ISSN 2176-5480

22nd International Congress of Mechanical Engineering (COBEM 2013)
November 3-7, 2013, Ribeirão Preto, SP, Brazil

ANALYSIS OF THE AXIAL BEHAVIOR OF A DRILLING RISER WITH A SUSPENDED MASS

Marcelo Anunciação Jaculli
José Ricardo Pelaquim Mendes
Celso Kazuyuki Morooka
Dept. of Petroleum Engineering - State University of Campinas - UNICAMP
Mendeleyev Street, 200, District of Barão Geraldo, Campinas, São Paulo, Brazil
mjaculli@gmail.com, jricardo@dep.fem.unicamp.br, morooka@dep.fem.unicamp.br

Kazuo Miura
PETROBRAS/CENPES
kazuo.miura@petrobras.com.br

Abstract. *For being used on deepwater, drilling risers are always subjected to extreme environment conditions, like sea waves and sea currents. Consequently, studies are done in this field to determine safe conditions to perform operations of equipment installation, like the BOP and the Christmas Tree. In this article, a simplified analytical approach for the axial problem of riser behaviors is presented, by using the damped wave equation. Based on the found solution, an operational map was generated for several wave amplitudes and wave angular frequencies. The operational map is useful while guiding the numerical simulations, by setting a specific range of values, since the simulations for a great range of values has a high computer cost.*

Keywords: *riser, drilling, Christmas Tree, BOP, vibration*

1. INTRODUCTION

A common situation in drilling activities and intervention on offshore oil wells is the installation of a blowout preventer (BOP) or a wet Christmas tree, on the bottom of the sea. This operation is done by a tool, called riser, which is basically made of a steel pipe, which connects the equipment with the floating drilling vessel. During the installation of the equipments, the riser is subjected to the action of sea currents, sea waves and the motion of the vessel. This way, the operation can only occurs when the environment conditions are favorable, in order to avoid mechanical collisions with the wellhead. Another situation, relatively new, is the navigation with the BOP suspended on the drilling riser. In these cases, there is always a risk of the BOP falling out to the bottom, which can damage the equipment and generate extra costs and delays.

In both cases, it is necessary the knowledge about the dynamic behavior of the system (riser and BOP) to avoid, or at least minimize, the risks involved. For such task, due to the complex nature of the problem, the numerical approach is more appropriate. However, for some simplified cases, it's possible to obtain analytical solutions. These solutions can provide a deep understanding of the system and, for example, guide the numerical simulations and also perform quick tests on the system behavior.

In this article, we present the analytical approach for the axial behavior of a riser, free of its transversal motions, with a suspended BOP. In some situations, this hypothesis is reasonable and allows us to obtain analytical solutions. The analytical approach here described explores the axial solution proposed by Chung and Whitney (1981), applied, at first, for ocean mining equipments. A numerical approach for the problem can be seen in Sevillano, *et al*., 2012.

2. PROBLEM DESCRIPTION

At first, it is necessary to define a coordinate system, as well the hypotheses that simplify the problem. The coordinate system is shown on Fig. 1, with the force balance in an infinitesimal element of the riser.

Here, the riser is characterized as a bar of length L and cross sectional area A, under the effects of only axial loads. The BOP is treated as a concentrated mass, bound to the lower end of the riser. The system is under the effect of heave motion on the top end, caused by the interaction between the sea waves and the vessel in which the riser is connected.

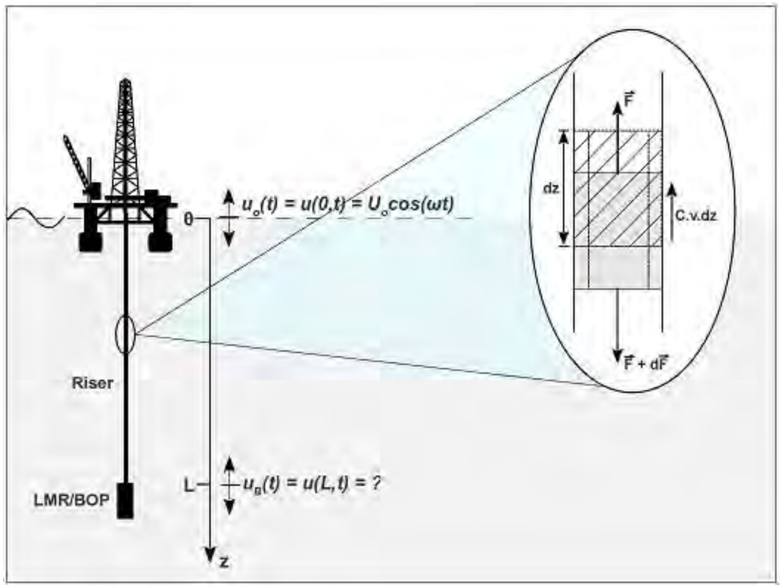

Figure 1. A scheme with the definition of the problem. Orientation of the coordinate system as well as the boundary conditions are presented, including details of the force balance on an infinitesimal element of the riser.

To simplify the problem and enable an analytical solution, the following hypotheses were adopted during the deduction of the equations on the next section:

- lateral motion of the riser was neglected, turning it to a problem purely axial;
- environment loads, like wind and sea currents, were neglected. Only the effect of sea waves was considered;
- the riser is considered fixed on the vessel and free at the other end;
- the BOP is approximated as a plate, with all its mass concentrated in its gravity center and with a negligible length when compared with the riser length;
- the damping along the riser is considered linear and purely viscous;
- the heave (vertical rig move caused by sea wave) is modeled as a wave with a constant amplitude and angular frequency.

3. BASIC EQUATIONS

3.1 Motion equations

When the riser oscillates, an infinitesimal element *dz* suffers the effect of internal axial forces *F* and $F + (\partial F/\partial z)dz$. Also, the sea water exerts a viscous damping force of magnitude $-C(\partial u/\partial t)$, which has the minus sign because it is always opposite to the velocity direction. This force is directly proportional to the velocity only in cases which the body moves through the fluid with low velocities, as explained by Weaver and Timoshenko (1990). Then, modeling the riser as a bar, a balance of forces on the infinitesimal element *dz*, as shown on Fig. 1, will result on the following motion equation:

$$\frac{\partial F}{\partial z}dz - C\frac{\partial u}{\partial t}dz = mdz\frac{\partial^2 u}{\partial t^2} \quad (1)$$

Where *m* is the linear mass of the riser, $\partial^2 u/\partial t^2$ and $\partial u/\partial t$ are the acceleration and velocity, respectively, *C* is the damping coefficient of the riser, ∂F is an infinitesimal element of axial force and *dz* is the infinitesimal element of length.

The axial force is related with the deformation on the riser by the following expression:

$$F = \sigma A = EA\frac{\partial u}{\partial z} \quad (2)$$

Where *E* is the Young's modulus, *A* is the cross sectional area of the riser and $\partial u/\partial z$ is the deformation on the riser. Substituting in Eq. (1) and simplifying:

$$m\frac{\partial^2 u}{\partial t^2} + C\frac{\partial u}{\partial t} - EA\frac{\partial^2 u}{\partial z^2} = 0 \quad (3)$$

ISSN 2176-5480

Equation (1) is known as the damped wave equation in one dimension. It is a second-order partial differential equation in both variables z and t. Thus, if we do a separation of variables $u(z,t) = U(z)T(t)$, the solution for $U(z)$ will require two boundary conditions. The solution will be valid for values of $0 < z < L$ and $t > 0$.

3.2 Boundary conditions

At the top end of the riser ($z = 0$), the displacement is solely a function of time:

$$u(z,t) = u_0(0,t) \tag{4}$$

Where $u_0(0,t)$ is the function for the displacement of a point of the ship in which the riser is connected. It is considered that the vessel has only heave motion and it transmits the movement to the top of the riser.

At the lower end of the riser ($z = L$), a balance of forces on the BOP will result on the equation:

$$-F - F_d = M_B \frac{\partial^2 u}{\partial t^2} \tag{5}$$

Where F_d is the drag force on the BOP, as follows:

$$F_d = \frac{1}{2}\rho A_B C_D \left(\frac{\partial u}{\partial t}\right)^2 \tag{6}$$

Where ρ is the ambient fluid density, A_B is the cross sectional area of the BOP, C_D is the drag coefficient on the BOP and M_B is the BOP mass.

Substituting Eq. (2) and Eq. (6) on Eq. (5), we reach:

$$-EA\frac{\partial u}{\partial z} - \frac{1}{2}\rho A_B C_D \left(\frac{\partial u}{\partial t}\right)^2 = M_B \frac{\partial^2 u}{\partial t^2} \tag{7}$$

The nonlinear term appearing in Eq. (7) may be linearized by the following approximation, according to Weaver and Timoshenko (1990) and Chung (1982):

$$\left(\frac{\partial u}{\partial t}\right)^2 \approx \frac{8}{3\pi} U_B \frac{\partial u}{\partial t}\omega \tag{8}$$

Where U_B is the displacement amplitude on the BOP, which is unknown at this point, and ω is the angular velocity.

This way, Eq. (7) becomes:

$$-EA\frac{\partial u}{\partial z} - \frac{4}{3\pi}\omega\rho A_B C_D U_B \frac{\partial u}{\partial t} = M_B \frac{\partial^2 u}{\partial t^2} \tag{9}$$

3.3 Equations on the frequency domain

Since we are interested on the permanent behavior of the system, which means, after the transitory terms become negligible, we can propose a solution:

$$u(z,t) = U(z)e^{i\omega t} \tag{10}$$

Substituting the proposed solution in Eq. (10) on the equations described in Eq. (3), Eq. (4) and Eq. (9), together with their derivates, we reach the equations:

$$U''(z) + \left(\frac{\omega^2 m - i\omega C}{EA}\right) * U(z) = 0 \tag{11}$$

$$U(0) = U_0 \tag{12}$$

$$U'(L) - \frac{\omega^2 (M_B - \frac{4}{3\pi} i\rho A_B C_D U_B)}{EA} * U(L) = 0 \tag{13}$$

4. SOLUTION OF THE MOTION EQUATION ON THE FREQUENCY DOMAIN

In order to solve Eq. (11) and Eq. (13), we propose a solution of the form:

$$U(z) = e^{rz} \tag{14}$$

Substituting in Eq. (11) and solving, we find a solution on the form:

$$U(z) = C_1 \cos(kz) + C_2 \sin(kz) \tag{15}$$

Where:

$$k = \sqrt{\frac{\omega^2 m - i\omega C}{EA}} \tag{16}$$

The constants C_1 and C_2 from Eq. (15) are obtained through the boundary conditions of the problem. Using the condition described in Eq. (12), we get:

$$C_1 = U_0 \tag{17}$$

To use the boundary condition in Eq. (13), it is necessary to substitute it in Eq. (15), evaluated at $z = L$, knowing that $C_1 = U_0$ and that k is given by Eq. (16).

$$U(L) = U_0\, cos(kL) + C_2\, sin(kL) \tag{18}$$

$$U\ (L) = -U_0 k\, sin(kL) + C_2 k\, cos(kL) \tag{19}$$

$$-U_0 k\, sin(kL) + C_2 k\, cos(kL) - \beta[U_0\, cos(kL) + C_2\, sin(kL)] = 0 \tag{20}$$

Where:

$$\beta = \frac{\omega^2 (M_B - \frac{4}{3\pi} i\rho A_B C_D U_B)}{EA} \tag{21}$$

Solving for C_2:

$$C_2 = U_0 \left[\frac{k\, sin(kL) + \beta \cos\,(kL)}{k\, cos(kL) - \beta sin\,(kL)}\right] \tag{22}$$

Finally, we replace the results obtained in Eq. (17) and Eq. (22) in the solution Eq. (15), to find:

$$U(z) = U_0 \left[\cos(kz) + \sin\,(kz) \left(\frac{k\, sin(kL) + \beta \cos\,(kL)}{k\, cos(kL) - \beta sin\,(kL)}\right)\right] \tag{23}$$

This equation is valid for any point between the top end of the riser ($z = 0$) and the BOP ($z = L$). To find the displacement on the BOP, we substitute $z = L$ in Eq. (23) and, after simplifying, we get the equation:

$$U(L) = U_0 \left[\frac{1}{cos(kL) - \frac{\beta}{k} sin\,(kL)}\right] \tag{24}$$

To obtain the displacements, it is necessary to solve the equation iteratively, since the amplitude U_B is unknown.

5. CASE STUDY

Using the previous equations, we can obtain results for the axial vibration of the BOP. For this purpose, we consolidated a case as shown in Tab. 1.

Table 1. Data used on the numeric example, consolidated from a real case.

Riser length	2000 m
Outer diameter	21 in (0.5334 m)

ISSN 2176-5480

Inner diameter	19.25 in (0.489 m)
Young's modulus	210 GPa
LMR/BOP mass	373000 kg
Riser linear mass	277 kg/m
Drag coefficient on the BOP	1
Linear damping coefficient along the riser	98 N.s/m²
BOP cross sectional area	16 m²
Displacement amplitude on the top	2.5 m

Figure 2 shows the results for the amplification (the ratio between the displacement amplitude on the BOP and the displacement amplitude on the top) versus the angular frequency, with the use of Eq. (24). In this case, the variation of angular frequency means heave variations. In other words, by using Eq. (24), we can note that some heave frequencies cause peaks on amplification, due to resonance phenomena. In Fig. 2, we can see the first three resonance peaks. It means that if real world heave has same frequency of one of those peaks, the riser will suffer that amplification specified on his motion.

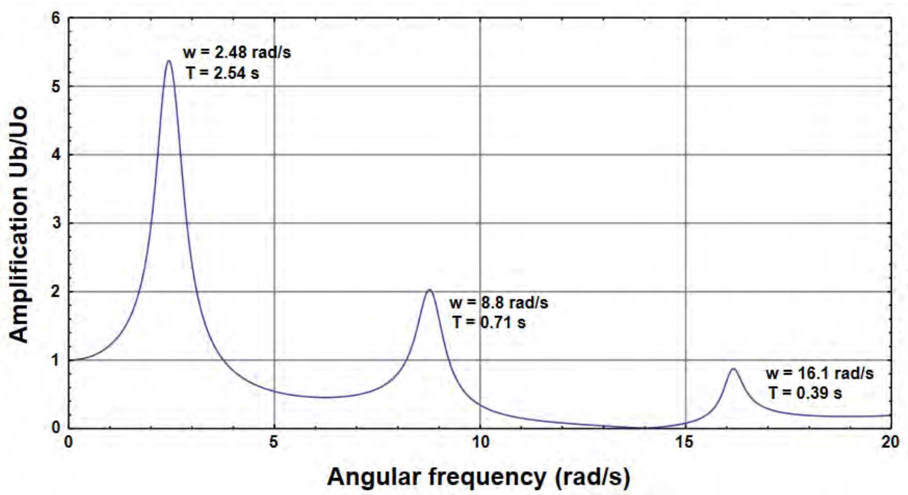

Figure 2. The riser displacement at the BOP (Amplification versus angular frequency, in radians per second).

By using Eq. (23) and the values for the angular frequencies in which those phenomena occur, we can generate an amplification profile (amplification versus depth graph) for each vibration mode of the riser. Figure 3 shows the amplification profile (in this case, the ratio between the displacement of a point on the riser and the displacement on the top) for the first three vibration modes.

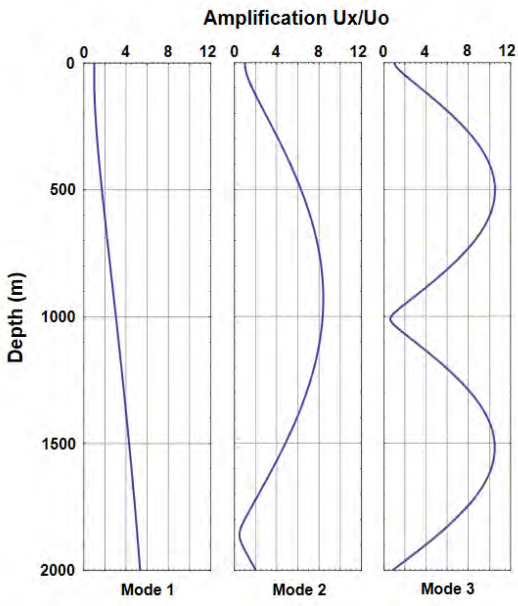

Figure 3. Amplification versus depth, in meters, for the displacement along the riser in its first three vibration modes.

From the analysis of the Fig. 3, we can conclude that the greatest amplifications do not occur necessarily on the BOP. For the second and third modes, there are points along the riser column in which the amplification has higher values than on the BOP. This can lead to the thought that it is necessary to analyze the riser on all its extension, with the aid of Eq. (23), and not only with the use of Eq. (24). The operational map described on the next topic is based on this idea.

6. OPERATIONAL MAP

Based on the previous results, we can make an operational map for the riser, for wave profiles with different amplitudes and angular frequencies. Figure 4 shows the maximum displacement of the riser as a function of both heave amplitude and angular frequency of the wave that excites the top end. The maximum displacement for a specific pair "heave amplitude and angular frequency" will not be necessarily on the BOP, as shown previously in Fig. 3.

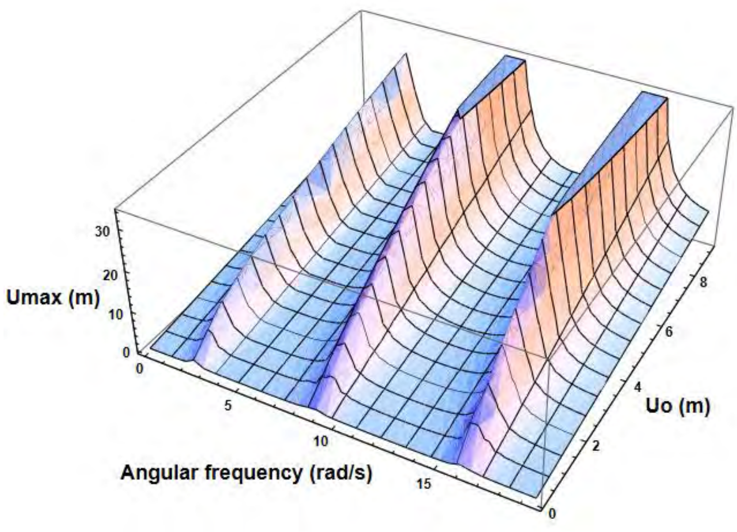

Figure 4. Maximum displacement of the riser, in meters, as a function of both heave amplitude, in meters, and angular frequency, in radians per second.

Taking the contour lines from Fig. 4, we finally obtain the operational map, showcased in Fig. 5. Having the necessary data from the wave profile, a person using the map can predict if the riser will be subjected to a dangerous displacement, which may cause its failure.

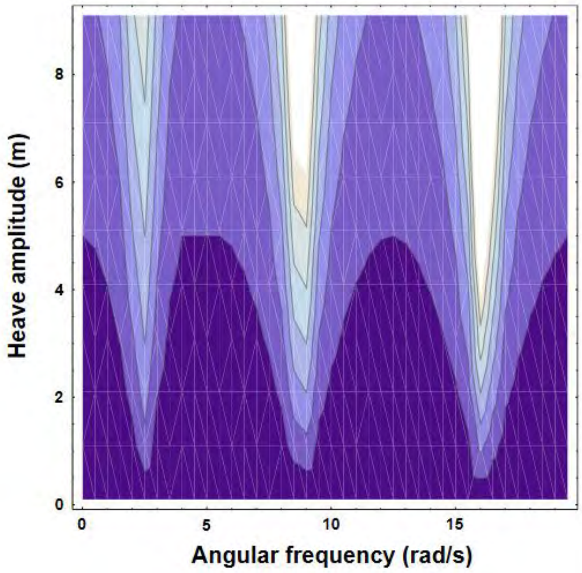

Figure 5. Operational map. The contour lines colors describe the maximum amplitude of displacements along the riser, for different heave amplitudes and wave angular frequencies.

7. CONCLUSION

An analytical approach for the axial behavior of a riser, free to move on its horizontal direction, was described. A case study using this analytical model and real world data was used to demonstrate the existence of resonance phenomena that can cause peak displacements. These displacements information were consolidated in a graph called "Operational map", where "peak displacement" problem generated by resonance frequency phenomena was highlighted. Besides the fact that environmental loads such as ocean waves and sea currents have been neglected, the analytical approach allows us to model the installation problem quickly, with good representation of the real operation. The operational map is very useful to be aware of new operational conditions when heave on the floating rig is approaching the resonance frequency of the suspended load system. It is also noted that the maximum displacement may occur at any point along its length, depending on the frequency of the sea wave excitation, leading to the conclusion that it is necessary to analyze the riser completely, and not only the BOP.

In order to go deeper into the problem, it is fundamental to include, in the analytical model, effects of the movement of the platform or ship ahead with the suspended BOP, which will result in lateral displacements. This will be the aim in a near future.

8. REFERENCES

Chung, J. S. and Whitney, A. K., 1981. “Dynamic Vertical Stretching Oscillation of an 18,000 ft Ocean Mining Pipe”. In *Offshore Technology Conference 4092*, May 4-7 1981, Houston, United States of America.

Chung, J. S., 1982. “Hydrodynamic Forces on a Marine Riser: A Velocity Potential Method”. In *Journal of Energy Resources Technology*, ASME Paper No. 76-Pet-81, Vol. 104, Boston, United States of America.

Sevillano, L. C., Morooka, C. K., Mendes, J. R. P. and Miura, K., 2012. “Estudo da Instalação de Equipamentos em Cabeça de Poço Submarino”. In 24º Congresso Nacional de Transporte Aquaviário, Construção Naval e Offshore – SOBENA2012, October 15-19 2012, Rio de Janeiro, Brazil.

Weaver, W. and Timoshenko, S., 1990. *Vibration problems in Engineering*. John Wiley & Sons, New Jersey, 5th edition.

9. RESPONSIBILITY NOTICE

The authors are the only responsible for the printed material included in this paper.