and the physical restrictions imposed by the network components. A numerical simulation carried out with and without failures has shown the relevance of the failures when accurate predictions are pursued.

7. ACKNOWLEDGEMENTS

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9. RESPONSABILITY NOTICE