ANALYTICAL STUDY OF MOTION EFFECTS OF A SUBMERGED SOLID ON SLOSHING INSIDE A RECTANGULAR TANK

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Abstract. *In this paper, a two-dimensional method of calculating forces on a submerged body in a rectangular tank partially filled and excited horizontally is proposed. The method was developed based on potential theory and uses the dimensions of the tank and the filling level to calculate the velocity of the fluid in the position where the object is located. By using the velocity, the dimensions and the drag coefficient of the body, the force due to sloshing can be calculated. The results showed the method provides a good approximation of the forces due to sloshing on the submerged body, and it contributes to better understanding of the phenomena.*

Keywords*: sloshing; submerged solid*

1. INTRODUCTION

Sloshing is a phenomenon of great interest in the design of liquid container like fuel tanks. Longitudinal lateral structures can be used to increase damping on the flow or longitudinal bulkheads can be installed to change the resonance frequency. However, these solutions only work at a limited range of filling levels of the tank.

In this work, aiming to reduce the undesirable sloshing for a wider range of filling level, a dynamic buffer that is connected to the tank structure by springs is considered. An analytical dynamics modeling is proposed to calculate lateral sloshing loads on a tank with a submerged body connected to the tank by horizontal springs.

The analytical model was developed from the connection of two mass-spring-damping system by a damping force: The first is the mechanical system which represents the submerged body with the springs connecting it to the tank and the second system is the sloshing motion, which is represented by a mechanical system of mass-spring.

2. ANALYTICAL MODEL

The system considered herein consists of a tank partially filled with liquid and a submerged body connected to the tank structure by springs, as shown in Figure 1. The tank is excited by a horizontal harmonics motion that results in sloshing motion.

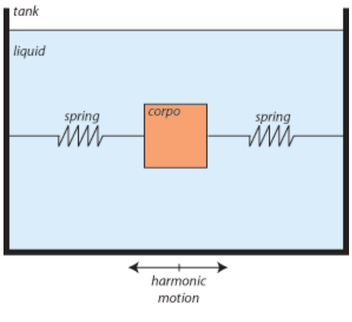

Figure 1 – Tank partially filled with liquid and a submerged body connected elastically to the tank.

This system can be interpreted as the composition of two different groups linked together by damping force. The first system is composed of the tank with fluid and the second is composed of the submerged body connected to the tank by springs as shown in Figure 2.

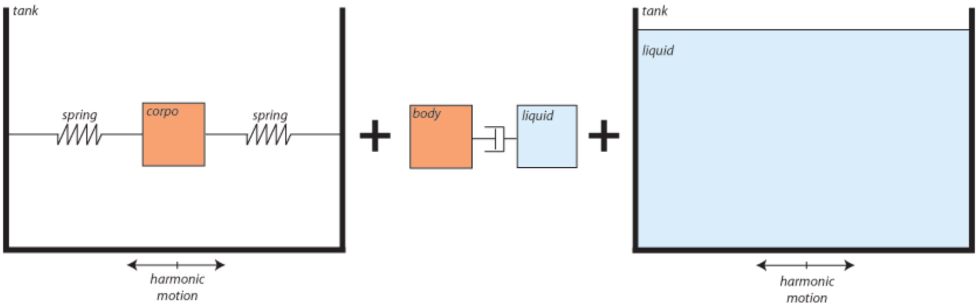

Figure 2 – Damping force used to connect two mechanical systems.

In this work, two different approaches were used. In the modeling of the phenomena, the first approach is based on the velocity field of the fluid using the potential theory. In the second approach, lateral forces are calculated considering the tank sloshing as a mechanical system.

The first approach is useful to identify positions that a suppressor may be more efficient and to calculate the force between the submerged object and the fluid.

The second approach is useful for the coupling all elements of the system to calculate the lateral force response on the tank in frequencies domain.

The analytical model presented herein to investigate the effects of the submerged body on the sloshing is based on the analytical model of sloshing proposed by (Graham & Rodriguez, 1952).

Details of the analytical development, assumptions and formulations will be presented in the following.

2.1. Potential theory of sloshing

As shown in (Graham & Rodriguez, 1952), assuming a two-dimensional rectangular tank with filling level h and width b (as illustrated in Figure 3 together with the coordinate system adopted herein), the incompressible, inviscit and irrotational flow in its interior can be described using the velocity potential ϕ that may be the solution of Laplace equation:

$$\frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0 \text{ ou } \nabla^2 \phi = 0 \tag{1}$$

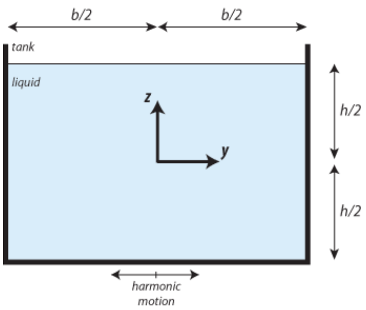

Figure 3 – Coordinate system and geometry of the rectangular tank

By assuming small amplitude motion, the free surface is given by the linearized condition:

$$g\eta\left(y, \frac{h}{2}, t\right) = \frac{\partial \phi}{\partial t}\left(y, \frac{h}{2}, t\right) \tag{2}$$

Further the kinematic boundary condition must be fulfilled:

$$\frac{\partial \eta}{\partial t}\left(y, \frac{h}{2}, t\right) = -\frac{\partial \phi}{\partial z}\left(y, \frac{h}{2}, t\right) \tag{3}$$

Where g is the gravity acceleration and η is the free surface equation.

Assuming $y_{tank}\ (t)$ as the horizontal motion imposed on the tank that is sinusoidal with amplitude A, we have:

$$y_{tank}(t) = A\sin(\omega t) \quad (4)$$

The boundary conditions on the tank lateral walls are:

$$\frac{\partial\phi}{\partial y}\left(\pm\frac{b}{2}, z, t\right) = A\cos(\omega t) \quad (5)$$

And on the bottom:

$$\frac{\partial\phi}{\partial z}\left(y, -\frac{h}{2}, t\right) = 0 \quad (6)$$

Leading to the solution:

$$\phi = -A\omega\cos\omega t\left\{y + \sum_{n=0}^{\infty}(-1)^n\frac{4b}{\pi^2(2n+1)^2}\left(\frac{\omega^2}{\omega_n^2-\omega^2}\right)\times\frac{\sin\left[(2n+1)\frac{\pi}{b}y\right]\cosh\left[(2n+1)\frac{\pi}{b}\left(z+\frac{h}{2}\right)\right]}{\cosh\left[(2n+1)\pi\frac{h}{a}\right]}\right\} \quad (7)$$

Where the frequencies of resonance ω_n are given by:

$$\omega_n^2 = (2n+1)\frac{\pi g}{b}\tanh\left[(2n+1)\frac{\pi h}{b}\right] \quad (8)$$

Deriving (7) in y and z directions, the velocity fields in each direction are:

$$\frac{\partial\phi}{\partial y} = -A\omega\cos(\omega t)\left[1 + \sum_{n=0}^{\infty}(-1)^n\frac{4}{\pi(2n+1)}\left(\frac{\omega^2}{\omega_n^2-\omega^2}\right)\frac{\cos\left[(2n+1)\frac{\pi}{b}y\right]\cosh\left[(2n+1)\frac{\pi}{b}\left(z+\frac{h}{2}\right)\right]}{\cosh\left[(2n+1)\pi\frac{h}{b}\right]}\right] \quad (9)$$

$$\frac{\partial\phi}{\partial z} = -A\omega\cos(\omega t)\left[\sum_{n=0}^{\infty}(-1)^n\frac{4}{\pi(2n+1)}\left(\frac{\omega^2}{\omega_n^2-\omega^2}\right)\frac{\sin\left[(2n+1)\frac{\pi}{b}y\right]\sinh\left[(2n+1)\frac{\pi}{b}\left(z+\frac{h}{2}\right)\right]}{\cosh\left[(2n+1)\pi\frac{h}{b}\right]}\right] \quad (10)$$

Figure 4 shows the plots of velocity amplitudes in directions y and z and the absolute values of the velocity obtained by applying the potential theory to a tank with a width of 1.0 m and filling level of 1.0 m and excitation amplitude of 1.0m.

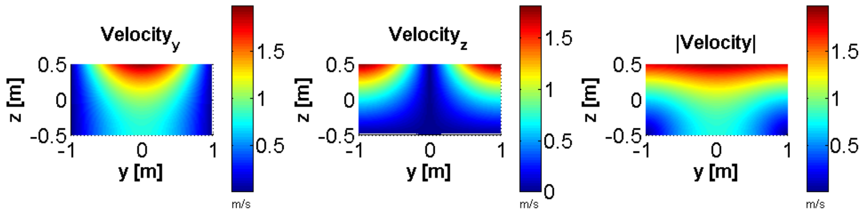

Figure 4 – y and z components and the absolute values of the velocity amplitude of the liquid in a tank with width of 1.0 m and filling level of 1.0 m and excitation amplitude of 1.0cm.

Analyzing the velocity field it may be observed that there are horizontal high speeds in the middle and near the free surface and high vertical speeds in the region near the wall and the free surface.

It is usual to place horizontal girders on the tanks side walls to reduce the sloshing motion due to higher fluid velocity in this region. Following the same reasoning, it is interesting to place an object in the middle and near the fluid surface to introduce additional damping reducing the sloshing motion.

2.2. Mass-spring system of sloshing

According to (Graham & Rodriguez, 1952), the lateral force due to sloshing can be calculated by integrating the pressure along the wall of the tank. This force is given dimensionless by:

$$\frac{F}{m_T g} = \frac{A}{h} f^2 \, sin\, \Omega t \left\{ 1 + \sum_{n=0}^{\infty} \frac{8 \tanh\left[(2n+1)\pi \frac{h}{b}\right]}{\pi^3 (2n+1)^3} \frac{1}{\left(\frac{f_n}{f}\right)^2 - 1} \right\} \qquad (11)$$

Where, m_T is the total mass of the fluid inside the tank, $f^2 = \frac{h\omega^2}{g}$ and $f_n^2 = \frac{h\omega_n^2}{g} = (2n+1)\pi \frac{h}{b} \tanh\left[(2n+1)\pi \frac{h}{b}\right]$.

The calculation of lateral forces due to sloshing under small amplitude excitation for different frequencies can be done considering a mechanical system composed by a static sloshing mass m_{ss} and an infinite set of dynamic mass-spring m_{sn}, k_{sn} that only move in the tank motion direction. The equations of motion of this system in response to sinusoidal motion of the tank are given by:

$$m_{sn}(\ddot{y} - A\omega^2 \sin \omega t) = -k_{sn} y \qquad (12)$$

Where, $k_{sn} = m_{sn}\omega_n^2$, n is the odd harmonic modes.

To calculate forces on the mechanical system described by (11) the sloshing dynamic masses m_{sn}, the sloshing static masse m_{ss} and the springs constants k_{sn} for n-th odd vibration mode may be formulated as follows:

$$\frac{m_{sn}}{m_T} = \frac{8 \tanh\left[(2n+1)\pi \frac{h}{b}\right]}{[(2n+1)\pi]^3 \frac{h}{b}} \qquad (13)$$

$$\frac{m_{ss}}{m_T} = 1 - \sum_{n=0}^{\infty} \frac{8 \tanh\left[(2n+1)\pi \frac{h}{b}\right]}{[(2n+1)\pi]^3 \frac{h}{b}} \qquad (14)$$

$$\frac{k_{sn} h}{m_T g} = \frac{\tanh^2\left[(2n+1)\pi \frac{h}{b}\right]}{[(2n+1)\pi]^2} \qquad (15)$$

Thus, the equivalent mechanical system can be represented as shown in Figure 5.

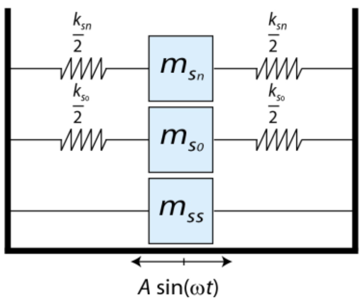

Figure 5 – Mechanical system diagram representing sloshing

Combining (11) with (14) we have:

$$\frac{F}{m_T} = \frac{A}{h} f^2 \sin \Omega t \left\{ \frac{m_{sf}}{m_T} + \sum_{n=0}^{\infty} \frac{m_{sn}}{m_T} + \sum_{n=0}^{\infty} \frac{m_{sn}}{m_T} \frac{1}{\left(\frac{f_n}{f}\right)^2 - 1} \right\} \qquad (16)$$

Figure 6 shows the graph of the dimensionless spring constant according to the aspect ratio given by Eq. (15).

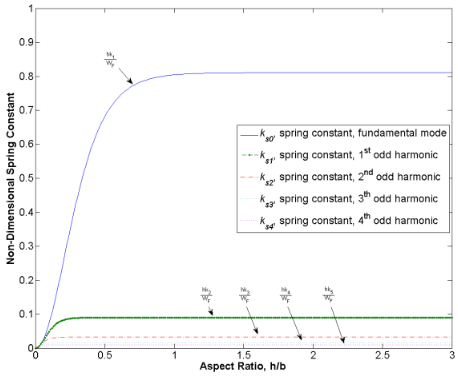

Figure 6 – Spring constants for sloshing mechanical system

Each spring constant k_{sn} is related to a dynamic mass m_{sn} given by Eq. (13). The graph of this function is given in Figure 7 along with the static mass obtained by Eq. (14).

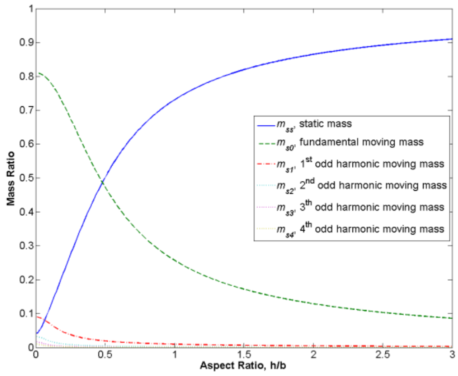

Figure 7 – Masses for sloshing mechanical system

From Figure 7, the two significant vibration modes masses in the range of aspect ratio between 0.3 and 1.5 are the static mass and the fundamental mode mass m_{s_0}. Therefore, only the static mass and the fundamental dynamic mass are considered in the analytical model of sloshing with the dynamic submerged body.

2.3. Mass-spring-mass system of sloshing with submerged body

Figure 8 illustrates a tank (width b and filling depth h) filled with fluid (total mass m_s) with a submerged body (mass m_b) connected to the tank structure by horizontal springs. The tank is excited harmonically with an amplitude A and frequency ω.

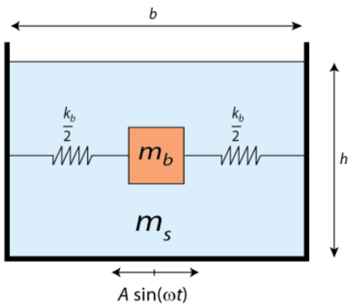

Figure 8 – Rectangular tank filled partially with a submerged body connected by horizontal springs.

To calculate the lateral force of this system, the fundamental dynamic motion of the sloshing mechanical model is coupled to the motion of the submerged body. The proposed coupling technique is to connect the dynamic sloshing mass to the submerged body mass through a hydrodynamic damping c_b.

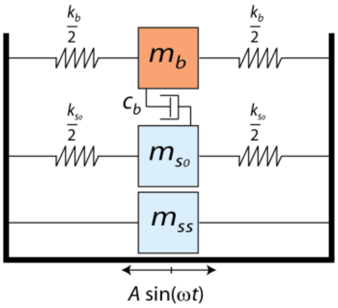

Figure 9 – Diagram of the mass-spring-damping system of the model.

By rearranging only the dynamic part of the system shown in Figure 9, it is possible to obtain a system of two degrees of freedom as shown in Figure 10, where the sloshing mass m_{s_0} is connected to the source of harmonic motion y_0 through spring which constant is k_{s_0} and damping c_{s_0}, by a damping c_b, m_{s_0} is also connected to the submerged body mass m_b, which is connected to y_0 by the spring which constant is k_b.

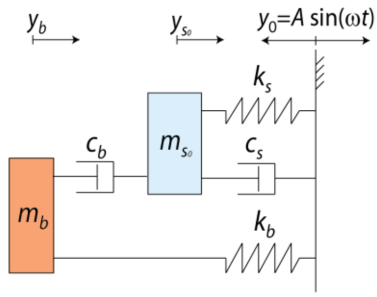

Figure 10 – Rearranged diagram of the mass-spring-damping system of the model.

The two equations show the balance of forces on m_{s_0} and m_b are given by:

$$m_{s_0}\ddot{y}_{s_0} = -k_{s_0}(y_{s_0} - y_0) - c_{s_0}(\dot{y}_{s_0} - \dot{y}_0) - c_b(\dot{y}_{s_0} - \dot{y}_b)$$
$$m_b\ddot{y}_b = -k_b(y_b - y_0) - c_b(\dot{y}_b - \dot{y}_{s_0}) \quad (17)$$

By developing these two equations, Eq. (18) can be obtained.

$$\frac{y_{s_0}}{A} = \frac{\left[\mu(f^2 - g^2) - 4\frac{c_b c_w}{c_c^2} g^2\right] + i\left[2\frac{c_b}{c_c} g(1 + \mu f^2) + 2\mu\frac{c_w}{c_c} g(f^2 - g^2)\right]}{\left[\mu(g^2 - 1)(g^2 - f^2) - 4\frac{c_b c_w}{c_c^2} g^2\right] + i\left\{2\frac{c_b}{c_c} g[1 - g^2 + \mu(f^2 - g^2)] + 2\mu\frac{c_w}{c_c} g(f^2 - g^2)\right\}} \quad (18)$$

Where: $= \frac{m_b}{m_{s0}}$, $\omega_{s_0 n}^2 = \frac{k_{s0}}{m_{s0}}$, $\omega_{bn}^2 = \frac{k_b}{m_b}$, $f = \frac{\omega_{bn}}{\omega_{s0n}}$, $g = \frac{\omega}{\omega_{s0n}}$ and $c_c = 2m_{s_0}\omega_{s_0 n}$.

3. RESULTS AND DISCUSSIONS

To analyze the developed model, the results of sloshing without the submerged body were compared to the results obtained by (Graham & Rodriguez, 1952). After that, The model with a dynamic submerged body was studied by varying damping coefficients.

3.1. Sloshing

The model used for comparisons of two-dimensional sloshing was a tank with 2.0 m wide, 2.0 m high and filled with water up to 1.0 m as shown in Figure 12. Harmonic horizontal motion was imposed to the tank to excite the fluid.

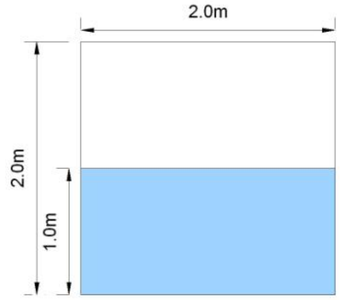

Figure 11 – Dimensions of the model used to compare results of sloshing.

The response of the analytical model developed in this work considering $c_b = 0$ is shown in Figure 12 together with the response of the model described in (Graham & Rodriguez, 1952). A third curve was drawn using the present work model with ratio between sloshing damping and critical damping equal to 0.03 $\left(\frac{c_{s0}}{c_c} = 0.03\right)$ which is the value obtained from experiments by (Warnitchai & Pinkaew, 1998).

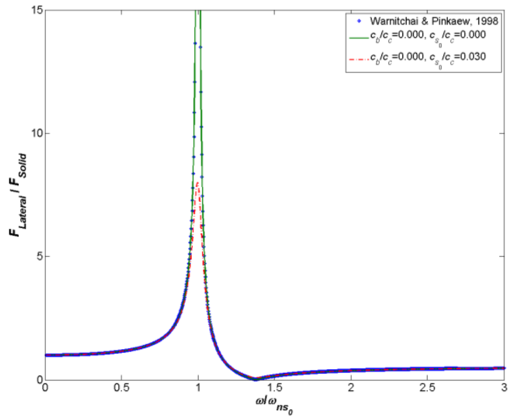

Figure 12 – Lateral forces due to sloshing as a function of the period of excitation of a tank with a width of 2.0m of water level of completion of 1.0m.

The curves of the response of the system without damping are exactly the same. The third curve shows that the inclusion of the damping of sloshing c_{s_0} affects the response consistently.

3.2. Sloshing with submerged body connected to the tank by springs

The model used for sloshing comparisons was a two-dimensional tank with 2.0 m wide, 2.0 m high and filled with water up to 1.0 m with a submerged square body linked to the tank by two lateral spring ($k = 8932.6$ N/m) as shown in Figure 13. Horizontal harmonic motion was imposed on the tank with amplitude of 3.0cm.

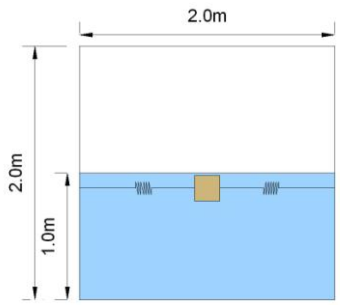

Figure 13 – Dimensions of the model used to compare results of sloshing with a submerged body.

Figure 14 gives the results of lateral forces on the tank with and without the submerged object of 0.2cm height and width. In the figure, the damping coefficients and natural frequencies are presented.

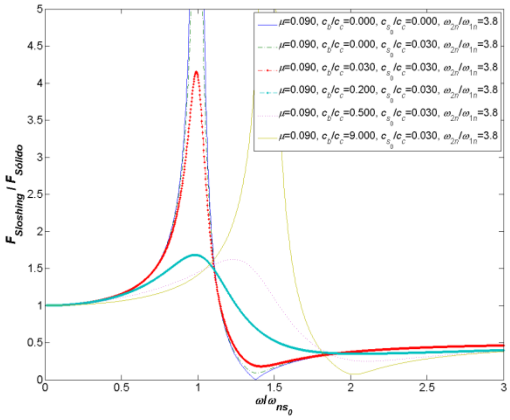

Figure 14 – Lateral forces due to sloshing depending on the period of excitation of a tank with a width of 2.0 m of fill level of 1.0 m of water with a submerged body.

The graph shows that the submerged dynamic body has small or no effect at low frequencies $\left(\frac{\omega}{\omega_{s0}} < 0.5\right)$.

Near the sloshing resonance frequency the amplitude declines proportionally to the damping of the body $\left(\frac{c_b}{c_c} < 0.2\right)$. For larger values of $\frac{c_b}{c_c}$ there is a shift of the natural frequency. This behavior is due to the high value of damping which decrease the relative motion between the two bodies making the whole $m_{s_0} + m_b$ to behave as a single body.

4. CONCLUSION AND FUTURE WORK

Analytical model to investigate the effects of the dynamics of a submerged body on the sloshing motion is presented in the present work.

The results show that the submerged body has small or no effect at low frequencies and near the resonance frequency of sloshing the amplitude declined due to the increase of damping. And, for higher values damping the two masses behave as a singles body.

5. ACKNOWLEDGEMENTS

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6. REFERENCES

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