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ASYMPTOTIC MODEL FOR LAMINAR ANNULAR FLOW WITH CASING'S ROTATION

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Abstract. *The cementing process is a very important stage during the drilling of a new oil well. During this process several liquids are displaced first through the casing and later through the annular space. Non-uniform liquid displacement through the annular space may lead to severe problems to the well structure and safety. Therefore, fluid properties and process conditions should be designed to minimize non-uniformities on the displacement front. In the oil industry it is common to use the rotation of the casing to improve the liquid displacement. However, there are few studies in the literature which comprehends the influence of the casing's rotation on the liquid displacement. A complete analysis of the annular flow that occurs during cementing is extremely complex, because of the presence of different liquids which often presents a non-Newtonian characteristic and the flow is three dimensional and transient. A complete model has a prohibitive high computational cost. Simplified models are available in the literature and are used by the oil industry in commercial simulation software for cementing. In this work, we extend the lubrication based model in cylindrical coordinates developed by Gomes and Carvalho, 2010, to include the casing's rotation for laminar flow.*

Keywords: *Cementing process, Casing's rotation, Lubrication theory, Annular space*

1. INTRODUCTION

The cementing process is a very important stage during the drilling of a new well. The cement not only ensures the adhesion between the casing and the rock, but also isolates and prevents the contamination of the interior of the well by water or other fluids that may be found in different zones.

During cementing, the cement slurry is pumped through the well, displacing the drilling fluids. In order to have a successful cementing operation it is necessary to completely remove the drilling or spacer fluids and avoid mixing between them. If failures occur the structure and the safety of the well may be compromised. The Figure 1 illustrates different steps of the well cementing.

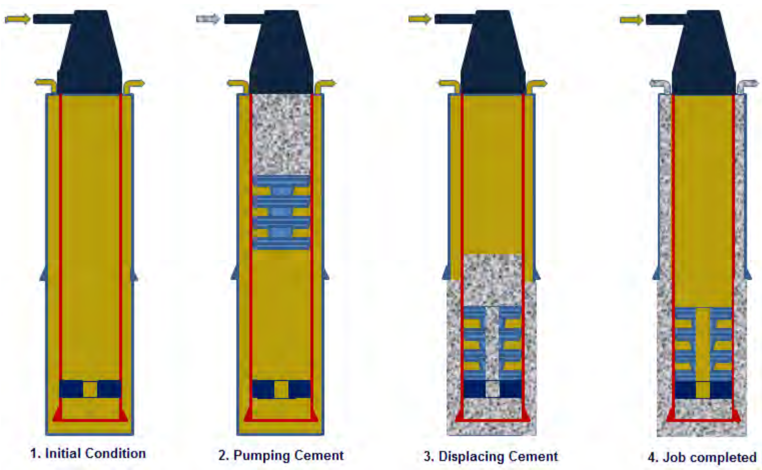

Figure 1. Stages during the cementing process.

There are several phenomena that can cause failures during cementing process. For example, the well geometry may complicate the full mud displacement. Besides, the reology of the fluids, the flow rate and the pumped volume also influence the process eficiency. Therefore, it is necessary to study the effect of the process conditions on the displacement in order to avoid undesired behavior.

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Modeling the cementing process is a complex task. This process involves different types of fluids, which often present non-Newtonian behavior. Moreover, the flow is three dimensional and transient. The full solution of the equation that describes the flow is extremely expensive. Hence, simplified models to describe the cementing process have been developed.

Bittleston *et al.*, 2002, developed a model that considers the eccentricity of a well and uses a cartesian coordinate system to represent the geometry of the annular space. The model is only accurate for radii ratio R_i/R close to 1. An asymptotic method solves the 2D problem as a sequence of 1D problems. As discussed in the work, the main focus was to solve the problem with low computational cost without a major concern with the accuracy of the solution.

The work of Pina and Carvalho, 2006, presented a model to describe the flow through annulus using lubrication theory and a cylindrical coordinate system. The model produces very accurate results for a larger range of radii ratio R_i/R. However, the analysis was restricted to simple phase laminar flow of Newtonian fluids.

The model developed by Gomes and Carvalho, 2010, extended the previous work to study the cementing process as a 2D flow of non-Newtonian fluids using lubrication theory and a cylindrical coordinate system to describe the annular space.

The previous work was extended by Araujo *et al.*, 2012, to study the displacement of different liquids through an annular space in turbulent conditions. It did not include the possible rotation of the casing.

In the oil industry it is common to use the rotation of the casing to improve the liquid displacement. However, there are few studies in the literature which comprehends the influence of the casing's rotation on the liquid displacement.

The goal of this work is to develop an asymptotic model to study the displacement of Newtonian liquids through an annular space considering the possible rotation of the casing. As done in previous models, lubrication theory is used to simplify the complete three dimensional set of differential equations to a single two dimensional equation that describes the pressure field.

2. MATHEMATICAL FORMULATION

As mentioned before the flow is three dimensional, transient and with moving boundaries. Therefore, a complete model has a prohibitive high computational cost.

Some aspects of the flow in the annular space that occur during cementing have been previously discussed by Gomes and Carvalho, 2010. As an extension of that work, the present research develops an asymptotic model based on the lubrication theory to study the displacement of different liquids through an annular space considering the casing's rotation.

The casing's rotation will affect the flow in differen aspects. First, there will be a contribution to the velocity profile of the flow. Additionally to the axial Poiseuille flow due to pressure gradient along the well, there will also be an azimuthal Couette flow due to the drag caused by the rotation of the casing.

Secondly, for non-Newtonian liquids, the casing's rotation will affect the liquids' viscosity. In order to evaluate the liquid's viscosity it will be necessary to consider an extra shear tension in the azimuthal direction. This effect has not been taken into account in this work since the goal of the present work is to model the flow in an annular space with casing's rotation for Newtonian liquids only.

The mathematical formulation developed considered the annular space between the casing and the formation as being an annular space between two cylinders with constant radius but variable eccentricity. Because of this and also in order to achieve better results, the equations used are in cylindrical coordinates. In order to simplify the governing equations, lubrication theory was applied. Therefore, we have the final equations for momentum conservation shown in Eqs. 1, 2 e 3.

$$-\frac{1}{\mu}\frac{\partial p^*}{\partial z} + \frac{1}{r}\left[\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right)\right] + \frac{\rho g z}{\mu} cos\alpha \frac{d\alpha}{dz} = 0 \tag{1}$$

$$-\frac{1}{\mu r}\frac{\partial p^*}{\partial \theta} + \frac{\partial}{\partial r}\left[\frac{1}{r}\left(\frac{\partial (rw)}{\partial r}\right)\right] - \frac{\rho g}{\mu}\cos\alpha\ \cos\theta = 0 \tag{2}$$

$$-\frac{\partial p^*}{\partial r} = 0 \tag{3}$$

Where p is the pressure field, u and w are the components of the velocity in the axial and in the tangential direction, g is the gravity acceleration, α is the angle that the well does with the vertical direction and r, θ and z are the directions in cylindrical coordinates.

From Eq. 3 we have that the pressure field is independent of the radius. Thus, we can integrate both Eq. 1 and 2 relative to the radius in order to find the velocity profile for the axial and tangencial directions. In addition to that, we also have to apply the boundary conditions for the velocity. These boundaries conditions state that the axial component of the velocity is zero at both walls - the inner and outer cylinders - and that the tangencial component is zero only at the outer cylinder, at the inner cylinder, or the casing, this component is given by Eq. 4.

$$w(R_i) = \Omega_i\ R_i \tag{4}$$

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Where Ω_i is the casing's rotation velocity in rad/s.

The final equations for the velocity at the axial and tangencial directions are shown in Eqs. 5 and 6.

$$u(r) = \frac{1}{4\mu}\left[r^2 + \frac{R_i^2 ln(r/R) - R^2 ln(r/R_i)}{ln(R/R_i)}\right]\left(\frac{\partial p}{\partial z} - \rho g z \cos\alpha \frac{d\alpha}{dz}\right) \tag{5}$$

$$w(r) = \frac{\Omega_i R_i^2}{R_i^2 - R^2}\left(r - \frac{R^2}{r}\right) + \frac{\rho g}{3\mu}\cos\theta\cos\alpha\left(r^2 - \left[\frac{R_i^2}{R_i + R} + R\right]r + \frac{R^2 R_i^2}{r\,(R_i + R)}\right) + \frac{1}{2\mu\,(R_i^2 - R^2)}\frac{\partial p}{\partial \theta}\left[r\left(R_i^2 \ln(r/R_i) - R^2 \ln(r/R)\right) - \frac{R^2 R_i^2}{r}\ln\frac{R}{R_i}\right] \tag{6}$$

Where R_i and R are the radius of the inner and outer cylinders, respectively.

The velocity profiles are used in the integrated mass conservation equation to obtain a differential equation that describes the pressure distribution of the flow $P(z, \theta)$:

$$\frac{\partial}{\partial z}\left[C_1 \frac{\partial P}{\partial z} + C_2\right] + \frac{\partial}{\partial \theta}\left[C_3 \frac{\partial P}{\partial \theta} + C_4\right] = 0 \tag{7}$$

Where the functions C_i are defined as a function of local geometry, liquid properties and the casing's rotation:

$$C_1 = \frac{1}{4\mu}\left[\frac{R_0^4 - R_i^4}{4} - \frac{\left(R_0^2 - R_i^2\right)\left(R_0^2 + R_i^2\right)}{2} + \frac{\left(R_0^2 - R_i^2\right)^2}{4\ln R_0/R_i}\right] \tag{8}$$

$$C_2 = -\frac{\rho g z}{4\mu}\cos\alpha\frac{d\alpha}{dz}\left[\frac{R_0^4 - R_i^4}{4} - \frac{\left(R_0^2 - R_i^2\right)\left(R_0^2 + R_i^2\right)}{2} + \frac{\left(R_0^2 - R_i^2\right)^2}{4\ln R_0/R_i}\right] \tag{9}$$

$$C_3 = \frac{1}{36\mu\,(R_0 + R_i)}\left[2R_0^4 + 2R_0^3 R_i + 12R_0^2 R_i^2 \ln R_i/R_0 - 2R_0 R_i^3 - 2R_i^4\right] \tag{10}$$

$$C_4 = \frac{\Omega_i R_i^2}{R_i^2 - R^2}\left[\frac{R^2 - R_i^2}{2} - R^2 \ln\frac{R}{R_i}\right] + \frac{\rho g \cos\alpha\cos\theta}{36\mu\,(R_0^5 + R_0^4 R_i - 8R_0^3 R_i^2 + 8R_0^2 R_i^2 - R_0 R_i^4 - R_i^5)} \tag{11}$$

From the equations above we can see that the casing's rotation contribution only influence the constant C_4.

In order to evaluate the evolution of the interface along the time, the concentration, ϕ, is convected as shown in Eq. 12.

$$\frac{\partial \phi}{\partial t} + v.\nabla\phi = 0 \tag{12}$$

Where the concentration gradient is given by:

$$\nabla\phi = u\frac{\partial C}{\partial z} + v\frac{\partial C}{\partial r} + w\frac{1}{r}\frac{\partial C}{\partial \theta} \tag{13}$$

When the casing's rotation were not considered the last term in Eq. 13 was zero and the concentration variation in azimuthal direction was neglected. The presence of a tangecial velocity due to the rotation of the casing will take into account the concentration variation in azimuthal direction and, consequently, help to stabilize the interface front.

3. RESULTS

The effect of casing's rotation is illustrated in the following example. For this example, it was considered a higher density Newtonian liquid ($\rho_1 = 1100\ kg/m^3$, $\mu_1 = 0.004\ Pa.s$) placed in a vertical eccentric annular space ($e_y = 0.01\ m$, $R_i = 0.04\ m$, $R = 0.05\ m$, $L = 200\ m$) that is displaced by a lower density Newtonian liquid ($\rho_1 = 750\ kg/m^3$, $\mu_2 = 0.004\ Pa.s$).

Figure 2 shows the evolution of the displacement front along the time. Also, the casing's rotation is increased from zero until 5 rad/s. Each subfigure in Figure 2 shows the mesh of θ versus z.

As the rotation increases, the interface is been gradually stabilized. For $\Omega = 0,1\ rad/s$ the displacement presents a better configuration for the interface than for the zero rotation case. Still, the flow finds a preferential path due to eccentricity and the displacement front is non-uniform. On the other hand, for $\Omega = 5\ rad/s$ even with the presence of the eccentricity along the well, the interface was completely stabilized and there were no preferential path.

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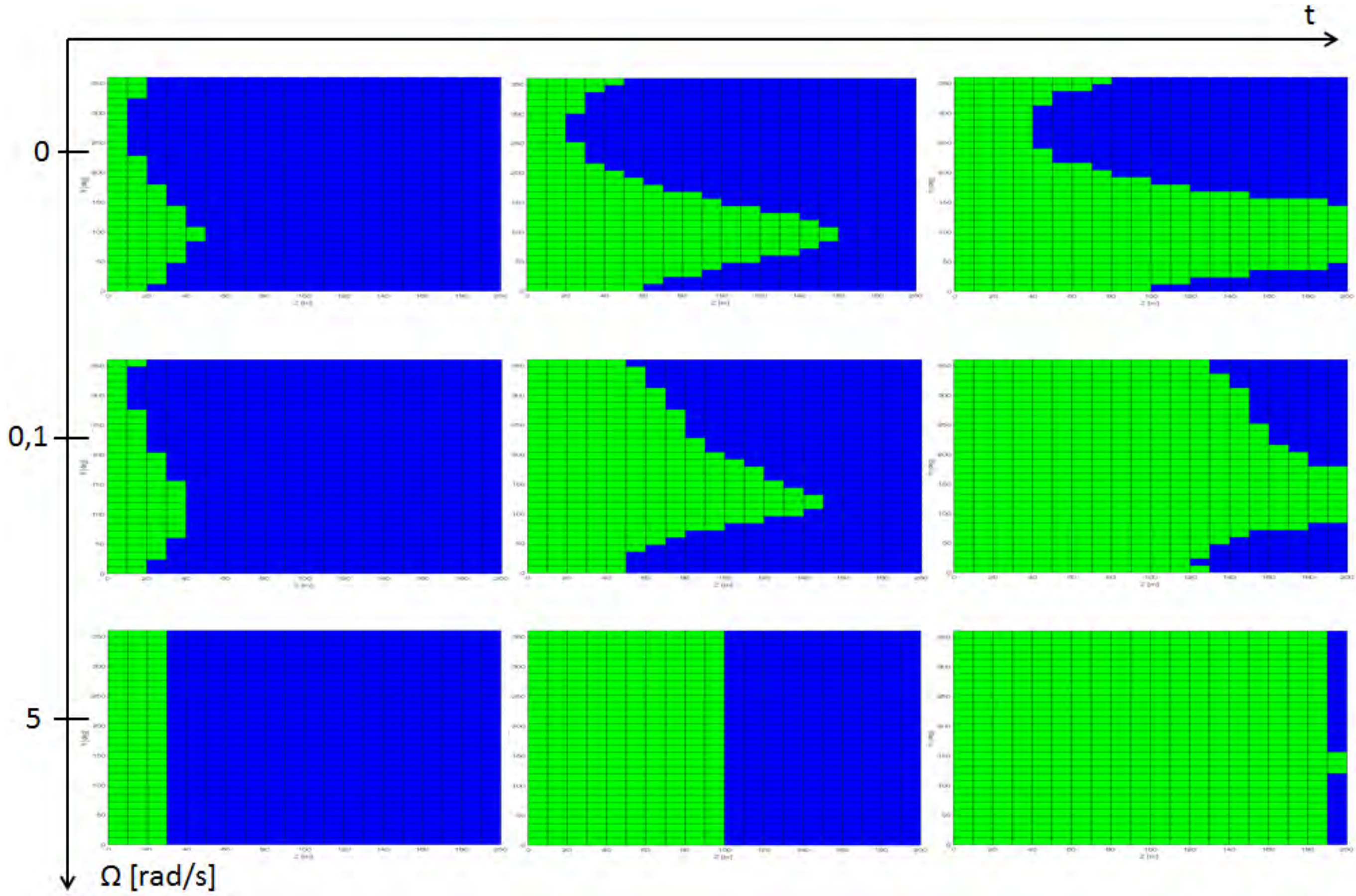

Figure 2. Evolution of the interface for two Newtonian liquids.

4. FINAL REMARKS

A lubrication based model for the displacement of a liquid inside an annular space considering the rotation of the casing is presented. It is an extension of previous development of a model for displacement of Newtonian liquids under laminar flow conditions. The results show the importance of the inclusion of the casing's rotation for the stabilization of the displacement front and, consequently, a better performance of the cementing process.

5. REFERENCES

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References must be listed in alphabetical order, according to the last name of the first author. See the following examples:

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