ISSN 2176-5480

22nd International Congress of Mechanical Engineering (COBEM 2013)
November 3-7, 2013, Ribeirão Preto, SP, Brazil

RESERVOIR PARAMETERS ESTIMATION AND FLOW RATE HISTORY RECONSTRUCTION USING DOWNHOLE PRESSURE DATA

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Abstract. *The analysis of long-term pressure data, as recorded by permanent downhole gauges (PDG), could contribute to characterize a reservoir and evaluate how their properties change during production. For example, the results of the interpretation of PDG data by simplified analytical models could be used for guidance in the development of the more complex reservoir numerical model. It would be necessary to establish a systematic procedure to analyze PDG data. This procedure would take into account the particularities of non-controlled production, as lack of flow rate records, the uncertainty of the existing records and also the proper nature of the physical phenomenon, which involves the changing in some of the reservoir properties with time. In this work, a nonlinear regression technique was used to analyze long-term pressure data to reconstruct flow rate history and to estimate well/reservoir parameters. Several analytical reservoir models were used, under the assumption of small changes on the fluid and rock properties during the, relatively short, selected flow period. The study focused, particularly, on the determination of the unknown or uncertain values of flow rate and the beginning and duration of production events. A discussion on the influence of the variable choices, parameter constraints and variants of the nonlinear regression method on the final match and convergence rate was presented.*

Keywords: *Well Test Analysis, Reservoir Characterization, Nonlinear Regression, Permanent Downhole Pressure Gauges*

1. INTRODUCTION

A method that has been used for many years to examine the well and reservoir conditions, to infer their properties and to evaluate the feasibility of producing an oil or gas reservoir is to change the well production (or injection) rate and monitoring the behavior of the wellbore flowing pressure as a function of time (Gringarten, 2008). This technique is known as pressure testing or well testing.

In a conventional pressure test, the pressure response due to a controlled variation of flow rate is carefully measured, usually in a short period of time. Analyzing this pressure response, it is possible to estimate well/reservoir parameters, such as permeability, skin factor, average and initial pressures, among others. It can also be useful to develop models to predict the future reservoir behavior and to make decisions to improve its performance using this information. However, to obtain more information about the well and the reservoir, it is necessary to make a continuous monitoring of their conditions. Therefore, in recent years, many wells have been completed with Permanent Downhole Gauges (PDGs). The PDGs are sensors, installed at the bottom of the oil production and water injection wells, which continuously record pressure and temperature. Its original purpose was the operational monitoring of pumps and downhole equipment but over the time it was noted that these registers could be a good source of information about the reservoir.

Data collection done with PDGs is not controlled as in a conventional pressure test. The PDG pressure record is subject to dynamic changes that may occur in the wellbore and the reservoir, such as sudden changes in flow temperature, among others. In addition, due to its high sampling rate, the volume of data recorded is extremely large to be processed, even for modern computers. Beyond these problems, the use of gauges which measure the flow rate on the formation face is not common. In most cases, the flow rates are measured on the surface and in irregular time intervals, which can be up to a week between two consecutive records, and the flow rate history may be incomplete (Athichanagorn *et al.*, 1999).

This study considers that the pressure data recorded by the PDGs have already gone through some stages of treatment. Thus, it is considered that the outliers have been removed. The noise present in the pressure records, which can be caused by different reasons, from tidal effects to measurement errors, have also been removed or minimized (Athichanagorn *et al.*, 1999; Marotti *et al.*, 2009).

After these processing stages, the long term pressure data is ready to be analyzed by a nonlinear regression algorithm. The technique employed is the least squares method with the Levenberg-Marquardt algorithm. This method was applied to different reservoir models, in synthetic and field data, with several production events, in order to reconstruct the flow rate history, to estimate the well and reservoir parameters and to refine the initial estimative of production times.

2. RESERVOIR MODELS

Three of the most commonly used well/reservoir flow models were considered in this study: a well with wellbore storage and skin factor effect in a homogeneous infinite reservoir, a well near a single sealing fault, with wellbore storage and skin factor effect, and a channel reservoir with skin factor effect. These models, with its analytical and semi analytical derivatives, were selected to study the influence of the numerical Laplace inversion method and two types of reservoir boundaries on the nonlinear regression algorithm. The next subsections are brief descriptions of each model.

2.1 Well with wellbore storage and skin factor in a homogeneous infinite reservoir

This is a classic case in petroleum reservoir engineering literature and probably the most widely used for transient pressure analysis. Assuming the basic hypothesis presented in Lee *et al.* (2003), the solution for this model, in Laplace space is given by:

$$\overline{\Delta p_{wf}} = \frac{K_0\left(\sqrt{\frac{u}{bk}}\right) + S\sqrt{\frac{u}{bk}}K_1\left(\sqrt{\frac{u}{bk}}\right)}{u\left\{\frac{k}{a}\sqrt{\frac{u}{bk}}K_1\left(\sqrt{\frac{u}{bk}}\right) + \frac{uC}{qB}\left[K_0\left(\sqrt{\frac{u}{bk}}\right) + S\sqrt{\frac{u}{bk}}K_1\left(\sqrt{\frac{u}{bk}}\right)\right]\right\}}, \tag{1}$$

where $\overline{\Delta p_{wf}}$ is the in Laplace transform of the flowing bottom-hole pressure drop, K_0 and K_1 are the modified Bessel functions of second species, of orders 0 and 1, respectively (Abramowitz and Stegun, 1972), u is the Laplace variable, k is the permeability, q is the flow rate, B is the formation volume factor, S is the skin factor, C is the wellbore storage coefficient, and a and b are the following constants:

$$a = \frac{qB\mu}{2\pi h} \qquad \text{and} \qquad b = \frac{1}{\phi\mu c_t r_{w+}^2}, \tag{2}$$

where h is the reservoir thickness, ϕ is the porosity, c_t is the system compressibility, μ is the viscosity and r_{w+} is the wellbore radius in its external face.

The derivatives of Eq. (1) with respect to the wellbore storage coefficient, to the permeability and to the skin factor are:

$$\frac{\partial\overline{\Delta p_{wf}}}{\partial C} = -\frac{u^2}{qB}\overline{\Delta p_{wf}}^2, \tag{3}$$

$$\frac{\partial\overline{\Delta p_{wf}}}{\partial k} = \frac{a\left[(1-2S)K_1\left(\sqrt{\frac{u}{bk}}\right)^2 - K_0\left(\sqrt{\frac{u}{bk}}\right)^2 - \frac{2K_1\left(\sqrt{\frac{u}{bk}}\right)K_0\left(\sqrt{\frac{u}{bk}}\right)}{\sqrt{\frac{u}{bk}}}\right]}{2bk\left\{k\sqrt{\frac{u}{bk}}K_1\left(\sqrt{\frac{u}{bk}}\right) + \frac{auC}{qB}\left[K_0\left(\sqrt{\frac{u}{bk}}\right) + S\sqrt{\frac{u}{bk}}K_1\left(\sqrt{\frac{u}{bk}}\right)\right]\right\}^2}, \tag{4}$$

$$\frac{\partial\overline{\Delta p_{wf}}}{\partial S} = \frac{aK_1\left(\sqrt{\frac{u}{bk}}\right)^2}{b\left\{k\sqrt{\frac{u}{bk}}K_1\left(\sqrt{\frac{u}{bk}}\right) + \frac{auC}{qB}\left[K_0\left(\sqrt{\frac{u}{bk}}\right) + S\sqrt{\frac{u}{bk}}K_1\left(\sqrt{\frac{u}{bk}}\right)\right]\right\}^2}. \tag{5}$$

2.2 Well with wellbore storage and skin factor near a sealing fault

A reservoir with a well located a distance L of a single sealing fault or barrier is a common case in well testing. The pressure response for this model is calculated using the image method to consider the effect of no-flow through the barrier, represented by an infinite vertical plane. This technique consists of setting how many image wells are needed to get the same streamline configuration of the actual configuration (Rosa *et al.*, 2006). For a reservoir near a sealing fault it is necessary to set only one image well located a distance $-L$ from the fault, that is, symmetric to the real well with respect to the fault. The pressure response will be calculated using the superposition principle. So, the pressure change solution for this model, in Laplace space, is:

$$\overline{\Delta p_{wf}} = \frac{K_0\left(r_w\sqrt{\frac{u}{ck}}\right) + K_0\left(2L\sqrt{\frac{u}{ck}}\right) + S}{u\left\{\frac{k}{a} + \frac{uC}{qB}\left[K_0\left(r_w\sqrt{\frac{u}{ck}}\right) + K_0\left(2L\sqrt{\frac{u}{ck}}\right) + S\right]\right\}}, \tag{6}$$

where $c = \frac{1}{\phi\mu c_t}$. The derivatives of Eq. (6) with respect to the wellbore storage coefficient, to the permeability, to the fault distance and to the skin factor are:

$$\frac{\partial\overline{\Delta p_{wf}}}{\partial C} = -\frac{u^2}{qB}\overline{\Delta p_{wf}}^2, \tag{7}$$

ISSN 2176-5480

$$\frac{\partial\overline{\Delta p_{wf}}}{\partial k}=\frac{a\left\{r_w\sqrt{\frac{u}{ck}}K_1\left(r_w\sqrt{\frac{u}{ck}}\right)-2\left[K_0\left(2L\sqrt{\frac{u}{ck}}\right)-L\sqrt{\frac{u}{ck}}K_1\left(2L\sqrt{\frac{u}{ck}}\right)+K_0\left(r_w\sqrt{\frac{u}{ck}}\right)+S\right]\right\}}{2u\left\{k+\frac{auC}{qB}\left[K_0\left(r_w\sqrt{\frac{u}{ck}}\right)+K_0\left(2L\sqrt{\frac{u}{ck}}\right)+S\right]\right\}^2}, \tag{8}$$

$$\frac{\partial\overline{\Delta p_{wf}}}{\partial L}=-\frac{2ak\sqrt{\frac{u}{ck}}K_1\left(2L\sqrt{\frac{u}{ck}}\right)}{u\left\{k+\frac{auC}{qB}\left[K_0\left(r_w\sqrt{\frac{u}{ck}}\right)+K_0\left(2L\sqrt{\frac{u}{ck}}\right)+S\right]\right\}^2}, \tag{9}$$

$$\frac{\partial\overline{\Delta p_{wf}}}{\partial S}=\frac{ak}{u\left\{k+\frac{auC}{qB}\left[K_0\left(r_w\sqrt{\frac{u}{ck}}\right)+K_0\left(2L\sqrt{\frac{u}{ck}}\right)+S\right]\right\}^2}. \tag{10}$$

2.3 Channel reservoir

In the channel reservoir model the well is located between two parallel sealing faults. The distances from the well to north and south faults are L_N and L_S, respectively. The channel width is $W = L_N + L_S$, but its length is considered infinite on the direction parallel to the faults. The two parallel sealing faults are simulated with an infinite series of image wells. The pressure caused by the real and image wells is calculated using the line-source solution in a homogeneous infinite reservoir. So, for a channel reservoir the wellbore pressure change is:

$$\Delta p_{wf}=\frac{qB\mu}{4\pi kh}\left\{\mathrm{E_i}\left(\frac{\phi\mu c_t r_w^2}{4kt}\right)+\sum_{j=1}^{\infty}\left[\mathrm{E_i}\left(\frac{\phi\mu c_t L_{S_j}^2}{4kt}\right)+\mathrm{E_i}\left(\frac{\phi\mu c_t L_{N_j}^2}{4kt}\right)\right]+2S\right\}, \tag{11}$$

where $\mathrm{E_i}$ is the exponential integral function, t is the time, L_{N_j} and L_{S_j} are the distances from the actual well to j-esime image on the north and south directions, respectively. To limit the terms evaluated in the sum of the Eq. (11), the pressure change from the image wells will be summed until $\Delta p_{wf_{image}} \leq 10^{-6}$. Thus, the derivatives of Eq. (11) with respect to the well offset $d = \frac{L_N}{W}$, to the permeability k, to the skin factor S and to the channel width W are:

$$\frac{\partial\Delta p_{wf}}{\partial d}=-\frac{qB\mu}{4\pi kh}\sum_{j=1}^{\infty}\left[\frac{2}{L_{N_j}}\exp\left(-\frac{\phi\mu c_t L_{N_j}^2}{4kt}\right)\frac{\partial L_{N_j}}{\partial d}+\frac{2}{L_{S_j}}\exp\left(-\frac{\phi\mu c_t L_{S_j}^2}{4kt}\right)\frac{\partial L_{S_j}}{\partial d}\right], \tag{12}$$

$$\frac{\partial\Delta p_{wf}}{\partial k}=-\frac{\Delta p_{wf}}{k}-\frac{qB\mu}{4\pi k^2 h}\left\{\exp\left(\frac{\phi\mu c_t r_w^2}{4kt}\right)+\sum_{j=1}^{\infty}\left[\exp\left(-\frac{\phi\mu c_t L_{N_j}^2}{4kt}\right)+\exp\left(-\frac{\phi\mu c_t L_{S_j}^2}{4kt}\right)\right]\right\}, \tag{13}$$

$$\frac{\partial\Delta p_{wf}}{\partial S}=\frac{qB\mu}{2\pi kh}, \tag{14}$$

$$\frac{\partial\Delta p_{wf}}{\partial W}=-\frac{qB\mu}{4\pi kh}\sum_{j=1}^{\infty}\left[\frac{2}{L_{N_j}}\exp\left(-\frac{\phi\mu c_t L_{N_j}^2}{4kt}\right)\frac{\partial L_{N_j}}{\partial W}+\frac{2}{L_{S_j}}\exp\left(-\frac{\phi\mu c_t L_{S_j}^2}{4kt}\right)\frac{\partial L_{S_j}}{\partial W}\right]. \tag{15}$$

3. VARIABLE FLOW RATE SOLUTIONS

For all the cases studied in this work, the pressure responses for the problem with variable flow rate were calculated using the superposition principle (Bourdet, 2002). According to this principle, a flow rate history with discrete changes can be represented by the simultaneous effect of several single flow rate histories. This equivalent flow rate history can be interpreted as the result of the production / injection of virtual wells located in the same position. Each well begins to produce at the time $t_{p,k}$ and has a constant flow rate $(q_{(k+1)} - q_k)$. This principle can be used for any number of virtual wells. Thus, the pressure change solution for a variable flow rate will be the sum of the solutions for each virtual well, considering each equivalent flow rate and initial production time:

$$\Delta p_{wf}=\Delta p_{wf_1}\left(t-t_{p,0},\,q_1-q_0\right)+\Delta p_{wf_2}\left(t-t_{p,1},\,q_2-q_1\right)+\cdots+\Delta p_{wf_{n+1}}\left(t-t_{p,n},\,q_{n+1}-q_n\right), \tag{16}$$

for $t > t_{p,n}$, where Δp_{wf_1}, Δp_{wf_2},...,$\Delta p_{wf_{n+1}}$ are the individual pressure changes for each constant flow rate.

Some of the analytical models for Δp_{wf} are known in the Laplace space and its analytical inversion are not possible due to the complexity of the expressions. Therefore, pressure change function and their derivatives with respect to the unknown parameters were calculated in Laplace space and numerically inverted to real space using the algorithm of Stehfest (1970). The derivatives with respect to the reservoir and well parameters, in general, were calculated using

$$\frac{\partial\Delta p_{wf}}{\partial x_i}=\mathcal{L}^{-1}\left\{\frac{\partial\overline{\Delta p_{wf}}}{\partial x_i}\right\}\qquad\text{and}\qquad\frac{\partial\Delta p_{wf}}{\partial p_i}=1.0, \tag{17}$$

ISSN 2176-5480

where p_i is the initial reservoir pressure. The derivatives with respect to the flow rate history parameters as the k-esime flow rate q_k and to the k-esime production time $t_{p,k}$, when the change in the flow rate occurs, are:

$$\frac{\partial \Delta p_{wf}}{\partial q_k} = \frac{\partial \Delta p_{wf_k}}{\partial q_k} + \frac{\partial \Delta p_{wf_{k+1}}}{\partial q_k} \quad \text{and} \quad \frac{\partial \Delta p_{wf}}{\partial t_{p,k}} = \frac{\partial \Delta p_{wf_{k+1}}}{\partial \tau}\frac{\partial \tau}{\partial t_{p,k}} = -\mathcal{L}^{-1}\left\{u\overline{\Delta p_{wf_{k+1}}}\right\}, \tag{18}$$

where $\tau = t - t_{p,k}$.

4. NONLINEAR REGRESSION ALGORITHM

To find the parameters that best fit the observed pressure response to an ideal theoretical model is the objective of the analysis of pressure data. These parameters can be reservoir and/or well features and their estimation are an optimization process known as nonlinear regression. The least square method is the nonlinear technique most commonly used and it consists in fit the observed data set y, with m observations, as close as possible to the response of a theoretical model $F(x,\, t)$. Therefore, the sum of the squares of the differences between the observed data y_j and the solution calculated by the model $F(x,\, t_j)$, called the residual, r_j, should be minimized. Thus, the objective function E is defined as:

$$E(x) = \frac{1}{2}\sum_{j=1}^{m}\left[y_j - F(x,\, t_j)\right]^2 = \frac{1}{2}\sum_{j=1}^{m} r_j^2. \tag{19}$$

The physical model $F(x,\, t_j)$ is a function of the unknown parameters vector x, with n elements, and of the independent variable t_j. Here, y_j is the pressure registered in the instant t_j and the regression parameters can be the wellbore storage coefficient, the permeability, the skin factor or the non-registered flow rates and production times, among others: $x = \{C, k, S, \cdots, L, p_i, \cdots, (qB)_k, \cdots, t_{p,k}\}$. A necessary condition to minimize the objective function is that the derivatives of Eq.(19) with respect to unknown parameters have to be equal to zero. There are many ways to minimize the objective function. Horne (1994) shows the more common methods in petroleum engineering. The most applied and recommended method is the Levenberg-Marquardt algorithm, with variables scaling (Moré, 1977; Nocedal and Wright, 1999).

The Levenberg-Marquardt algorithm gives good results in practice, however it is important to emphasize that its convergence is slow when the residual is large or when the regression parameters are strongly correlated.

During the minimization of the objective function, the regression parameters can assume any value. Thus, it is possible that some of these parameters take on values that do not have physical meaning. For example, nothing prevents that during the nonlinear regression to estimate a negative permeability or negative wellbore storage coefficient. To maintain the properties within limits physically admissible is necessary to make use of constraints. In this work two techniques were used to prevent unphysical values of the regression parameters: the transformation of the parameters, suggested by Vieira (1992) and Dastan and Horne (2011), the penalty functions based on fractions, suggested by Carvalho (1993), and the penalty functions based on exponentials, suggested by Vieira (1992).

It is important to evaluate the quality of the results after the nonlinear regression. Therefore, confidence intervals were calculated for each unknown parameter, according to the work of Dogru *et al.* (1977). To make use of the confidence intervals as criteria in this study, the limits suggested by Horne (1995) were employed to analyze if a confidence interval was acceptable. Additional details could be found in Junqueira (2011).

5. METHODOLOGY

Synthetic data were generated, using the reservoir models equations presented in section 2, to simulate data registered by PDGs. These data comprehend 550 hours of production, divided into 13 periods with constant flow rates, as shown in Figure (1). The pressures were calculated at equal time intervals, with a total of 2200 records. Noise was added to the synthetic data to represent the behavior observed in field data. The resulting synthetic pressure data was filtered using the techniques presented in Marotti *et al.* (2009). Thus, a relatively low level of noise remains in the synthetic data after this stage. The pressure resulting from this preprocessing was analyzed with the nonlinear regression algorithm.

A well with wellbore storage and skin factor in an infinite reservoir was considered in case S_1. The reservoir model near a sealing fault, with wellbore storage and skin factor was used for case S_2. Finally, the records in the case S_3 were simulated using the channel reservoir model with skin factor and without wellbore storage coefficient. In Fig. (2) a plot of the pressure history, for the S_1 case, is presented as example.

The use of field data would be interesting to complement the analysis, however records representing actual pressures collected by PDG's were not found in the literature. So, the field data set published by Clark and Golf-Racht (1985) was used in this paper as case F_1. In this case the well was produced for 13.067 hours before shutting in a buildup, where 39 pressure measurements were recorded in 18 hours. In Figure (3), the pressure recorded is represented by circles and the line represents the pressure history, reconstructed with the parameters found by Clark and Golf-Racht (1985). The vertical dashed lines indicate the times at which the flow rate changed, according to the reference. In the absence of evidence showing the contrary, this case was analyzed using the model reported in the reference: wellbore with storage and skin factor effect in a homogeneous infinite reservoir.

ISSN 2176-5480

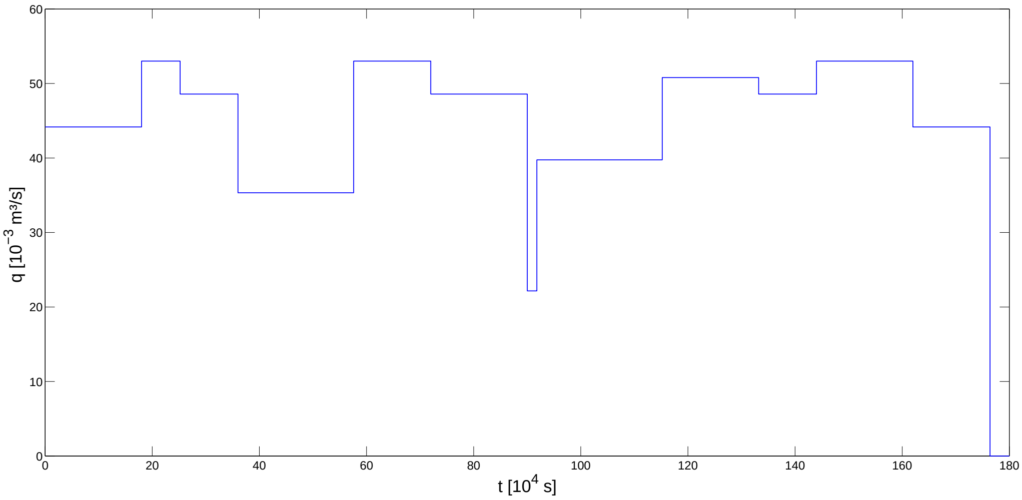

Figure 1. Flow rate history

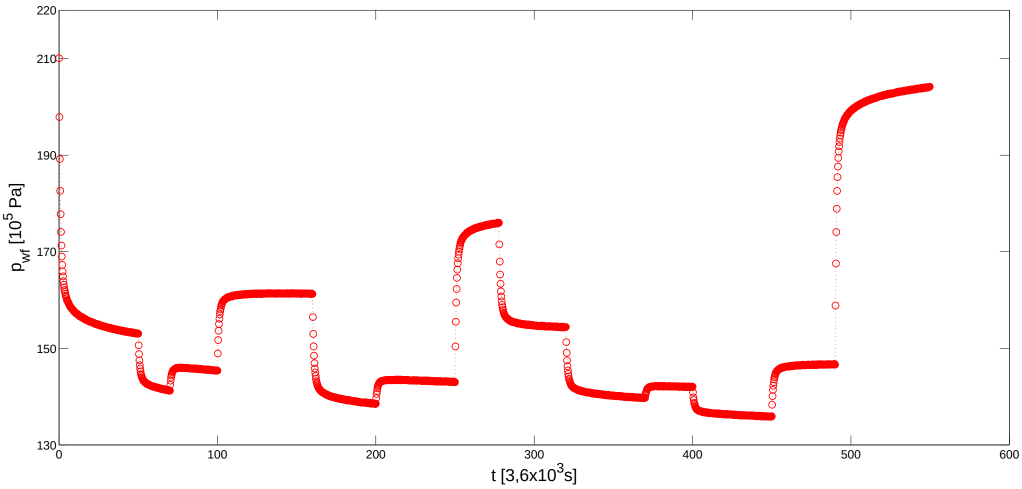

Figure 2. Pressure history for S_1 case

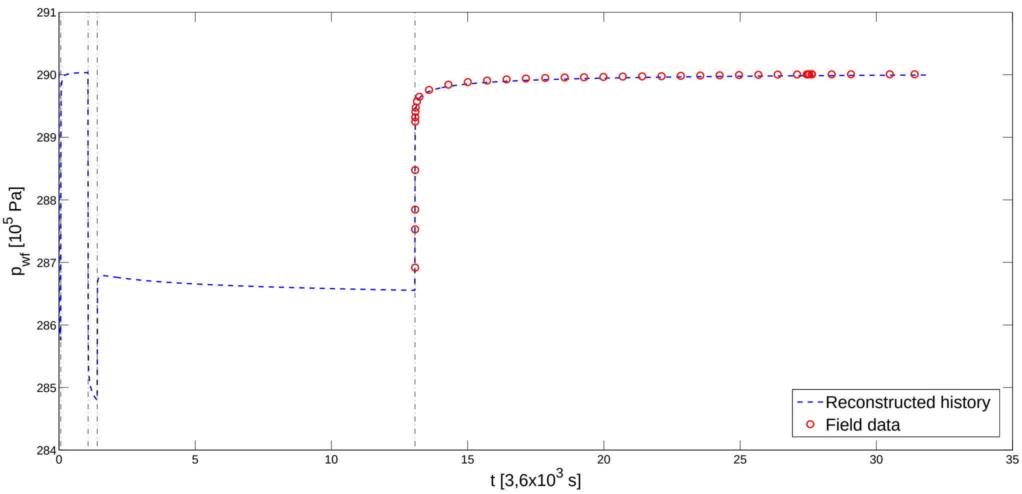

Figure 3. Pressure history for F_1 case

6. RESULTS

The tests were performed on the data where, besides well/reservoir properties and flow rates, at least two production times were also considered as unknown. In practice, initial estimative of production times is more accurate, since these points are well characterized by the sudden change in pressure. Thus, the intention to use this variable as a parameter of the nonlinear regression is to refine its initial estimative. Several combinations of the initial estimative of the well/reservoir parameters were tested for nonlinear regression with the original objective function, Eq. (19), the penalty functions and the transformed parameters, with and without variables scaling. In general, models S_1 and S_3 with the permeability, the skin factor, the wellbore storage coefficient, the initial pressure, some flow rates and production times considered unknowns converge, with acceptable confidence intervals, when transformed parameters were used. Similarly, realizations employing penalty functions converged in almost all cases, being the number of iterations the main difference between them. As an example see the match on Figure (4).

The same tests were performed with the S_2 pressure data, generated with the reservoir model near a sealing fault. Initially, tests were performed considering as unknown parameters the permeability, the skin factor, the storage coefficient, the initial pressure, the distance to the fault, some flow rates and production times. In this analysis was not possible to estimate these properties together, because the parameters C, S and L are strongly correlated. In this model the pressure response is a function of the group of variables $\frac{C}{L}\exp(S)$ therefore, it is not possible to obtain an acceptable fit when all these parameters are unknown.

To solve this problem, it was considered that the distance to the fault was estimated by another method, such as by a conventional technique. This parameter was chosen because the properties of well-reservoir system vary with time, although they were considered constant in this work, but the distance to fault can be said constant with time. Some selected results for this case are shown on the Table (1). Also in this case the confidence intervals are calculated within the limits suggested by Horne (1995), which makes the result acceptable. The value of the group $\frac{C}{L}\exp(S)$ for the actual parameters is 0.2289 and it is 0.2334 for the estimated parameters, a percentual difference of 1.97%. Similarly to the case of models S_1 and S_3, the use of penalty functions and transformed parameters considerably improves the convergence.

Table 1. Some non-linear regression results for the S_2 case. k in md, C in $10^{-2}\,m^3/kPa$, $t_{p,k}$ in $10^4\,s$ and $(qB)_k$ in $10^{-3}\,m^3/s$.

Parameter	S	k	C	$t_{p,3}$	$t_{p,10}$	$(qB)_1$	$(qB)_4$	$(qB)_7$	$(qB)_8$	$(qB)_{11}$
Actual value	2.60	50.00	0.0202	36.00	144.00	53.0	53.00	39.75	50.79	44.16
Initial estimative	0.00	500.00	0.0100	36.18	143.82	55.65	47.70	43.72	48.25	43.72
Final estimative	2.59	50.97	0.0208	36.00	143.99	52.54	53.12	39.78	51.41	45.09
Difference %	0.19	1.95	2.97	0.00	0.008	0.85	0.23	0.08	1.23	1.47
CI	4.7E-4	0.031	0.0013	0.01	0.042	3.40	3.40	3.47	3.47	3.86
CI %	0.018	0.061	0.0031	0.01	0.01	1.80	1.78	2.42	1.88	2.38

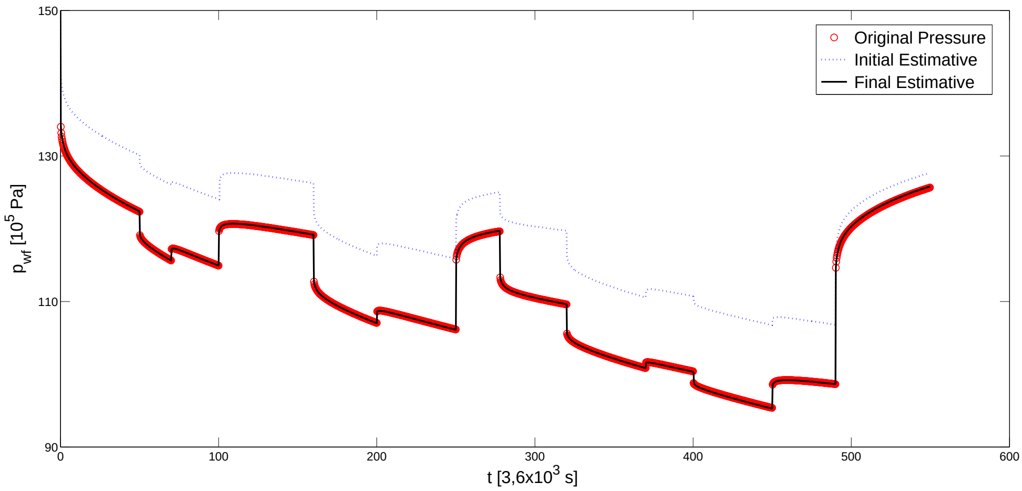

Figure 4. Pressure history and nonlinear regression results for S_3 case

For the field data analysis, case F_1, the permeability, the skin factor, the wellbore storage coefficient and the initial pressure were considered as unknown parameters. No matter what the initial estimates, regression converges to a set of parameters quite different from those obtained by the conventional analysis reported on the reference. To check the quality of this analysis, the confidence intervals for the parameter set of the reference were calculated. These results appear in Table (2), and clearly fall outside the acceptable range.

In other tests, some of the production times and flow rates were included as parameters on the nonlinear regression, using its reference values as initial estimative. As an example, Table (3) shows the results for a case where $t_{p,1}$ and the flow rates q_1 and q_4 were considered unknowns. Similar results were obtained with several combination of the production

ISSN 2176-5480

Table 2. Confidence intervals for the reference parameters of the F_1 case. k in md, C in $10^{-2}\,m^3/kPa$ and p_i in MPa.

Parameter	S	k	C	p_i
Reference value	9.30	3115.00	0.0154	290.04
CI	29.35	456.11	0.0839	0.0754
CI %	315.64	146.42	522.77	0.0259

history times and flow rates. Comparing the confidence intervals calculated for this analysis to the limits suggested by Horne (1995) it is concluded that these results are not acceptable. The data set was analyzed using the sealing fault model, as suggested by Clark and Golf-Racht (1985), with similar results.

Table 3. Non-linear regression results for the F_1 case. k in md, C in $10^{-2}\,m^3/kPa$, p_i in MPa, $t_{p,k}$ in $10^4\,s$ and $(qB)_k$ in $10^{-3}\,m^3/s$.

Parameter	S	k	C	p_i	$(qB)_1$	$(qB)_4$	$t_{p,1}$
Reference value	9.30	3115.00	0.0154	290.04	284.27	197.41	0.024
Initial estimative	0.0	1000.00	0.0100	290.04	284.27	197.41	0.024
Final estimative	19.4	2542.40	0.0187	290.03	284.13	118.19	0.004
Difference %	108.6	18.4	21.4	0.0019	0.0508	40.1	84.5
CI	5.2752	1813.50	0.0069	0.0140	0.3172	0.00056	0.0026
CI %	27.2	71.3	37.0	0.0048	0.0310	0.00013	25.1

It can be observed that the results in Table (3) shows narrower ranges than the results in Table (2), therefore, more reliable estimations. It is also important to observe that for q_4 the confidence interval calculated in the Table (3) is small and the percentage difference from the reference value for the estimated value is high, suggesting uncertainty in the record of this parameter. Furthermore, the results of a conventional semilog analysis and a nonlinear regression with the time superposition function as the independent variable are in close agreement with the parameters reported on the reference. Thus, some hypotheses can be raised to the fact that it was not possible to estimate the permeability, the wellbore storage coefficient and the skin factor with acceptable confidence intervals. The first hypothesis is that the data was recorded and/or reported inaccurately, so that it cannot get a proper adjustment of the pressure to a theoretical model. Another hypothesis is that the analytical model used is not appropriate for this set of pressures. In addition, only 39 measures of pressure were taken in 18 hours, which may influence the quality of the fit.

7. CONCLUSIONS

Pressure data fittings for long periods with a variable flow rate history using the nonlinear regression method were successful on synthetic data. However, it is important to note that in this work the well/reservoir properties were considered constant over time. It is also important to say that the noise added in synthetic data does not cover all possible problems that may occur during the data record made by PDGs. In addition, flow rates were considered constant during a time interval, which may not happen in actual production.

In all the studied cases the quality of the fitting and the performance of the nonlinear regression algorithm were better when the transformed parameters in addition to variable scaling were used. A comparative analysis of the performance of the two types of penalty functions employed shows that the functions based on fractions have a poorer performance than the based on the exponential function.

Examining the tests of a reservoir near to a sealing fault, it was observed that, when the effect of the boundaries appears, the pressure response becomes a function of the group of variables $\frac{C}{L}\exp(S)$. So, the curves of pressure collapse when the production time is long. Thus, the nonlinear regression performed on all the data was not able to estimate the skin factor, the permeability, the storage coefficient and the distance to the fault together. In this situation, one of these parameters must be estimated by a conventional analysis and the others by the nonlinear regression method.

As many authors reported, the number of terms on the numerical inversion algorithm could affect the precision and stability of the results. The stability issue is particularly important for cases with negative skin factors, even when the equivalent radius concept is employed. For the cases analyzed here, eight to sixteen terms were used, without severe stability problems, being twelve terms a conservative choice.

The analysis made in this study for channel reservoir model without wellbore storage effects shows no appreciable differences in the performance of a model with an explicit solution in comparison to the models with non-analytically invertible Laplace space solutions.

It is worth mentioning that it is not possible to make the regression when all flow rates are unknown, because the problem becomes indeterminate and it cannot estimate a single solution for the unknown parameters. In the case of

ISSN 2176-5480

production time, it was possible to use nonlinear regression to fit this parameter, even when initial estimates were distant to the actual value. However, in practice the time production value is well characterized by the sudden change in pressure. Thus, the analysis of production time with the nonlinear regression method will be done only to make its value more accurate.

The field data presented in this paper does not represent a PDG record, but it serves to illustrate some of the difficulties that can be found in a field data analysis.

There are still issues to be investigated in the use of nonlinear regression for long pressure data study. It is suggested, for example, to analyze data where the parameters, like, k or S, may vary over time. Also others reservoir models can be analyzed to identify the groups of properties, which cannot be estimated together. There are also other nonlinear regression methods in the literature, as the least absolute values method, that are recommended for analysis of conventional tests and that can be used for the study of long term pressure data.

8. ACKNOWLEDGEMENTS

The authors wish to thank to CAPES, MEC and UENF/LENEP for the financial support.

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