FINITE ELEMENT MODELING OF EDDY-CURRENT COUPLERS

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Abstract. Eddy-current systems have been employed in many industrial applications such as in braking, transmission or damping systems. For coupling applications, the main advantage with respect to other devices is the possibility of working in complete absence of mechanical contact, eliminating wear problems and reducing the need for high accuracy alignment techniques, typically required in mechanical coupling systems. In this work, the analysis of an eddy-current coupler is performed using both analytical and numerical methodologies. The analytical solution is developed using a lumped parameter model of a conductor moving in a magnetic field. The construction and analysis of the numerical model is done in a partial differential equation solver, and the results are obtained by the insertion of the Maxwell’s equations in terms of the magnetic vector potential directly into the solver. The studies are developed by means of simplified models, and the validity of the approximations is shown by comparison of the results with experimental data obtained from bibliography.

Keywords: Eddy-current, coupling systems, finite element, electromagnetism

1. INTRODUCTION

The working principle of eddy-current couplers relies on the interaction between a conductor in motion with respect to a magnetic field. This motion determines an electric field in the conductor that induces eddy currents. The eddy-currents interact with the same magnetic field that induced them to create the coupling effect. Usually a coupler is constituted by a magnetic circuit with several magnetic pole pairs oriented in attraction, and a conductor disk that spins in the air gap. Figure 1 shows a possible configuration for the coupler with 4 pole pairs, where \( R_m \) is the radial position of the center of the magnets and \( R_d \) is the conductor disk radius. The main advantage of this kind of configuration is the possibility of working without mechanical contact, eliminating wear problems and reducing the need for high accuracy alignment techniques typically required in mechanical coupling systems. Furthermore, the employment of high energy product rare earth permanent magnets and high conductivity materials allow good coupling performance.

The design techniques and analysis of eddy-current couplers are already well consolidated, and many works on the subject can be found in the literature. The solution approach to the problem can be purely numerical (Nehl et al., 1994), purely analytical (Tonoli, 2007) or hybrid (Canova and Vusini, 2003). In order to get deeper insight, experimental characterization can be carried out, and in a recent work Amati and Tonoli (2008) showed the procedure to obtain design parameters from experimental tests and the importance of these parameters for design of eddy-current dampers and couplers in dynamic conditions.

In this work a finite element scheme is used to solve a 3D model of an eddy current coupler. The analytical solution of a conductor in linear motion in a magnetic field is introduced based on the equivalent mechanical model developed by Amati et al. (2008) in order to obtain the mechanical parameters of the eddy current coupler. Next, the governing equations are directly introduced in a partial differential equation solver.

Figure 1. Typical configuration of an eddy current coupler.
2. LUMPED PARAMETER MODEL OF A LINEAR EDDY-CURRENT COUPLER

The relative motion between a conductor and a magnetic field generates an electric field that is governed by Faraday’s law and is oriented according to Lenz law. In its simpler form an eddy-current coupler can be represented as a lumped parameter electric system of a $N$ turns coil shunt-connected to a resistance $R$ and an inductance $L$, with velocity $v$ relative to a perpendicular magnetic field $B$, as shown in Fig. 2. For this system, the induced electric field is proportional to the velocity of the relative motion and results in an induced electromotive force $\varepsilon_{emf}$ between the circuit terminals.

![Figure 2. Lumped parameter model of an eddy-current coupler](image)

As the circuit is closed an electric current $i$ will flow, and its interaction with the constant magnetic field generates a force $F$ proportional to the current and in opposite direction with respect to the velocity. This phenomenon is governed through the state equation given by Eq. (1) and the output expression is specified by Eq. (2).

\[
\frac{di}{dt} = -\omega_{RL} i + \frac{K_m}{L} v \\
F = K_m i
\]

(1)  
(2)

where the electromechanical constant $K_m$ is equal to $NBL$, and the frequency of the electric pole of the system $\omega_{RL}$ is equal to $R/L$.

Equations (1) and (2) correspond to the same formalism of a mechanical system formed by a spring in series with a viscous damper. A mechanical equivalent representation is shown in Fig. 3.

![Figure 3. Equivalent mechanical representation of an eddy-current coupler](image)

This analogy allows to obtain more straightforward results because of the coupler can be best described as a mechanical system, and Eqs. (3) and (4) show the new set of equations to be solved.

\[
\dot{x} = -\omega_{RL} x + v \\
F = k x
\]

(3)  
(4)

where the natural frequency $\omega_{RL}$ is equal to $k/c$, the equivalent stiffness $k$ is equal to $K_m^2/L$ and the damping coefficient $c$ is equal to $K_m^2/R$, all corresponding to the mechanical representation, (Tonoli, 2007).

In the eddy-current coupler, the relative velocity between the conductor and the magnetic field can be approximated as the tangential speed of the conducting disk at the mean radius of the magnets $R_m$. When the conductor is in motion with constant slip speed $\Omega$, the magnetic field seen by each point of it at radius $R_m$ is approximately sinusoidal. This
allows that Eq. (1) can be solved considering that the coil in Fig. 2 moves at a velocity \( v \) with harmonic profile and angular frequency \( \Omega \), in the magnetic field frame of reference.

Then, Eq. (5) gives the torque obtained from the coupler as function of the slip speed. This expression has the classic form of the curve torque versus slip speed of an induction machine running at constant speed, and it is obtained by multiplying Eq. (2) times \( R_m \).

\[
T = \frac{c \Omega R_m^2}{1 + \left( \frac{\Omega}{\omega_{RL}} \right)^2}
\]  

Two design parameters are necessary to build the torque - slip speed curve of an eddy current coupler using Eq. (5). They are the angular frequency \( \omega_{RL} \) of the electric pole and the equivalent damping coefficient \( c \), which can be obtained by either numerical simulations or experimental tests.

### 3. FINITE ELEMENT SCHEME

For a given coupler, a finite element model can be used for obtaining more approximate design parameters. The eddy-current coupler, as seen before, behaves as a typical induction machine and for simulating this kind of behavior a finite element scheme using a partial differential equation (PDE) solver is used. In this section, an approaching method using the direct insertion of Maxwell equations into the solver is presented.

#### 3.1. Formulation of the eddy-current problem

An induction problem can be characterized by the solution of Eqs. (6) and (7), that represent the laws of Faraday and Ampere respectively.

\[
\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}
\]

\[
\nabla \times \vec{B} = \mu \vec{J}
\]  

These two equations show that time-varying solution of electromagnetic problems with finite elements require the discretization of both the problem regions and the time domain, using time-stepping methods (Henneberger et al., 2000). According to Rodger et al. (2002), moving conductor eddy-current problems can be solved in steady state conditions by means of a Minkowski transformation provided that the conductor has invariant cross-section at right angles to the direction of motion (smooth conductor). This approach uses the quasi-static approximation, and Lorentz current density terms accounts for the motional eddy-currents. This procedure eliminates the need for time-stepping solutions.

The utilization of a magnetic vector potential \( \vec{A} \) formulation for writing Eqs. (6) and (7) gives the governing equations for a moving smooth conductor eddy-current problem in a formalism more adequate to our purposes. If only linear materials are used, Eq. (8) gives the equation to be solved in the conductor disk and Eq. (9) gives the equation to be solved in all other domains of the problem.

\[
-\frac{1}{\mu} \nabla^2 \vec{A} = -\sigma \vec{v} \cdot \nabla \vec{A} + \vec{A} \times \omega
\]  

\[
-\frac{1}{\mu} \nabla^2 \vec{A} = 0
\]  

where \( \vec{v} \) is the conductor’s speed and \( \omega \) is its angular velocity.

#### 3.2. Setting the PDE problem

To solve the eddy-current coupler problem, a finite element solver for partial differential equations was chosen. In this kind of approach a general form equation is given, and many types of physical problems can be solved directly by introducing the properties of the problem in the general equation. Equation (10) shows the expression to be solved in each region of the coupler problem. In this case, the representative coefficients \( c, \gamma, a \) and \( \beta \) must be obtained by manipulation of Eqs. (8) and (9), resulting in a direct insertion of Maxwell’s equations into the solver.
\[ \nabla \cdot \left( -c \nabla \tilde{u} + \gamma \right) + a \tilde{u} + \beta \cdot \nabla \tilde{u} = 0 \]  (10)

From the conceptual point of view, the eddy-current coupler contains only three different regions. These are the conductor disk, where the induction phenomenon takes place; the permanent magnets that generate the magnetic field; and the back iron and surrounding air, where the last two (back iron and surrounding air) can be represented using the same coefficients. Equations (11), (12), (13) and (14) give the values of the coefficients that characterize the governing equations for each region in terms of classical electric and magnetic quantities for coordinates \( x, y \) and \( z \). Using these coefficients it’s possible to do modeling in two or three dimensions.

\[
\begin{align*}
    c_{x,y,z} &= \frac{1}{\mu} \\
    \gamma_x &= \{ 0, -\mu_0 M_z, \mu_0 M_y \} \\
    \gamma_y &= \{ \mu_0 M_x, 0, -\mu_0 M_z \} \\
    \gamma_z &= \{ -\mu_0 M_y, \mu_0 M_x, 0 \} \\
    a_x &= \{ 0, \omega_z, -\omega_y \} \\
    a_y &= \{ -\omega_z, 0, \omega_x \} \\
    a_z &= \{ \omega_y, -\omega_x, 0 \} \\
    \beta_{x,y,z} &= -\sigma \ddot{v}
\end{align*}
\]  (11-14)

From this setting the solution for the magnetic vector potential is obtained. More meaningful quantities such as the magnetic flux density \( \vec{B} \), the electric current density \( \vec{J} \) and the Lorentz force \( \vec{F} = \int \vec{J} \times \vec{B} \, dv \) can be calculated as post-processing quantities.

3.3. Finite element model

The finite element model of the eddy-current coupler is constructed using the anti-symmetry plane in the middle of the conductor and a sector of 90° is considered to take advantage of the model’s cyclic symmetry, resulting in a simulation model of one eighth of the total problem geometry.

The geometric dimensions and properties of the problem are given in Tab. 1. All characteristics were considered to be linear. No thermal or saturation effects are taken into account as they are considered to be irrelevant in this case.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Geometric parameter</th>
<th>Value [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>( g )</td>
<td>Air gap</td>
<td>1</td>
</tr>
<tr>
<td>( S_d )</td>
<td>Disk thickness</td>
<td>7</td>
</tr>
<tr>
<td>( S_m )</td>
<td>Magnet thickness</td>
<td>6</td>
</tr>
<tr>
<td>( M_r )</td>
<td>Magnet radius</td>
<td>30</td>
</tr>
<tr>
<td>( R_d )</td>
<td>Conductor radius</td>
<td>80</td>
</tr>
<tr>
<td>( R_m )</td>
<td>Radial position of the magnets</td>
<td>55</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>Electric conductivity of copper</td>
<td>5.998E7 [AV⁻¹]</td>
</tr>
<tr>
<td>( B_r )</td>
<td>Remanent magnetic flux density</td>
<td>1.22 [T]</td>
</tr>
<tr>
<td>( \mu_0 )</td>
<td>Magnetic permeability of vacuum</td>
<td>4\pi\times10^{-7}</td>
</tr>
<tr>
<td>( \mu_r )</td>
<td>Relative magnetic permeability of iron</td>
<td>4000</td>
</tr>
</tbody>
</table>

Figure 4 shows the eddy-current distribution in the conductor disk as a consequence of its relative motion in the magnetic field. It is true that the proposed scheme is capable of taking into account the coupling between the two directions of motion and the static magnetic field. Also the cyclic boundary conditions ensure the continuity of the quantities across them.
As the angular velocity grows, the velocity currents increase and their distribution in the conductor change due to the $RL$ dynamics. Figures 5a and 5b show the Lorentz forces and magnetic flux density distribution at low frequency and at the electric pole frequency respectively.

![Figure 4. Three dimensional magnetic flux density and eddy-current distribution showed as arrows at 10 rpm](image1)

![Figure 5. Lorentz force distribution showed as arrows in the conductor, a) at low frequency (10 rpm), b) at electric pole frequency (860 rpm)](image2)

To obtain the coupling torque, the integration of the Lorentz force over the disk domain is carried out and Fig. 6a shows the comparison of the obtained results in the numerical model with experimental data obtained from the bibliography (Amati and Tonoli, 2008). The perfect correspondence between the two sets of data confirms the validity of the hypothesis assumed and the accuracy of the finite element scheme used to solve Maxwell’s equations. The equivalent mechanical design parameters can be obtained from the analytical model. A curve fitting between the numerical data and the analytical curve is done and Fig. 6b shows the comparison between these curves. An equivalent viscous damping of $c = 392.5 \text{ [Nsm}^{-1}]$ and a pole frequency of $\omega_{RL} = 860 \text{ [rpm]}$ is obtained.

At higher frequencies the analytic and numerical curves deviate a little probably due to the bi-dimensional nature of the analytical model that is not capable of taking into account some of the effects that are inherently three-dimensional.
4. CONCLUSIONS

In this work a finite element scheme using the direct insertion of the governing equations in a partial differential equation solver is presented and used to solve a 3D model of an eddy current coupler. The procedure is validated for the simulation of such components by comparison of the obtained results with experimental data obtained from literature.

The analytical solution of a conductor in linear motion in a magnetic field is developed as an equivalent lumped parameters model of the eddy-current coupler and a curve fitting of the analytical curve is used to obtain the mechanical equivalent design parameters.

Concluding, both the analytical model and the numerical scheme allow for good results and can be used as design tools for eddy-current couplers and similar devices.

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6. REFERENCES


5. RESPONSIBILITY NOTICE

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