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A DIMENSIONLESS CORRELATION FOR THE FROST DENSITY

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Abstract. *A first-principles model for predicting the evolution of the porosity of a frost layer over time was advanced. The theoretical model was used together with experimental data obtained elsewhere to put forward a semi-empirical correlation for the frost porosity as a function of the time, and the modified Jakob number, which carries information of the air stream and plate surface conditions. When compared to experimental data, the proposed equation showed errors for the frost density within ±10% bounds. Comparisons with other models available in the open literature are also reported.*

Keywords: *frost density; dimensionless correlation; semi-empirical approach*

1. INTRODUCTION

Frosting phenomena take place in a wide range of engineering applications, spanning from aerospace to HVAC-R products. In most cases, frost formation has to be avoided as it may deplete the equipment performance or even lead it to collapsing. Thus predicting frost growth and densification through simulation models is an up-to-date issue for modern engineering design. Different frost growth and densification models have been proposed in the past decades. Their main features are summarized in Table 1. Albeit the literature related with the modeling of frost formation processes is abundant, most models lack of generality as they rely on empirical correlations for the frost density, which narrows they range of application.

Table 1. Summary of some influential studies of frost formation modeling

Author	Year	Origin	Geometry	Approach		Initial conditions	
				Domain	Porous Medium	Thickness [mm]	Density [kg/m^3]
O'Neal	1982	USA	Channel	Lumped	Yes	0.05	40
Sami and Duong	1989	Canada	Plate	Lumped	No	-	-
Le Gall et al.	1996	France	Plate	1-D	Yes	0.1	25
Lee et al.	1997	S. Korea	Plate	Lumped	No	-	-
Cheng and Cheng	2001	China	Plate	Lumped	No	-	Hayashi et al. (1977)
Na and Webb	2004	USA	Plate	1-D	Yes	0.02	30
Lenic et al.	2009	Croatia	Channel	2-D	Yes	0.02	30
Hermes et al.	2009	Brazil	Plate	Lumped	Yes	0.001	Authors' own
Kandula	2011	USA	Plate	Lumped	No	0	Author's own
Cui et al.	2011	China	Channel	2-D	Yes	0	Nucleation model
Hermes	2012	Brazil	Plate	Analytical	No	0	Hermes et al. (2009)

Despite the importance of the frost density to the frost formation models, just a few frost correlations can be found in the open literature (Iragorry et al., 2004). In a pioneering study, Biguria and Wenzel (1970) gathered experimental data for cryogenic conditions, $-95.5<T_w<-28.9$°C, and proposed a first-order empirical correlation based on a 2^5 experimental factorial design considering the air stream velocity and humidity, the plate and frost surface temperatures, and the boundary layer characteristics (laminar or turbulent). A few years later, Hayashi et al. (1977) put forward a correlation for the frost density as a function of the frost surface temperature only. The correlation is based on an exponencial fit, as follows:

$$\rho_f = 650\exp(0.277T_f) \qquad (1)$$

which is valid for $-18.6<T_f<-5$°C, with T_f in [°C]. The tests were carried out at room temperature, for air velocities ranging from 2 to 6 m/s.

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Mao et al. (1992) proposed a dimensionless approach for correlating the frost density as a function of the Reynolds and Fourier numbers, the dimensionless temperature difference based on the frost and plate surface temperatures and the triple point of water, and the absolute humidity of the air stream. Such a formulation was later revisited by Yang and Lee (2004), who proposed:

$$\frac{\rho_f}{\rho_i} = 1.54\times10^{-4}\left(\frac{u_a L}{\nu_a}\right)^{0.351}\left(\frac{\alpha_a t}{L^2}\right)^{0.311}\omega_a^{-0.368}\left(\exp\left(\frac{T_a - T_{tp}}{T_a - T_w}\right)\right)^{2.4} \tag{2}$$

which is valid for $-35<T_w<-15$°C, $5<T_f<15$°C, $3.22<\omega_a<8.47$ g/kg, and $1<u_a<2.5$ m/s.

In order to account for the plate temperature, Hermes et al. (2009) introduced a third coefficient into the correlation of Hayashi et al. (1977). The new correlation was then fitted to 24 experimental data points gathered by the same authors, when the following equation was achieved:

$$\rho_f = 207\exp\left(0.266T_f - 0.0615T_w\right) \tag{3}$$

which is valid for $-15<T_w<-5$°C, $16<T_a<22$°C, $0.5<\phi_a<0.8$, and u_a=0.7 m/s, with both T_f and T_w in [°C].

The correlation of Hayashi et al. (1977) was also revisited by Wang et al. (2012), who proposed two multiplying factors to account for the air and plate temperatures, as follows:

$$\rho_f = 650\exp\left(0.277T_f\right)c_1 c_2 \tag{4}$$

where $c_1=0.70132-0.11346T_w-0.00203T_w^2$ and $c_2=1.4333-0.17389T_a-0.00722T_a^2$, with $-16<T_w<-8$°C, $11<T_a<19$°C, $0.42<\phi_a<0.8$, and u_a=5 m/s.

Recently, Kandula (2011) proposed the following dimensionless correlation based on experimental data obtained elsewhere,

$$\frac{\rho_f}{\rho_i} = 0.5\left(\frac{T_f - T_w}{T_m - T_w}\right)\exp\left(-0.376+1.5\left(1-\left(\frac{T_f - T_w}{T_m - T_w}\right)\right)\right)\left(1-\sqrt{\frac{u_a L}{\mathrm{Re}_c \nu_a}}\right) \tag{5}$$

where T_m is the melting temperature of ice, and $Re_c=5\times10^5$ is the critical Reynolds value for laminar-turbulent transition.

In all cases, the frost density correlations have an empirical background which limits its applications to the range of the experimental data used to fit them. In addition, most correlations depend on the frost surface temperature; therefore, they cannot be used as an evolving equation to predict the frost density over time without a conjugated model for the time evolution of the thickness of the frost layer. The present study is aimed at advancing a theoretical equation for the frost porosity that can be fitted to experimental data to come up with an evolving equation for the frost density without the need of calculating the properties of the frost surface.

2. PROPOSED CORRELATION

Frost is defined as a porous medium comprised of moist air and ice crystals, so that the density of a frost layer can be related to the medium porosity, as follows:

$$\rho_f = \rho_a\varepsilon + \rho_i(1-\varepsilon) \tag{6}$$

where ρ_i and ρ_a are the densities of ice and moist air, respectively, and ε is the frost porosity, defined as the ratio between the volume occupied by moist air, V_a, and the total volume of the porous medium, $V=V_a+V_i$.

For conditions of high supersaturation degree and low surface temperature, the ice crystals assume the form of needles (Fletcher, 1970) and, therefore, the frosted medium can be modelled as a random array of ice columns, as depicted in Fig. 1. Assuming the heat conduction through an ice column is the only heat transfer mechanism to produce the vapor-to-solid phase change at the column tip, the following energy balance can be written (Schneider, 1978),

$$\rho_i i_{sv} d\delta = \frac{k_i}{\delta}\Delta T dt \tag{7}$$

where q is the heat flux flowing through the ice column, i_{sv} is the latent heat of desublimation of water vapor, k_i is the heat conductivity of ice, and ΔT is the temperature difference along the column of length δ.

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Equation (7) can be re-arranged into the form,

$$\delta d\delta = \lambda dt \tag{8}$$

where $\lambda = k_i \Delta T / \rho_i i_{sv}$ has diffusivity dimensions [m^2/s]. Assuming λ is nearly constant over time, eq. (8) can be solved analytically, yielding

$$\delta = \sqrt{2\lambda t} \tag{9}$$

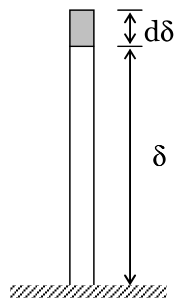

Figure 1. Schematic representation of the growth of a frost column

Substituting eq. (9) into eq. (8), and noting that

$$\varepsilon = 1 - A_c N \delta \tag{10}$$

where A_c is the cross-section area of one column, and N is the number of columns per volume unit, it can be shown that

$$\varepsilon = 1 - C\sqrt{t} \tag{11}$$

where C is a constant to be determined either theoretically or fitted to the experimental data. In the present analysis, the second approach has been chosen. Experimental data of Hermes et al. (2009), transcripted in Table 1 for porosity rather than density as published originally, was adopted for this purpose. It is worth noting that most of porosity data showed values higher than $\pi/4$, which is the figure for the porosity of a porous medium made of a compact array of cylinders (Bear, 1972).

Table 2. Experimental data of Hermes et al. (2009) written here for the frost porosity

Test #	T_a, °C	ϕ	T_w, °C	ε (1 h)	ε (2 h)	1/Ja	C
1	22	0.8	-15	0.853	0.777	1.033	0.00250
2	22	0.8	-10	0.806	0.725	1.157	0.00307
3	22	0.8	-5	0.756	0.654	1.299	0.00389
4	22	0.5	-15	0.903	0.858	0.778	0.00163
5	22	0.5	-10	0.871	0.830	0.884	0.00195
6	22	0.5	-5	0.861	0.811	1.005	0.00238
7	16	0.8	-15	0.908	0.863	0.822	0.00176
8	16	0.8	-10	0.887	0.834	0.932	0.00211
9	16	0.8	-5	0.852	0.792	1.057	0.00260
10	16	0.5	-15	0.912	0.884	0.631	0.00128
11	16	0.5	-10	0.909	0.874	0.729	0.00151
12	16	0.5	-5	0.879	0.852	0.840	0.00181

Recalling that $\varepsilon(t=0)=1$, eq. (11) was fitted to each of the 12 conditions tested by Hermes et al. (2009). The curve fitting is illustrated in Fig. 2, where one can note that 3 data points were used for each test condition. The C-values obtained for the 12 conditions are also reported in Table 1. In all cases, the lowest fitting coefficient was $R^2=0.990$, suggesting that eq. (11) is representing the physics embedded in the experimental data quite well.

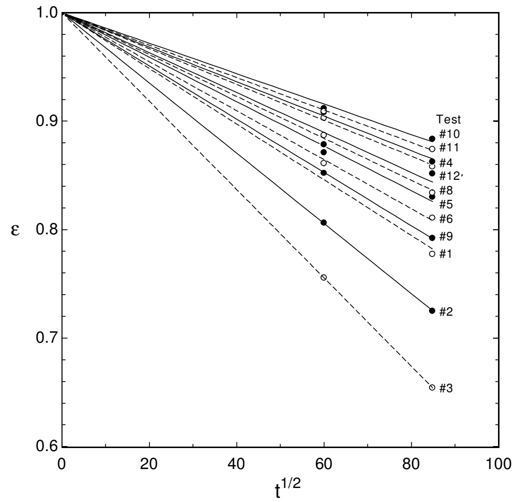

Figure 2. Best fitting of Hermes et al. (2009) data using eq. (11)

In order to express the *C*-constant as a function of the running conditions, such as air and plate temperatures and humidities, a second fitting was carried out considering the following dimensionless group, which recalls the Jakob number (Hermes, 2012),

$$Ja = \frac{c_p}{i_{sv}} \frac{T_{sat}(\omega_a) - T_w}{\omega_a - \omega_{sat}(T_w)} \tag{12}$$

Figure 3 illustrates the variation of *C* with 1/*Ja*, where an exponencial relationship can be observed. Therefore, the following exponencial fit was adopted,

$$C = C_0 \exp\left(\frac{C_1}{Ja}\right) \tag{13}$$

where C_0=0.000448 and C_1=1.663. Eqs. (11) and (13) can then be used to predict the frost porosity over time. Noting that $\rho_i >> \rho_a$, and defining the dimensionless time as $\tau = Dt/L^2$, where *D* is the water vapor diffusivity into air [m^2/s], eqs. (11) and (13) may be substituted into eq. (6) to provide the following dimensionless correlation for the frost density as a function of the *Ja* and τ:

$$\frac{\rho_f}{\rho_i} = 0.00882\, exp\left(\frac{1.663}{Ja}\right)\sqrt{\tau} \tag{14}$$

3. DISCUSSION

Figure 4 compares the predictions of eq. (11) with the experimental data of Hermes et al. (2009), where one can see that the proposed correlation agrees to all the experimental data points within a ±10% error band. The predictions of eqs. (1) to (5) were also included in Fig. 4, where it can be seen that the correlation due to Hermes et al. (2009) agrees to the experimental data within ±15% error bands, whereas the other models show poorer preditions. The model of Kandula (2011) underpredicts the frost density, while the dimensionless correlation of Yang and Lee (2004) shows the opposite behavior, overpredicting the experimental values of the frost density. The correlations of Hayashi et al. (1977) and Wang et al. (2012), being the latter based upon the former, showed the worst predictions, with differences higher than 50% when compared with the experimental data.

Figure 5 assesses the sensitivity of the frost density, calculated by means of eq. (14), with regard to the working conditions, summarized in the form of the modified Jakob number. A parabolic behavior can be observed, which is due to the $Fo^{1/2}$ scale in the model, which is typical of diffusive dominant mass transfer problems (Hermes, 2012). The lower the Jakob number, the higher is the density, as the latent heat transfer predominates over the sensible heat transfer in such a condition.

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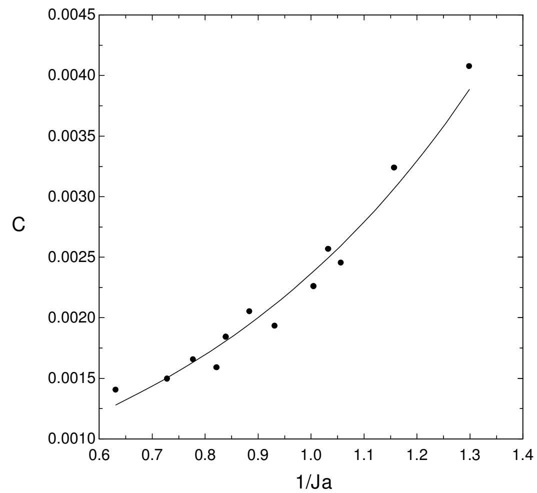

Figure 3. Functional relationship between *C* and *Ja*

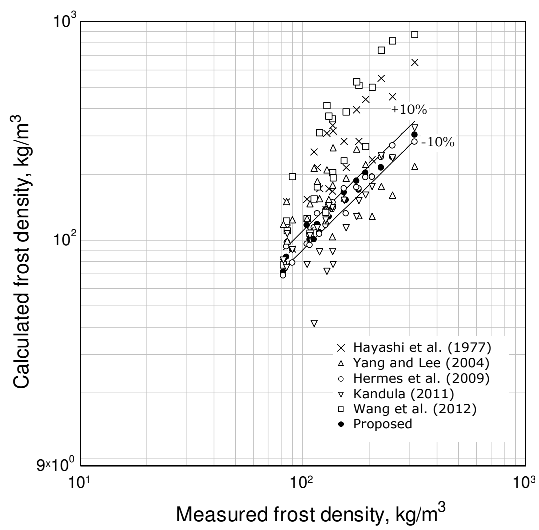

Figure 4. Comparison between experimental and calculated frost density

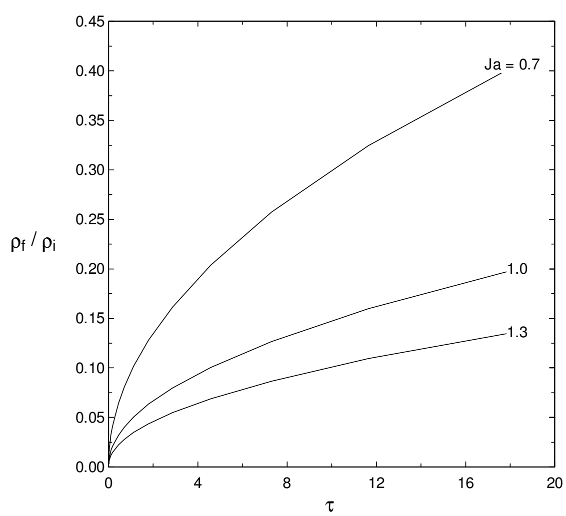

Figure 5. Evolution of frost density over τ for different *Ja* values

4. FINAL REMARKS

A first-principles model for predicting the time evolution of the porosity of a frost layer over a flat surface was proposed. The theoretical model was fitted to experimental data obtained elsewhere, being dependent on the modified Jakob number, which carries information of air stream and surface conditions, and the time. The model validation exercise revealed that the proposed correlation was able to predict the experimental data for the frost density within ±10% error bounds. It is worth noting that most empirical correlations depend upon the frost surface temperature, which is obtained by means of a more sophisticated simulation model, thus meaning that the frost correlations based on the frost surface temperature have no stand-alone application. The proposed semi-empirical model, summarized by eq. (14), not only provides an explicit relationship between the frost density and the time (in the form of the Fourier number), but is also independent on the frost surface temperature.

5. ACKNOWLEDGMENTS

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