MODELING PLASTIC STRAIN LOCALIZATION PROMOTED BY THERMOMECHANICAL COUPLING IN METALLIC MATERIALS

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Abstract. *Thermomechanical coupling is an important phenomenon in different engineering problems. Inelastic cyclic strain promotes heating of metallic structural elements, and a considerable amount of heat can be generated in situations where high loading rates and/or high amplitudes of inelastic strain are of concern. The temperature rise of mechanical component depends on the loading amplitude, frequency and temperature boundary conditions. Nevertheless, traditional low-cycle fatigue models neglect the material temperature variation due to thermomechanical coupling and unreal life predictions may be obtained. Indeed, there are situations where such couplings cannot be neglected and a physically more realistic model must take it into account. In this paper, a continuum mechanics model with internal variables is proposed to study the thermomechanical coupling effects of metallic components subjected to inelastic loadings. A thermodynamic approach allows a proper identification of the thermomechanical coupling in the mechanical and thermal equations. A numerical procedure is developed based on an operator split technique associated with an iterative numerical scheme in order to deal with the non-linearities in the formulation. With this assumption, coupled governing equations are solved involving three uncoupled problems: thermal, thermoelastic and elastoplastic behaviors. Classical finite element method is employed for spatial discretization in all uncoupled problems. Numerical simulations of steel tensile and fatigue test specimens subjected to inelastic loadings are presented and analyzed. Experimental analysis with infrared thermal camera is developed to measure the temperature distribution on the steel bars. Results suggest that the proposed model is capable of capturing important localization phenomena related to plastic strain localization due thermomechanical coupling.*

Keywords: *Thermomechanical Coupling, Modeling, Numerical Simulation, Elastoplasticity*

1. INTRODUCTION

When metallic structural elements are submitted to inelastic cyclic strain, a very important phenomenon must be considered: the thermomechanical coupling. This kind of solicitation promotes heating of the elements and a considerable amount of heat can be generated in situations where high loading rates and/or high amplitudes of inelastic strain are of concern (Simo and Miehe, 1992; Pacheco, 1994; Barbosa *et al.*, 1995; Pacheco and Costa-Mattos, 1997; Stabler and Baker, 2000, Rosakis *et al*., 2000; Longère and Dragon, 2008; Costa-Mattos and Pacheco, 2009). The temperature rise of mechanical component depends on the loading amplitude, frequency and temperature boundary conditions. It promotes a mechanical properties decrease, which in turn promotes a plastic strain increase. This phenomenon is known as thermomechanical coupling and can accelerate the structural degradation process.

Usually, the material temperature variation due to thermomechanical coupling is not taken into account in traditional low-cycle fatigue models, so unreal life predictions may be obtained. Since there are situations where such couplings cannot be neglected and a physically more realistic model must take it in consideration, this paper presents a continuum mechanics model with internal variables to study the thermomechanical coupling effects of metallic components submitted to inelastic loadings (Pacheco, 1994; Lemaitre and Chaboche, 1990). Figure 1 shows the feedback phenomenon that can be observed in metallic elements subjected to inelastic strain loadings (Nolte, 2007; Nolte *et al.*, 2007).

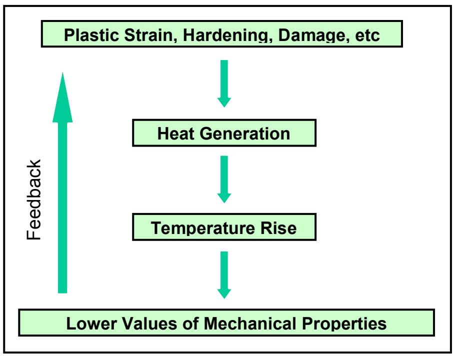


Figure 1. Thermomechanical coupling in metallic elements subjected to inelastic strain loadings.

A thermodynamic approach allows a proper identification of the thermomechanical coupling in the mechanical and thermal equations, while a numerical procedure is developed based on an operator split technique associated with an iterative numerical scheme in order to deal with the non-linearities in the formulation. Three uncoupled problems are involved to solve coupled governing: thermal, thermoelastic and elastoplastic behaviors. Classical finite element method is employed for spatial discretization in all uncoupled problems and numerical simulations of steel plates with stress concentrators subjected to inelastic loadings are presented and analyzed. Experimental analysis with infrared thermal camera has been developed to measure the temperature distribution on the specimens. Results suggest that the proposed model is capable of capturing important localization phenomena related to plastic strain localization due thermomechanical coupling.

2. CONSTITUTIVE MODEL

By considering thermodynamic forces, defined from the Helmholtz free energy, ψ, and thermodynamic fluxes, defined from the pseudo-potential of dissipation, ϕ, it is possible to formulate constitutive equations within the framework of continuum mechanics and the thermodynamics of irreversible processes, by (Lemaitre and Chaboche, 1990; Pacheco, 1994).

For this, a Helmholtz free energy is proposed as a function of total strain, ε_{ij}, temperature, T and observable variables. Besides, the following internal variables are considered: plastic strain, ε_{ij}^p, kinematic hardening, c_{ij}, and isotropic hardening, p. Therefore, the following free energy is proposed, employing indicial notation where summation convention (i = 1,2,3) is evoked (Eringen, 1967), except when indicated:

$$\rho\psi(\varepsilon_{ij},\varepsilon_{ij}^p,c_{ij},p,T)=\left[W_e(\varepsilon_{ij}-\varepsilon_{ij}^p,T)+W_a(c_{ij},p,T)\right]-W_T(T) \tag{1}$$

where ρ is the material density, W_e is the elastic energy density, W_a is the energy density associated to the hardening and W_T is the energy density associated with the temperature, defined as:

$$\begin{aligned} &W_e(\varepsilon_{ij}-\varepsilon_{ij}^p,T)=\frac{E}{2(1+\nu)}\left[(\varepsilon_{ij}-\varepsilon_{ij}^p)(\varepsilon_{ij}-\varepsilon_{ij}^p)+\frac{\nu}{1-2\nu}(\varepsilon_{jj}-\varepsilon_{jj}^p)^2\right]-\frac{\alpha E}{1-2\nu}(\varepsilon_{jj}-\varepsilon_{jj}^p)\\ &W_a(c_{ij},p,T)=\tfrac{1}{2}a\,c_{ij}c_{ij}+b\left[p+(1/d)e^{-dp}\right]\\ &W_T(T)=\rho\int_{T_0}^{T}C_1\log(\xi)\,d\xi+\frac{\rho}{2}C_2T^2 \end{aligned} \tag{2}$$

where T_0 is a reference temperature, E is the Young modulus, ν is the Poisson ratio, a is a material parameter associated with kinematic hardening, while b and d are material parameters associated with isotropic hardening. C_1 and C_2 are positive constants. The increment of elastic strain is defined as follows:

$$d\varepsilon_{ij}^e=d\varepsilon_{ij}-d\varepsilon_{ij}^p-\alpha_T dT\delta_{ij} \tag{3}$$

The last term is associated with thermal expansion and the parameter α_T is the coefficient of linear thermal expansion.

The general formulation of this model was developed and previously applied to the study of various related problems (Pacheco, 1994; Pacheco and Mattos, 1997; Pacheco *et al.*, 2001; Oliveira *et al.*, 2003; Oliveira, 2004; Silva *et al.*, 2004). A detailed description of this constitutive model may be obtained in the cited references.

This contribution considers life prediction of metallic plane truss structures subjected to cyclic inelastic loadings. From the mechanical point of view, it is assumed that the specimen is submitted to uniaxial strain. Concerning thermal characteristics, it is assumed that the specimen experiments a heat conveccion through its surface. Under these assumptions, a one-dimensional model is formulated and tensor quantities presented in the general formulation may be replaced by scalar quantities. For this situation the thermodynamics forces (σ_{ij}, P_{ij}, B_{ij}^c, B^c, B^p, s), respectively associated with state variables $(\varepsilon_{ij},\varepsilon_{ij}^p,c_{ij},p,T)$, are defined as follows:

$$\begin{aligned} &\sigma_{ij}=\rho\frac{\partial\psi}{\partial\varepsilon_{ij}}=\mathrm{E}(\varepsilon_{ij}-\varepsilon_{ij}^p)-E\delta_{ij}\alpha_T(T-T_0)\quad;\quad P_{ij}=-\rho\frac{\partial\psi}{\partial\varepsilon_{ij}^p}=\sigma_{ij}\\ &B_{ij}^c=-\rho\frac{\partial\psi}{\partial c_{ij}}=-(2/3)X_{ij}=-a\,c_{ij}\quad;\quad B^p=-\rho\frac{\partial\psi}{\partial p}=-R=-(1-D)b\left[1-e^{-dp}\right] \end{aligned} \tag{4}$$

$$B^D = -\rho \frac{\partial \psi}{\partial D} = W_e(\varepsilon_{ij} - \varepsilon_{ij}^p, T) + W_a(c_{ij}, p, T) \quad ; \quad s = -\rho \frac{\partial \psi}{\partial T}$$

where X and R are auxiliary variables directly related to kinematic and isotropic hardenings, respectively. In order to describe dissipation processes, it is necessary to introduce a potential of dissipation $\phi(\dot{\varepsilon}^p, \dot{c}, \dot{p}, q)$, which can be split into two parts: $\phi(\dot{\varepsilon}_{ij}^p, \dot{c}_{ij}, \dot{p}, q) = \phi_I(\dot{\varepsilon}_{ij}^p, \dot{c}_{ij}, \dot{p}) + \phi_T(q)$. This potential can be written through its dual $\phi^*(P_{ij}, X_{ij}, R, g) = \phi_I^*(P_{ij}, X_{ij}, R,) + \phi_T^*(g)$, as follows:

$$\phi_I^* = I_f^*(P_{ij}, X_{ij}, R) \quad ; \quad \phi_T^* = \frac{T}{2} \Lambda \, g^2 \tag{5}$$

where $g = (1/T)\ \partial T/\partial x$ and Λ is the coefficient of thermal conductivity; $I_f^*(P_{ij}, X_{ij}, R)$ is the indicator function associated with elastic domain (Lemaitre and Chaboche, 1990),

$$f(\sigma_{ij}, X_{ij}, R) = \sqrt{\frac{3}{2}(\sigma_{ij}^d - X_{ij}^d)(\sigma_{ij}^d - X_{ij}^d)} - (S_Y + R) \le 0 \tag{6}$$

where S_Y is the material yield stress, $\sigma_{ij}^d = \sigma_{ij} - \delta_{ij}(\sigma_{kk}/3)$ and $X_{ij}^d = X_{ij} - \delta_{ij}(X_{kk}/3)$. A set of evolution laws obtained from ϕ^* characterizes dissipative processes,

$$\dot{\varepsilon}_{ij}^p = \frac{\partial \phi^*}{\partial P_{ij}} = \lambda \,\mathrm{sign}(\sigma_{ij} - X_{ij}) \; ; \qquad \dot{c}_{ij} = \frac{\partial \phi^*}{\partial B_{ij}^c} = \dot{\varepsilon}_{ij}^p + \frac{\varphi}{a} B_{ij}^c \dot{p} \quad ; \qquad \dot{p} = \frac{\partial \phi^*}{\partial B^p} = \lambda$$

$$q = -\frac{\partial \phi^*}{\partial g} = -\Lambda T \, g = -\Lambda \frac{\partial T}{\partial x} \tag{7}$$

where λ is the plastic multiplier (Lemaitre and Chaboche, 1990) from the classical theory of plasticity, sign(x) = x / |x|, φ is a material parameter associated with kinematic hardening and q is the heat flow. By assuming that the specific heat is $c_p = -(T/\rho)\, \partial^2 W / \partial T^2$ and also considering the set of constitutive Eqs. (4) and (7), the energy equation can be written as (Pacheco, 1994):

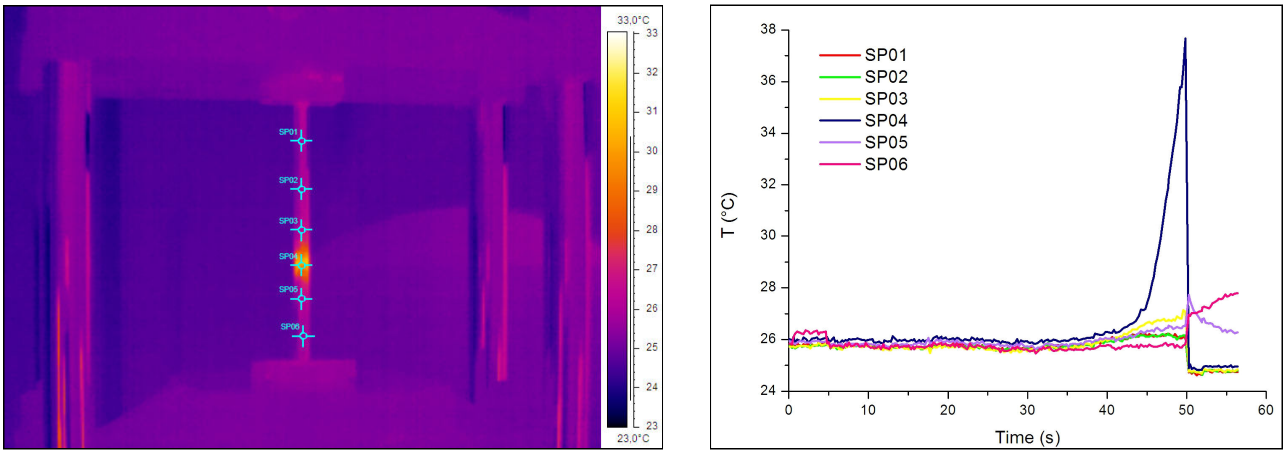
$$\frac{\partial}{\partial x_i}\left(\Lambda \frac{\partial T}{\partial x_i}\right) - h\frac{Per}{A}(T - T_\infty) - \rho c_p \dot{T} = -a_I - a_T \qquad \text{where} \begin{cases} a_I = \sigma_{ij}\dot{\varepsilon}_{ij}^p - X_{ij}\dot{c}_{ij} - R\dot{p} \\ a_T = T\left(\frac{\partial \sigma_{ij}}{\partial T}\left(\dot{\varepsilon}_{ij} - \dot{\varepsilon}_{ij}^p\right) + \frac{\partial X_{ij}}{\partial T}\dot{c}_{ij} + \frac{\partial R}{\partial T}\dot{p}\right) \end{cases} \tag{8}$$

where h is the convection coefficient, T_∞ is the surrounding temperature, Per is the perimeter and A is the cross section area. Terms a_I and a_T are, respectively, internal and thermal coupling. The first one appears in the right hand side of the energy equation and is called internal coupling. It is always positive and has a role in the energy equation similar to a heat source in the classical heat equation for rigid bodies. The last term in the right hand side of the energy equation can be positive or negative and is called the thermal coupling.

3. EXPERIMENTAL RESULTS

Experimental analysis with infrared thermal camera is developed to measure the temperature distribution on round steel bars subjected to a tensile test. Temperature rise is expected due heat generation due thermomechanical coupling promoted by plastic strain. A 30 tons capacity tensile machine Amsler is used to test AISI 1045 steel round specimens with a diameter of 12.5 mm and a length of 120 mm. The test loading rate is adjusted to result in a total duration of approximately 50 s.

Figure 2*a* shows an infrared (IR) thermal image of steel specimen obtained with a FLIR A-320 camera for a tensile test. Six spots used to assess the temperature evolution during the test are shown in the figure. Figure 2*b* presents the temperature evolution for the six spots. Spot 4 is positioned at the rupture region where the necking develops and the plastic strain localization occurs. At this position a maximum temperature of approximately 38ºC is observed.



(*a*) (*b*)

Figure 2. Thermal data from a tensile test: (*a*) infrared (IR) thermal image and (*b*) temperature evolution at 6 positions.

4. NUMERICAL PROCEDURE

The numerical procedure here proposed is based on the operator split technique (Ortiz *et al.*, 1983; Pacheco, 1994) in order to deal with nonlinearities in the formulation. With this assumption, coupled governing equations are solved from two uncoupled problems: thermal and thermo-elastoplastic. In this article, finite element method is employed to perform spatial discretization of governing equations. Therefore, the following moduli are considered:

Thermal Problem - Comprises a conduction problem with surface convection. Themomechanical coupling is considered as a heat source. Material properties depend on temperature and, therefore. Classical finite element method is employed for spatial discretization.

Thermo-elastoplastic Problem - Stress and strain fields are evaluated from temperature distribution obtained in the thermal problem and from the mechanical loading. Classical finite element method is employed for spatial discretization.

5. NUMERICAL SIMULATIONS

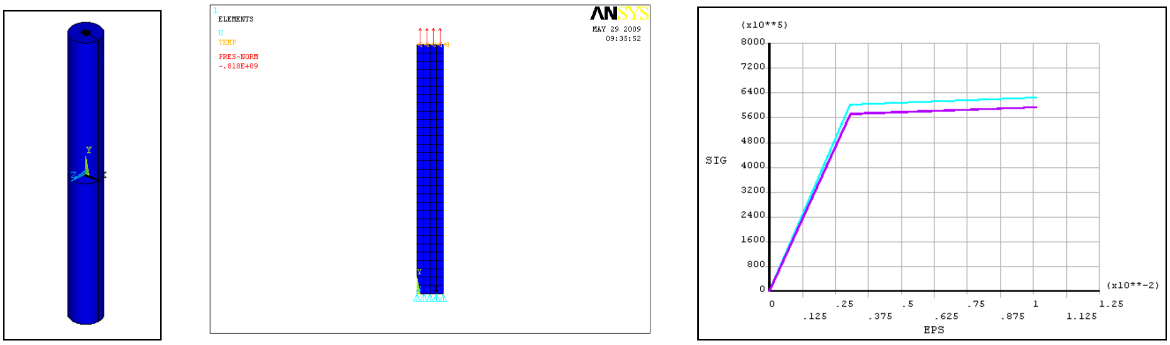
The proposed model is applied to study the thermomechanical coupling effects of metallic components submitted to inelastic loadings. A non-linear finite element model with temperature dependent properties is presented to study the effect of thermomechanical coupling in mechanical components subjected to inelastic deformation. Numerical simulations are performed with commercial finite element code ANSYS (ANSYS, 2006), employing coupled thermal and mechanical fields element PLANE13 (4 nodes bidimensional element with displacement and temperature degrees of freedom) for spatial discretization. The final meshes are defined after a convergence analysis.

To implement the operator split technique and the themomechanical coupling as a heat source, a program developed in APDL (ANSYS Parametric Design Language) is used. Through this approach the internal coupling associated to plastic deformation and kinematic hardening is calculated for each time step from stress and plastic strain fields obtained from the thermo-elastoplastic problem results of the previous step. A small step and a convergence analysis guarantee the convergence of the results.

Two geometries are analyzed: a tensile test specimen (with the same characteristics of the one described on Section 3) and a hour-glass fatigue test specimen. Axissymmetric models are adopted for both geometries to reduce the computational cost.

5.1. Tensile Test Specimen

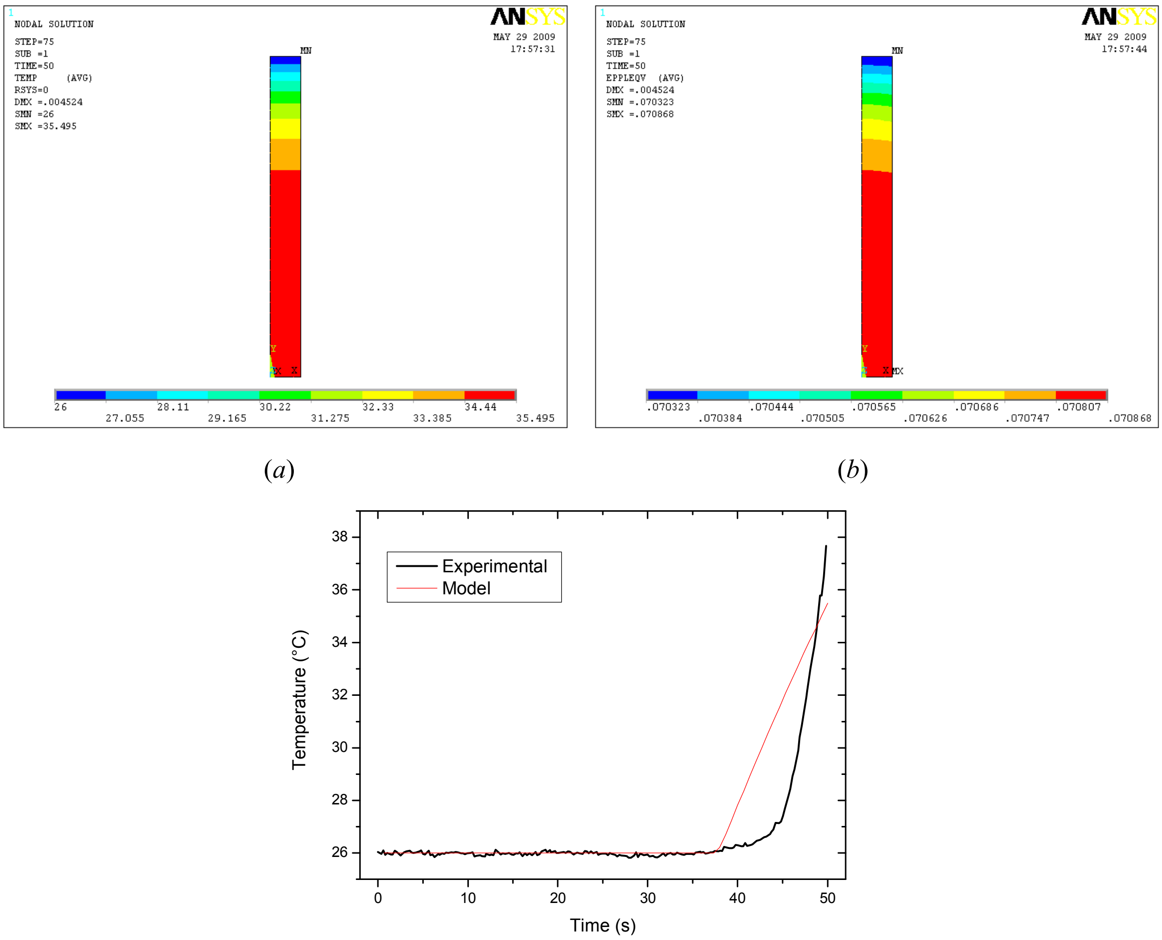
To establish a comparison with experimental data numerical simulations are developed considering a cold-drawn AISI 1045 steel tensile test round specimen with the same characteristics described in Section 3 (Fig 3*a*). The following constant thermal properties are adopted (Smithell, 1976): heat capacity, c_p, of 480 J/Kg K, thermal conductivity, Λ, of 52 W/m K and coefficient of linear expansion, α, of 11.21 x 10^{-6}/K. Besides the axissymmetry condition, a symmetry condition is also adopted and only half bar is considered. Figure 3*b* shows the mesh obtained after a convergence analysis with the boundary conditions and the loading. A prescribed monotonic stress load is applied to the specimen end. Figure 3*c* shows the temperature dependent stress-strain curve for two different temperatures. A linear kinematic hardening is considered. It is assumed that the specimen is at an initial temperature of 26 ºC. This temperature is similar to the ambient temperature and a convection coefficient, h, of 10 W/m^2 °C is used. Convection boundary condition is prescribed at the specimen surface. It is considered that the grips temperature remains constant and therefore a constant temperature condition is adopted at the specimen end.



(*a*) (*b*) (*c*)

Figure 3. Tensile test specimen geometry (*a*), boundary conditions and mechanical loading (*b*) and material stress-strain curve (*c*).

Figure 4*a* and 4*b* presents, respectively, temperature and equivalent plastic strain distributions for the final time instant. The thermal boundary conditions and the temperature depended mechanical properties promotes the localization of thermal and plastic strain processes at the middle of the specimen. Figure 4*c* shows a comparison between experimental and numerical temperature evolution at the middle of the specimen (spot 4 is adopted for the experimental data) indicating a good agreement between experimental and numerical results, indicating that the model captures the localization phenomenon promoted by plastic strain and thermal boundary conditions.



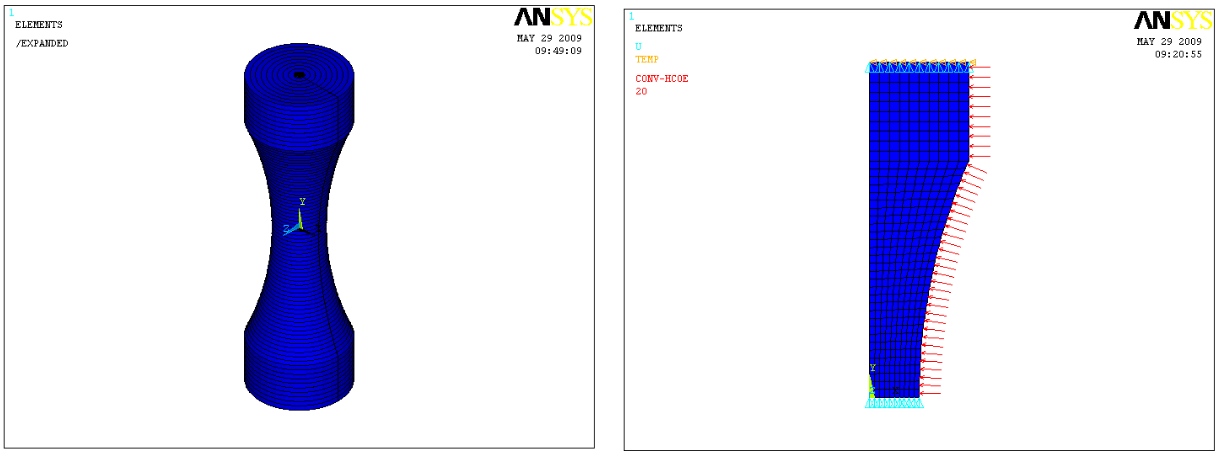
(*a*) (*b*)

(*c*)

Figure 4. Temperature (*a*) and equivalent plastic strain (*b*) distributions for the final time instant. Experimental and numerical temperature evolution at the middle of the specimen (*c*).

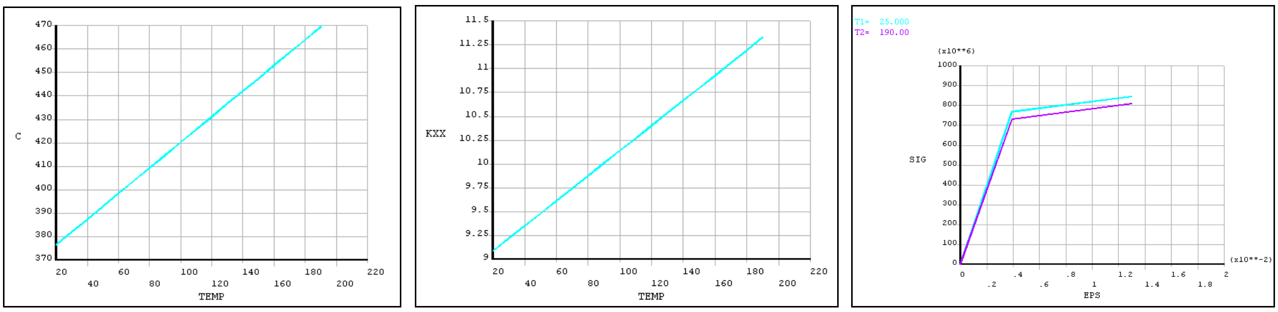
5.2. Hour-glass Fatigue Test Specimen

To study the behavior of a more complex geometry numerical simulations are developed considering a AISI 4140 steel hour-glass fatigue test specimen with a minimum diameter of 8 mm and a length of 38.4 mm (Fig 5*a*). Besides the axissymmetry condition, a symmetry condition is also adopted and only half bar is considered. Figure 5*b* shows the mesh obtained after a convergence analysis with the boundary conditions and the applied loading. A prescribed senoidal displacement condition is adopted. A linear kinematic hardening is considered. It is assumed that the specimen is at an initial temperature of 26 ºC. This temperature is similar to the ambient temperature and a convection coefficient, *h*, of 10 W/m^2 °C is used. Convection boundary condition is prescribed at the specimen surface. It is considered that the grips temperature remains constant and a constant temperature condition is adopted at the specimen end. Figure 6 shows the temperature dependent thermomechanical properties adopted (Oliveira, 2004, 2008).



(*a*) (*b*)

Figure 5. Hour-glass fatigue test specimen (*a*) and axissymetric mesh with boundary conditions and loadings (*b*).



(*a*) (*b*) (*c*)

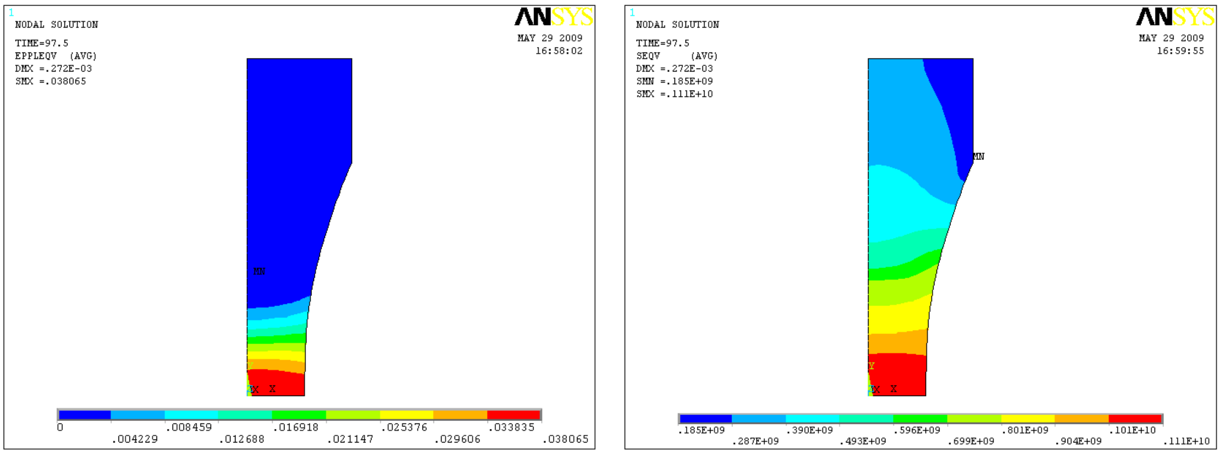
Figure 6. AISI 4140 temperature dependent thermomechanical properties: (*a*) specific heat, (*b*) thermal conductibility and (c) stress-strain curve.

In order to allow the evaluation of the thermomechanical effects in the plastic strain localization, three models are considered:

Uncoupled model: neglects the thermomechanical coupling terms present in Eq. (8) and therefore, the thermal problem is solved as a rigid body;

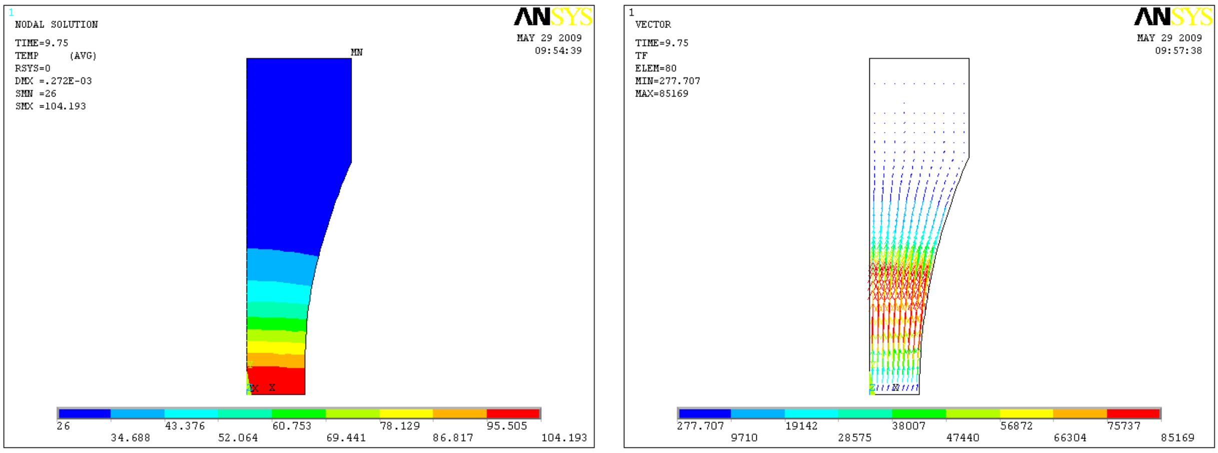
Coupled model: considers the thermomechanical coupling terms for two loading frequencies (1 Hz and 0.1 Hz).

Figures 7 and 8 presents, respectively, results for *uncoupled* and *coupled* (1 Hz) models at the last loading cycle. Temperature and plastic strain localization can be observed at the middle of the specimen where a stress concentrator exists. The localization phenomenon is promoted by the feedback effect due to thermomechanical coupling. The thermal boundary conditions and the temperature depended mechanical properties promotes the localization of thermal and plastic strain processes at the middle of the specimen.

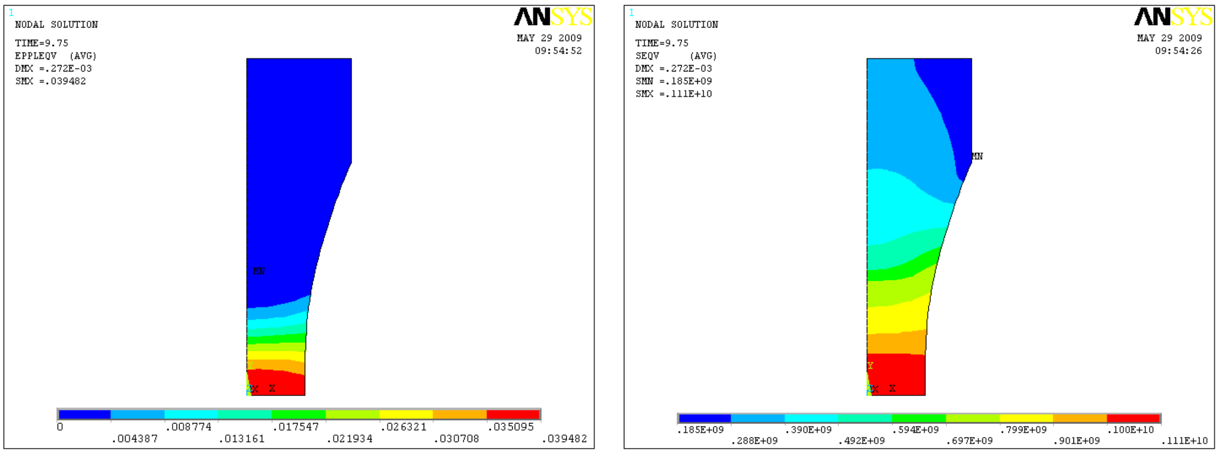


(*a*) (*b*)

Figure 7. Equivalent plastic strain distribution (*a*) and *von Mises* equivalent stress distribution (*b*). *Uncoupled model*.



(*a*) (*b*)



(*c*) (*d*)

Figure 8. Temperature distribution (*a*), thermal flux (*b*), equivalent plastic strain distribution (*c*) and *von Mises* equivalent stress distribution (*d*). *Coupled model* for 1 Hz.

Figure 9 shows the plastic strain and temperature distribution through *x* and *y* axis for the *coupled model* with a 1 Hz loading frequency for cycles $N = 2, 4, 6, 8, 10$.

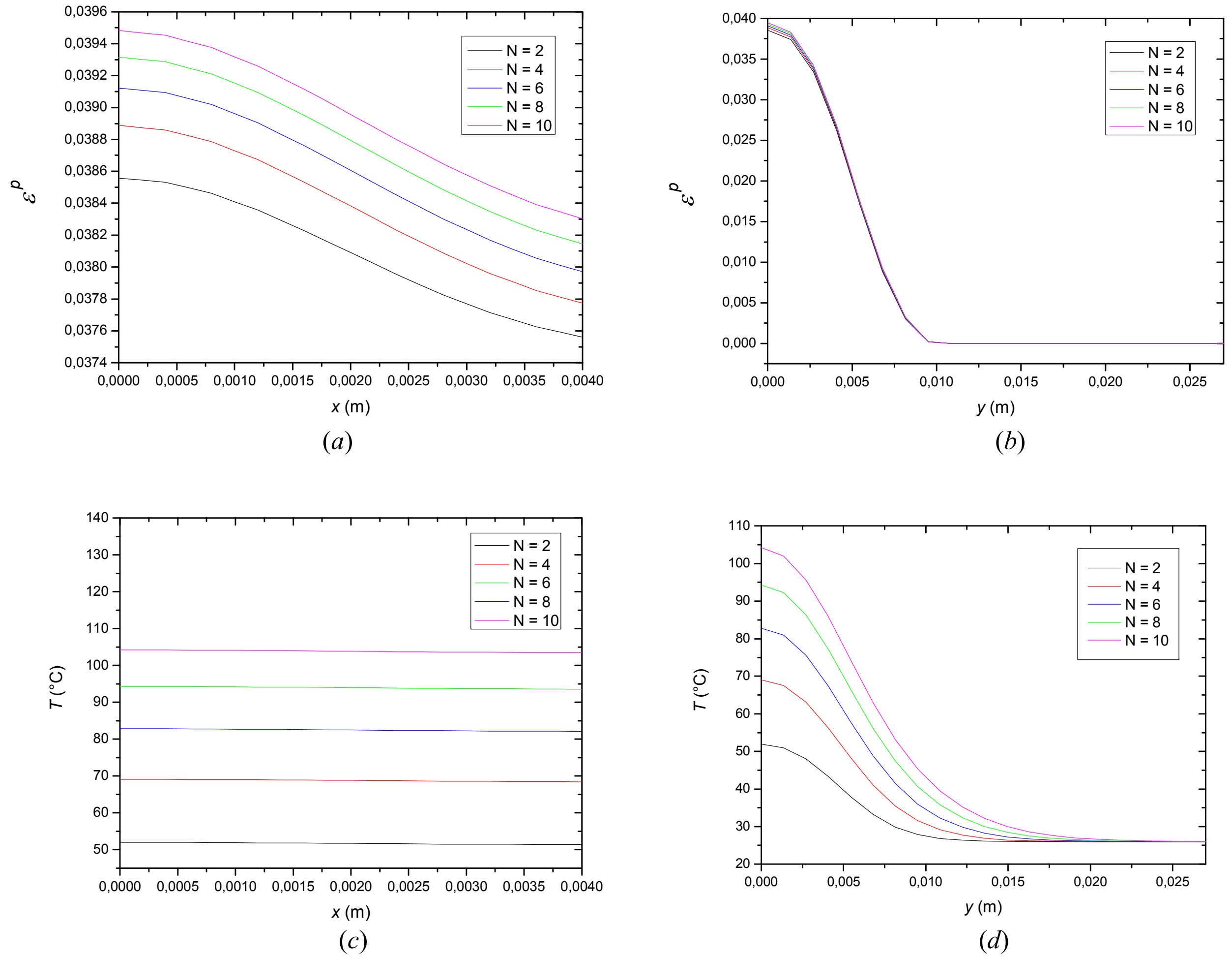


Figure 9. Plastic strain and temperature distribution through *x* and *y* axis. *Coupled model* for 1 Hz.

Figure 10 presents the plastic strain distribution through *x* and *y* axis for the *uncoupled* and *coupled* models at the last cycle. Results show that the *coupled* models present higher values of plastic strain.

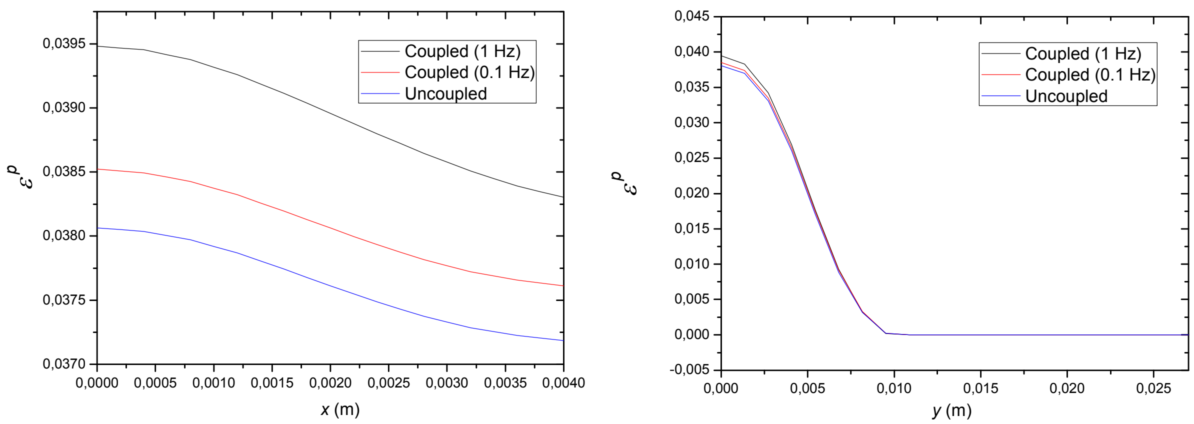


Figure 10. Plastic strain and temperature distribution through *x* and *y* axis. *Uncoupled* and *coupled models.*

Figure 11 presents the evolution of the stress-strain curve, temperature, *von Mises* equivalent stress and equivalent plastic strain for *uncoupled* and *coupled* models. Results show that the *uncoupled* model predicts a stabilized cycle for all variables whereas for the *coupled* model maximum temperature and maximum plastic strain presents a continuous rise.

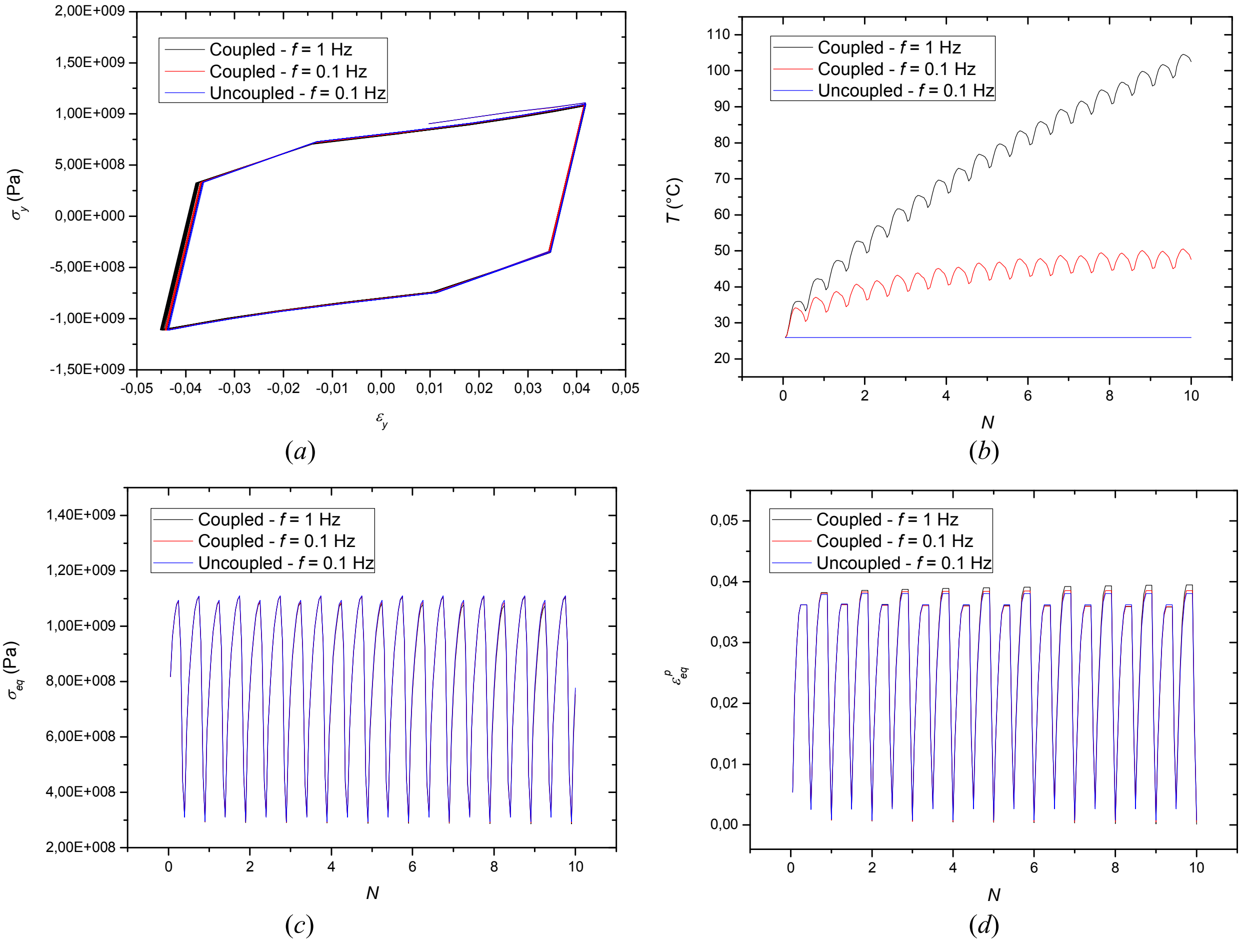


Figure 11. Stress-strain curve (*a*). Temperature (*b*), *von Mises* equivalent stress (*c*) and equivalent plastic strain (*d*) evolution.

6. CONCLUSION

In this paper an anisothermal constitutive model with internal variables based on continuum damage mechanics is proposed to study the thermomechanical coupling effects in elastoplastic round specimens subjected to inelastic mechanical loadings. This formulation provides a rational methodology to study complex phenomena like the amount of heat generated during plastic strain of metals and how it affects its structural integrity. The numerical procedure developed is based on the operator split technique and allows one to deal with the nonlinearities in the formulation using traditional tested classical numerical methods, as the finite element method which is used for spatial discretization. Numerical simulations considering a tensile test and an hour-glass fatigue test specimens subjected to inelastic loadings are presented and analyzed. Experimental analysis with infrared thermal camera is developed to measure the temperature distribution. Results suggest that the proposed model is capable of capturing important localization phenomena related to plastic strain evolution.

7. ACKNOWLEDGEMENTS

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