INTEGRAL TRANSFORM ANALYSIS OF DRYING IN CYLINDRICAL CAPILLARY POROUS MEDIA: COMPARISON OF TWO-DIMENSIONAL AND LUMPED-DIFFERENTIAL FORMULATIONS

Lucilia B. Dantas, luciliabdp@gmail.com
Departamento de Ciências Exatas – Campus Petrolina -UPE –BR 203, Km2, Campus Universitário,
Caixa postal 66, Cep 56300-000. Petrolina – PE – Brasil.

Renato M. Cotta, cotta@serv.com.ufrj.br
Helcio R. B. Orlande, helcio@serv.com.ufrj.br
DEM/EE & PEM/COPPE/UFRJ - Cidade Universitária, Cx. Postal 68503
Rio de Janeiro, RJ, 21945-970

Claudivania de Alencar de Aquino, claudivania123@yahoo.com.br
Departamento de Ciências Exatas – Campus Petrolina -UPE Br 203, Km2, Campus Universitário,
Caixa postal 66, Cep 56300-000. Petrolina – PE – Brasil

Abstract. *This work deals with the solution of the heat and mass transfer problem during drying of capillary porous media. The physical problem considered is described by the linear Luikov's equations in cylindrical coordinates. The two-dimensional problem is solved with the Generalized Integral Transform Technique (GITT). A comparison of the 2D solution with the two approximate solutions considered in this work will establish their ranges of validity in terms of the radial Biot number, for different values of Lu, Pn, Ko, Biq, Bim and ε. The Generalized Integral Transform Technique is a powerful hybrid numerical-analytical approach, which has been used for the solution of different heat transfer problems. In such an approach, the original partial differential equation governing the physical problem is transformed in at least one of its spatial independent variables.*

Keywords: *Generalized Integral Transform Technique, Luikov's equations, heat and mass transfer and drying.*

1. INTRODUCTION

The present paper illustrates, through examples on drying in capillary porous media, the hybrid tools that have been developed along the last few years aimed at enhancing the simulation process in thermal sciences and engineering. Following the physical model construction, hybrid tools for development of the mathematical model and of the solution methodology are investigated. The hybrid nature is explored by making use of lumped-differential formulations, numerical-analytical methods, and symbolic-numerical computations.

The phenomena of heat and mass transfer in capillary porous media has practical applications in several different areas including, among others, drying and the study of moisture migration in soils and construction materials (Luikov,1975; Luikov, 1980). For the mathematical modeling of such phenomena, Luikov (1975, 1980) has proposed his widely known formulation, based on a system of coupled partial differential equations, which takes into account the effects of the temperature gradient on the moisture migration. A few approaches of analytical nature have been used for the solution of Luikov's equations in one-dimensional and multi-dimensional problems (Ribeiro and Cotta, 1993; Ribeiro and Cotta, 1995; Guigon *et al.,* 1999; Thum *et al.,*2001; Lobo *et al.*, 1987; Mikhailov and Özisik, 1994). Nevertheless, several multidimensional heat transfer problems might involve small gradients along a specific spatial direction or even inside the whole body.

A common engineering approach in such cases is to integrate the governing equations in the directions with smaller gradients. Afterwards, the well-established Generalized Integral Transform Technique (GITT) is employed, as a hybrid numerical-analytical solution methodology for diffusion and convection-diffusion problems (Cotta, 1990; Cotta, 1993; Cotta, 1994; Cotta, 1998; Cotta and Orlande, 2003; Cotta and Mikhailov, 2004; Cotta *et al.*, 2004; Cotta *et al.*, 2005). The relative merits of such approach over purely numerical procedures, in light of its hybrid nature, are also discussed, such as the automatic global accuracy control feature and the mild increase on computational costs for multidimensional nonlinear situations. The use of the *Generalized Integral Transform Technique* in drying problems with simple eigenvalue problems involving analytical eigenfunctions, can avoid the calculation of complex eigenvalues for this class of heat and mass transfer problem based on Luikov's formulation.

Thus, for the sake of illustration, in this paper we examine the solution of a two-dimensional drying problem in cylindrical coordinates. The coupled heat and mass transfer in the capillary-porous body is formulated with Luikov's equations. Temperature and moisture content gradients along the radial direction are supposed small, so that the governing equations are integrated in this direction. Both the lumped and the CIEA approximations are considered in this work to approximate the dependent variables at the surface of the cylinder. The resulting two-dimensional problem is solved with the Generalized Integral Transform Technique (GITT).

Despite all the progress achieved in the computational solution of drying problems formulated by the original set of Luikov's equations, these methodologies are still quite too complex for engineering-type work in the realm of applications. In particular due to the multidisciplinary aspects of this physical problem, appearing within various sciences branches where a profound mathematical background might not be *a priori* required, the development of simplified formulations becomes of major relevance. One such possibility of simplification is the classical lumped system analysis, based on the assumption of uniform distribution of the associated potentials over the whole problem domain or along selected coordinates. The alternative technique of producing approximate formulations described in this section, based on the use of Hermite-type approximations for integrals, was examined. Therefore, the present example is aimed at illustrating improved lumped- differential formulations developed in the context of drying problems Cotta and Mikhailov, 1997; Cheroto *et al.*, 1997, Dantas *et al.*, 2000, Dantas *et al.*, 2007), starting from the Luikov system of partial differential equations (Luikov, 1975; Luikov, 1980). The integral transform solution of the original Luikov system provides the reference results (Ribeiro and Cotta, 1993; Ribeiro and Cotta, 1995; Guigon et al., 1999; Thum et al., 2001; Lobo *et al.*, 1987) to illustrate the accuracy and applicability limits of the approximate formulations.

2. PHYSICAL PROBLEM AND MATHEMATICAL FORMULATION

The physical problem under picture involves a cylindrical capillary porous medium of radius R_0 and length l, initially at uniform temperature and uniform moisture content. One of the boundaries, which is impervious to moisture transfer, is in contact with a heater. The other boundary is in contact with the dry surrounding air, thus resulting in a convective boundary condition for both the temperature and the moisture content. The lateral surface of the cylinder is also supposed to be impervious to mass transfer, but heat losses at this boundary are taken into account through a convective boundary condition. The linear system of equations proposed by Luikov (1966), for the modeling of such physical problem involving the drying of a capillary porous media, can be written in dimensionless form as (Luikov, 1966, Mikhailov and Özisik, 1994, Cotta, 1993, Ribeiro, 1993 and Cotta, Ribeiro and Lobo, 1998, Dantas et al.., 2007 Younsi et al., 2006):

$$\frac{\partial\theta(R,Z,\tau)}{\partial\tau}=\alpha\frac{\partial^2\theta(R,Z,\tau)}{\partial Z^2}-\beta\frac{\partial^2\phi(R,Z,\tau)}{\partial Z^2}+\frac{r_a^2\alpha}{R}\frac{\partial}{\partial R}\left[R\frac{\partial\theta(R,Z,\tau)}{\partial R}\right]-\frac{r_a^2\beta}{R}\frac{\partial}{\partial R}\left[R\frac{\partial\phi(R,Z,\tau)}{\partial R}\right]$$

$$\frac{\partial\phi(R,Z,\tau)}{\partial\tau}=Lu\frac{\partial^2\phi(R,Z,\tau)}{\partial Z^2}-LuPn\frac{\partial^2\theta(R,Z,\tau)}{\partial Z^2}+$$
$$+\frac{r_a^2 Lu}{R}\frac{\partial}{\partial R}\left[R\frac{\partial\phi(R,Z,\tau)}{\partial R}\right]-\frac{r_a^2 LuPn}{R}\frac{\partial}{\partial R}\left[R\frac{\partial\theta(R,Z,\tau)}{\partial R}\right]$$

in $0<R<1$ and $0<Z<1$, for $\tau>0$ (1.a,b)

$$\theta(R,Z,0)=0,\quad \phi(R,Z,0)=0,$$ for $\tau=0$, in $0<R<1$ and $0<Z<1$ (1.c,d)

$$\frac{\partial\theta(0,Z,\tau)}{\partial R}=0,\quad \frac{\partial\phi(0,Z,\tau)}{\partial R}=0,$$ at $R=0$ and $0<Z<1$, for $\tau>0$ (1.e,f)

$$\frac{\partial\theta(1,Z,\tau)}{\partial R}-Bi_{qr}[1-\theta(1,Z,\tau)]=0,$$ at $R=1$ and $0<Z<1$ for $\tau>0$ (1.g)

$$\frac{\partial\phi(1,Z,\tau)}{\partial R}=Pn\frac{\partial\theta(1,Z,\tau)}{\partial R},$$ at $R=1$ and $0<Z<1$ for $\tau>0$ (1.h)

$$\frac{\partial\theta(R,1,\tau)}{\partial Z}-Bi_q[1-\theta(R,1,\tau)]+(1-\varepsilon)KoLuBi_m[1-\phi(R,1,\tau)]=0,$$

$$\frac{\partial\phi(R,1,\tau)}{\partial Z}+Bi_m^*\phi(R,1,\tau)=Bi_m^*-PnBi_q[\theta(R,1,\tau)-1],$$ at $Z=1$ and $0<R<1$, for $\tau>0$ (1.i,j)

The various dimensionless groups appearing above are defined as

$$\theta(R,Z,\tau)=\frac{T(\gamma,z,t)-T_0}{T-T_0},\ \phi(R,Z,\tau)=\frac{u_0-u(\gamma,z,t)}{u_0-u_s},\ Q=\frac{ql}{k(T_s-T_0)},\ \tau=\frac{at}{l^2} \tag{2.a-d}$$

$$Lu=\frac{a_m}{a},\ Pn=\delta\frac{T_s-T_0}{u_0-u_s},\ Bi_q=\frac{hl}{k},\ Bi_m=\frac{h_m l}{k_m},\ Ko=\frac{\lambda}{c}\frac{u_0-u_s}{T_s-T_0},\ Bi_{qr}=\frac{h_r R_0}{k}, \tag{2.e-j}$$

$$r_a=\frac{l}{r},\qquad R=\frac{r}{R_0},\qquad Z=\frac{z}{l},\qquad Bi_m^*=Bi_m\left[1-(1-\varepsilon)Pn\ Ko\ Lu\right], \tag{2.k-n}$$

$$\alpha=1+\varepsilon\ KoLuPn,\qquad \beta=\varepsilon\ KoLu \tag{2.o,p}$$

where a is the thermal diffusivity of the porous medium, a_m is the moisture diffusivity in the porous medium, c is the specific heat of porous medium, h *and* h_r are the heat transfer coefficients at the top and lateral surfaces, respectively, h_m is the mass transfer coefficient, k is the thermal conductivity, k_m is the moisture conductivity, l is the thickness of porous medium, q is the prescribed heat flux, λ is the latent heat of evaporation of water, T_s is the temperature of the surrounding air, T_o is the uniform initial temperature in the medium, u_s is the moisture content of the surrounding air, u_o is the uniform initial moisture content in the medium, δ is the thermogradient coefficient and ε is the phase conversion factor. *Lu, Pn* and *Ko* denote the Luikov, Posnov and Kossovitch numbers, respectively.

3. HYBRID METHODS: THE GENERALIZED INTEGRAL TRANSFORM TECHNIQUE (GITT)

Within the last two decades, the classical integral transform method (Mikhailov and Özisik, 1994) gained a hybrid numerical-analytical structure, offering user controlled accuracy and quite efficient computational performance for a wide variety of *a priori* non transformable problems (Cotta, 1990; Cotta, 1993; Cotta, 1994; Cotta, 1998; Cotta and Orlande, 2003; Cotta and Mikhailov, 2004; Cotta *et al.*, 2004; Cotta *et al.*, 2005), including the nonlinear formulations of interest in heat and fluid flow applications. Besides being an alternative computational method on itself, this hybrid approach is particularly well suited for benchmarking purposes, in light of its automatic error control feature, retaining the same characteristics of a purely analytical solution. In addition to the straightforward error control and estimation, an outstanding aspect of this method is the direct extension to multidimensional situations, with a moderate increase in computational effort with respect to one-dimensional applications. Again, the hybrid nature is responsible for this behavior, since the analytical part in the solution procedure is employed over all but one independent variable, and the numerical task is always reduced to the integration of an ordinary differential system in this one single independent variable.

The application under study here involves simultaneous heat and mass transfer during drying of a capillary porous body under the Luikov model, according to the one-dimensional problem formulation that results from the CIEA reformulation previously developed.

We use in this work the GITT for the solution of the one-dimensional problem (1), following the approach advanced in (Ribeiro *et al.,* 1993; Ribeiro and Cotta, 1995). In order to reduce the effects of the non-homogeneties on the convergence of the series solution, we filter problem (1) by writing its solution as

$$\theta(R,Z,\tau)=\theta_s(Z)+\theta_h(R,Z,\tau)\qquad\qquad \phi(R,Z,\tau)=\phi_s(Z)+\phi_h(R,Z,\tau) \tag{3.a,b}$$

where the filtering solutions are obtained from the following steady-state problem

$$\alpha\frac{d^2\theta_s(Z)}{dZ^2}=\beta\frac{d^2\phi_s(Z)}{dZ^2}\qquad \text{in } 0<Z<1, \tag{4.a}$$

$$\frac{d^2\phi_s(Z)}{dZ^2}=Pn\frac{d^2\theta_s(Z)}{dZ^2}\qquad \text{in } 0<Z<1, \tag{4.b}$$

$$\frac{d\theta_s(0)}{dZ}=-Q,\qquad \frac{d\phi_s(0)}{dZ}=-PnQ,\qquad \text{at } 0<Z<1, \tag{4.c,d}$$

$$\frac{d\theta_s(1)}{dZ}+Bi_q\ \theta_s(1)=Bi_q-(1-\varepsilon)KoLuBi_m[1-\phi_s(1)],\qquad \text{at } Z=1, \tag{4.e}$$

$$\frac{d\phi_s(1)}{dZ}+Bi_m^*\phi_s(1)=Bi_m^*-PnBi_q[\theta_s(1)-1],\qquad \text{at } Z=1, \tag{4.f}$$

in the form

$$\theta_l(Z) = 1 + Q\left(1 + \frac{1}{Bi_q} - Z\right) \qquad \theta_s(Z) = 1 + PnQ(1 - Z) \tag{5.a,b}$$

By substituting equations (3.a,b) into equations (1) and using equations (4), we obtain the filetered problem as:

$$\frac{\partial\theta_h(R,Z,\tau)}{\partial\tau} = \alpha\frac{\partial^2\theta_h(R,Z,\tau)}{\partial Z^2} - \beta\frac{\partial^2\phi_h(R,Z,\tau)}{\partial Z^2} + \frac{r_a^2\alpha}{R}\frac{\partial}{\partial R}\left[R\frac{\partial\theta_h(R,Z,\tau)}{\partial R}\right] - \\ - \frac{r_a^2\beta}{R}\frac{\partial}{\partial R}\left[\frac{\partial\phi_h(R,Z,\tau)}{\partial R}\right]$$

in 0<*R*<1 and 0<*Z*<1, for τ>0 (6.a)

$$\frac{\partial\phi_h(R,Z,\tau)}{\partial\tau} = Lu\frac{\partial^2\phi_h(R,Z,\tau)}{\partial Z^2} - LuPn\frac{\partial^2\theta_h(R,Z,\tau)}{\partial Z^2} + \frac{r_a^2 Lu}{R}\frac{\partial}{\partial R}\left[\frac{\partial\phi_h(R,Z,\tau)}{\partial R}\right] - \\ - \frac{r_a^2 LuPn}{R}\frac{\partial}{\partial R}\left[\frac{\partial\theta_h(R,Z,\tau)}{\partial R}\right]$$

in 0<*R*<1 and 0<*Z*<1, for τ>0 (6.b)

$$\theta_h(R,Z,0) = -\theta_s(Z), \quad \phi_h(R,Z,0) = -\phi_s(Z), \qquad \text{at } \tau=0, \text{ in } 0<R<1 \text{ and } 0<Z<1 \tag{6.c,d}$$

$$\frac{\partial\theta_h(0,Z,\tau)}{\partial R} = 0, \quad \frac{\partial\phi_h(0,Z,\tau)}{\partial R} = 0, \qquad \text{at } R=0,\ 0<Z<1 \text{ and } \tau>0 \tag{6.e,f}$$

$$\frac{\partial\theta_h(R,0,\tau)}{\partial Z} = 0, \quad \frac{\partial\phi_h(R,0,\tau)}{\partial Z} = 0, \qquad \text{at } Z=0,\ 0<R<1 \text{ and } \tau>0 \tag{6.g,h}$$

$$\frac{\partial\theta_h(1,Z,\tau)}{\partial R} + Bi_{qr}\,\theta_h(1,Z,\tau) = Bi_{qr}[1 - \theta_s(Z)], \qquad \text{at } R=1,\ 0<Z<1 \text{ and } \tau>0 \tag{6.i}$$

$$\frac{\partial\phi_h(1,Z,\tau)}{\partial R} = Pn\frac{\partial\theta_h(1,R,\tau)}{\partial R} = PnBi_{qr}\left[1 - \theta_h(1,Z,\tau) - \theta_s(Z)\right], \qquad \text{at } R=1,\ 0<Z<1 \text{ and } \tau>0 \tag{6.j}$$

$$\frac{\partial\theta_h(R,1,\tau)}{\partial Z} + Bi_q\,\theta_h(R,1,\tau) = (1-\varepsilon)KoLuBi_m\phi_h(R,1,\tau), \qquad \text{at } Z=1,\ 0<R<1 \text{ and } \tau>0 \tag{6.k}$$

$$\frac{\partial\phi_h(R,1,\tau)}{\partial Z} + Bi_m^*\phi_h(R,1,\tau) = -PnBi_q\theta_h(R,1,\tau), \qquad \text{at } Z=1,\ 0<R<1 \text{ and } \tau>0 \tag{6.l}$$

The following eigenvalue problems are used in order to define the integral transform/inverse formula pairs for temperature:

$$\frac{d^2\varphi_j(\gamma_j Z)}{dZ^2} + \xi_j^2\varphi_j(\gamma_j Z) = 0 \qquad \text{in } 0<Z<1 \tag{7.a}$$

$$\frac{d\varphi_j(0)}{dZ} = 0; \quad \frac{d\varphi_j(\gamma_j 1)}{dZ} + Bi_q\varphi_j(\gamma_j 1) = 0 \tag{7.b,c}$$

$$\frac{d^2\Omega_i(\eta_i R)}{dR^2} + \frac{1}{R}\frac{d\Omega_i(\eta_i R)}{dR} + \eta_i^2\Omega_i(\eta_i R) = 0 \qquad \text{in } 0<R<1 \tag{8.a}$$

$$\frac{d\Omega_i(0)}{dR}=0\,;\qquad \frac{d\Omega_i(\eta_i 1)}{dR}+Bi_{qr}\Omega_i(\eta_i 1)=0 \tag{8.b,c}$$

Similarly, the following eigenvalue problems are used for the moisture content:

$$\frac{d^2\Gamma_j(\xi_j Z)}{dZ^2}+\xi_j^2\Gamma_j(\xi_j Z)=0 \qquad \text{in } 0<Z<1 \tag{9.a}$$

$$\frac{d\Gamma_j(0)}{dZ}=0\,;\qquad \frac{d\Gamma_j(\xi_j 1)}{dZ}+Bi_m^*\Gamma_j(\xi_j 1)=0 \tag{9.b,c}$$

And

$$\frac{d^2\Pi_i(\sigma_i R)}{dR^2}+\frac{1}{R}\frac{d\Pi_i(\sigma_i R)}{dR}+\sigma_i^2\Pi_i(\sigma_i R)=0 \qquad \text{in } 0<R<1 \tag{10.a}$$

$$\frac{d\Pi_i(0)}{dR}=0\,;\qquad \frac{d\Pi_i(\sigma_i 1)}{dR}=0 \tag{10.b,c}$$

The eigenfunctions, normalization integrals and transcendental equations for the determination of the eigenvalues are given by:

$$\varphi_j(Z)=\cos(\gamma_j Z) \qquad N_1(\gamma_j)=\frac{1}{2}\left[1+\frac{Bi_q}{\gamma_j^2+Bi_q^2}\right] \qquad (\gamma_j)\tan(\gamma_j)=Bi_q \tag{11.a-c}$$

$$\Omega_i(\eta_i R)=J_0(\eta_i R) \qquad N_3(\eta_i)=\left[\frac{J_0^2(\eta_i 1)}{2}\frac{Bi_{qr}^2+\eta_i^2}{\eta_i^2}\right] \qquad \eta_i J_0'(\eta_i)+Bi_{qr}J_0(\eta_i)=0 \tag{12.a-c}$$

$$\Gamma_j(Z)=\cos(\xi_j Z) \qquad N_2(\xi_j)=\frac{1}{2}\left[1+\frac{Bi_m^*}{\xi_j^2+Bi_m^{*2}}\right] \qquad (\xi_j)\tan(\xi_j)=Bi_m^* \tag{13.a-c}$$

$$\Pi_i(\sigma_i R)=\begin{cases} J_0(\sigma_i R) & , \ for\ i\neq 0 \\ 1 & , \ for\ i=0 \end{cases} \qquad N_4(\sigma_i)=\begin{cases} \dfrac{J_0^2(\sigma_i)}{2} & , \ for\ i\neq 0 \\[2ex] \dfrac{1}{2} & , \ for\ i=0 \end{cases} \tag{14.a,b}$$

$$\begin{cases} J_0'(\sigma_i)=0 & , \ for\ i\neq 0 \\ \sigma_0=0 & , \ for\ i=0 \end{cases} \tag{14.c}$$

The integral transform / inverse formula pairs for temperature and moisture content are defined, respectively, as

$$\theta_h(R,Z,\tau)=\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\overline{\Omega}_i(\eta_i R)\overline{\varphi}_j(\gamma_j Z)\tilde{\overline{\theta}}_{ij}(\tau) \qquad \text{(inverse)} \tag{15.a}$$

$$\tilde{\overline{\theta}}_{ij}(\tau)=\int_{R=0}^{1}\int_{Z=0}^{1}R\overline{\Omega}_i(\eta_i R)\overline{\varphi}_j(\gamma_j Z)\theta_h(R,Z,\tau)\,dZ\,dR \qquad \text{(transform)} \tag{15.b}$$

and,

$$\phi_h(R,Z,\tau)=\sum_{i=0}^{\infty}\sum_{j=1}^{\infty}\overline{\Pi}_i(\sigma_i R)\overline{\Gamma}_j(\xi_j Z)\tilde{\overline{\phi}}_{ij}(\tau) \qquad \text{(inverse)} \tag{16.a}$$

$$\tilde{\overline{\phi}}_{ij}(\tau)=\int_{R=0}^{1}\int_{Z=0}^{1}R\overline{\Pi}_i(\sigma_i R)\overline{\Gamma}_j(\xi_j Z)\phi_h(R,Z,\tau)\,dZ\,dR \qquad \text{(transform)} \tag{16.b}$$

The integral transformation of problem (6) results on the following system of coupled ordinary differential equations:

$$\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\sum_{l=1}^{\infty}\sum_{m=1}^{\infty}\delta_{ijlm}\frac{d\tilde{\tilde{\theta}}_{ij}(\tau)}{d\tau}+\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\sum_{l=1}^{\infty}\sum_{m=1}^{\infty}C_{ijlm}\tilde{\tilde{\theta}}_{ij}(\tau)+\sum_{i=0}^{\infty}\sum_{j=1}^{\infty}\sum_{l=0}^{\infty}\sum_{m=1}^{\infty}D_{ijlm}\tilde{\tilde{\phi}}_{lm}(\tau)=\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}G_{ij} \tag{17.a}$$

$$\sum_{i=0}^{\infty}\sum_{j=1}^{\infty}\sum_{l=0}^{\infty}\sum_{m=1}^{\infty}\delta_{ijlm}\frac{d\tilde{\tilde{\phi}}_{ij}(\tau)}{d\tau}=\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\sum_{l=1}^{\infty}\sum_{m=1}^{\infty}E_{ijlm}\tilde{\tilde{\theta}}_{lm}(\tau)+\sum_{i=0}^{\infty}\sum_{j=1}^{\infty}\sum_{l=0}^{\infty}\sum_{m=1}^{\infty}F_{ijlm}\tilde{\tilde{\phi}}_{ij}(\tau) \tag{17.b}$$

$$\tilde{\tilde{\theta}}_{ij}(0)=-\int_{R=0}^{1}\int_{Z=0}^{1}R\,\overline{\Omega}_i(\eta_i R)\overline{\varphi}_j(\gamma_j Z)\,\theta_s(Z)\,dZ\,dR \tag{17.c}$$

$$\tilde{\tilde{\phi}}_{ij}(0)=-\int_{R=0}^{1}\int_{Z=0}^{1}R\overline{\Pi}_i(\sigma_i R)\overline{\Gamma}_j(\xi_j R)\phi_s(Z)\,dZ\,dR \tag{17.d}$$

Where

$$A_{ijlm}=\int_{R=0}^{1}\int_{Z=0}^{1}R\,\overline{\Omega}_i(\eta_i R)\overline{\varphi}_j(\xi_j Z)\overline{\Pi}_l(\sigma_l R)\overline{\Gamma}_m(\xi_m Z)\,dZ\,dR \tag{18.a}$$

$$B_{ijlm}=\int_{R=0}^{1}\int_{Z=0}^{1}R\,\overline{\Pi}_i(\sigma_i R)\overline{\Gamma}_j(\xi_j Z)\overline{\Omega}_l(\eta_l R)\overline{\varphi}_m(\gamma_m Z)\,dZ\,dR \tag{18.b}$$

$$C_{ijlm}=-\delta_{ijlm}\alpha(r_a^2\eta_i^2+\gamma_j^2)+\delta_{il}\beta Pn\,Bi_q\overline{\varphi}_j(\gamma_j)\overline{\varphi}_m(\gamma_m)+\delta_{jm}r_a^2\beta PnBi_{qr}\overline{\Omega}_i(\eta_i)\overline{\Omega}_l(\eta_l) \tag{18.c}$$

$$D_{ijlm}=\beta(r_a^2\eta_i^2+\gamma_j^2)A_{ijlm}+\left[KoLuBi_m-\beta Bi_q\right]I_{1,il}\overline{\varphi}_j(\gamma_j)\overline{\Gamma}_m(\xi_m)-r_a^2\beta Bi_{qr}\overline{\Omega}_i(\eta_i)\overline{\Pi}_l(\sigma_l)I_{2,jm} \tag{18.d}$$

$$E_{ijlm}=LuPn(r_a^2\sigma_i^2+\xi_j^2)B_{ijlm}-LuPnBi_m^{*}I_{3,il}\overline{\Gamma}_j(\xi_j)\overline{\varphi}_m(\gamma_m) \tag{18.e}$$

$$F_{ijlm}=-\delta_{ijlm}Lu(r_a^2\sigma_i^2+\xi_j^2)-\delta_{il}Lu^2Pn(1-\varepsilon)Ko\,Bi_m\,\overline{\Gamma}_j(\xi_j)\overline{\Gamma}_m(\xi_m) \tag{18.f}$$

$$G_{ij}=r_a^2Bi_{qr}\overline{\Omega}_i(\eta_i)I_{4,j} \tag{18.g}$$

$$I_{1,il}=\int_{R=0}^{1}R\,\overline{\Omega}_i(\eta_i R)\overline{\Pi}_l(\sigma_l R)\,dR=\frac{\eta_i J_0(\sigma_l)J_1(\eta_i)-\sigma_l J_0(\eta_i)J_1(\sigma_l)}{(\eta_i^2-\sigma_l^2)\sqrt{N_3(\eta_i)}\sqrt{N_4(\sigma_l)}} \tag{18.h}$$

$$I_{2,jm}=\int_{Z=0}^{1}\overline{\varphi}_j(\gamma_j Z)\overline{\Gamma}_m(\xi_m Z)\,dZ=\frac{\gamma_j\cos(\xi_m)sen(\gamma_j)-\xi_m sen(\xi_m)\cos(\gamma_j)}{\sqrt{N_1(\gamma_j)}\sqrt{N_2(\xi_m)}(\gamma_j^2-\xi_m^2)} \tag{18.i}$$

$$I_{3,jm}=\int_{Z=0}^{1}\overline{\Gamma}_j(\xi_j Z)\overline{\varphi}_m(\gamma_m Z)\,dZ=\frac{\gamma_m\cos(\xi_j)sen(\gamma_m)-\xi_j sen(\xi_j)\cos(\gamma_m)}{\sqrt{N_1(\gamma_m)}\sqrt{N_2(\xi_j)}(\gamma_m^2-\xi_j^2)} \tag{18.j}$$

$$I_{4,j}=\int_{Z=0}^{1}\overline{\varphi}_j(Z)\left[1-\theta_s(Z)\right]dZ \tag{18.k}$$

The solution for the system (17), truncated to a sufficiently large order to reach convergence, is obtained with the subroutine DIVPAG of the IMSL (1987). Then, the temperature and moisture content along the axial and radial direction can be computed by using the inverse formula given by Eqs. (15a), (16a).

4. RESULTS AND DISCUSSION

We examine below the effects of the Biot number in the radial direction on the approximate solutions obtained *via* lumped and $H_{1,1}/H_{0,0}$ (Dantas et al., 2007) approaches and two dimensional solution via GITT. Figures 1.to 3 show the results for $Bi_{qr} = 0, 1.0$ and 10, as well as the results obtained with the exact one-dimensional solution *via* GITT (Guigon *et al.,* 1999), for the average temperature and average moisture content at the position $Z=0, 0.5$ *and* 1. Other parameters of importance for the analysis were taken as: $Lu=0.4, Pn=0.6, Ko=5.0, Bi_q=Bi_m=2.5, \varepsilon=0.2$ *and* $Q=0.9$.

As expected, the lumped, CIEA and 2Dsolutions are in perfect agreement with the 1D solution for $Bi_{qr}=0$. As Bi_{qr} increases, the average temperatures obtained with the approximate solutions tend to be smaller than that for the 1D solution, due to the lateral heat losses. The same behavior is observed for the average moisture content. By comparing the 2D approximate solutions, we can notice that the average temperatures and the average moisture contents tend to be larger with the $H_{1,1}/H_{0,0}$ approximation than with the lumped approach. It can be observed from Figs. 1a e 1b at Z=0, the CIEA solution are in agreement with the 2D solution for $Bi_{qr}=1$ *and* 10.0. The solution *via* GITT of the two-dimensional problem given by equations (1) was implementation here, allowing for the verification of the progressive accuracy loss in the classical lumped analysis. The comparison of the two approximate solutions proposed in Dantas et al. (2007) with such a 2D solution identifies ranges of validity in terms of the radial Biot number, for different values of $Lu, Pn, Ko, Bi_q, Bi_m, Q$ and ε.

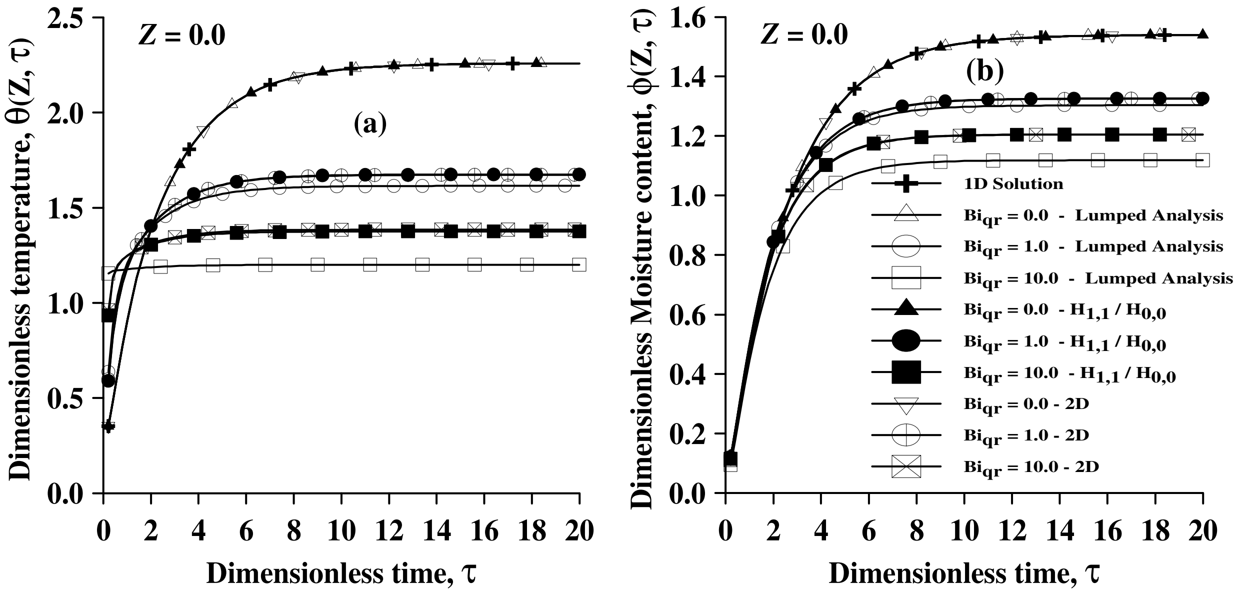

Figure 1. Comparison of lumped, improved lumped and two-dimensional solutions for temperature and moisture potential profiles with different thermal Biot numbers.

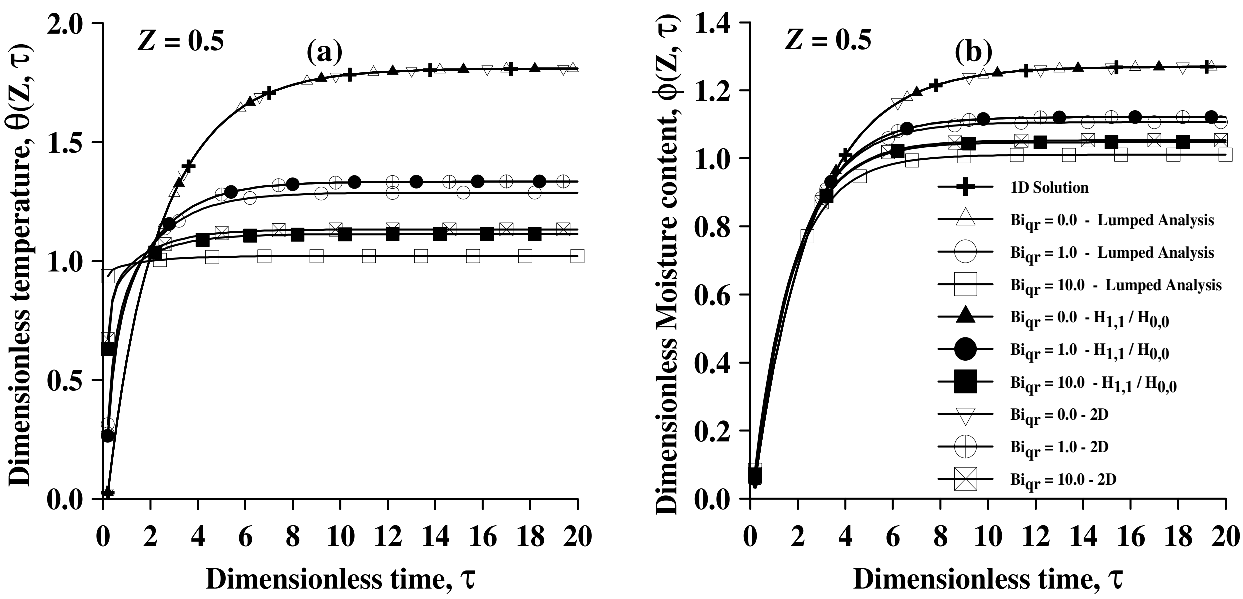

Figure 2 – Comparison of lumped, improved lumped and two-dimensional solutions for temperature and moisture potential profiles with different thermal Biot numbers.

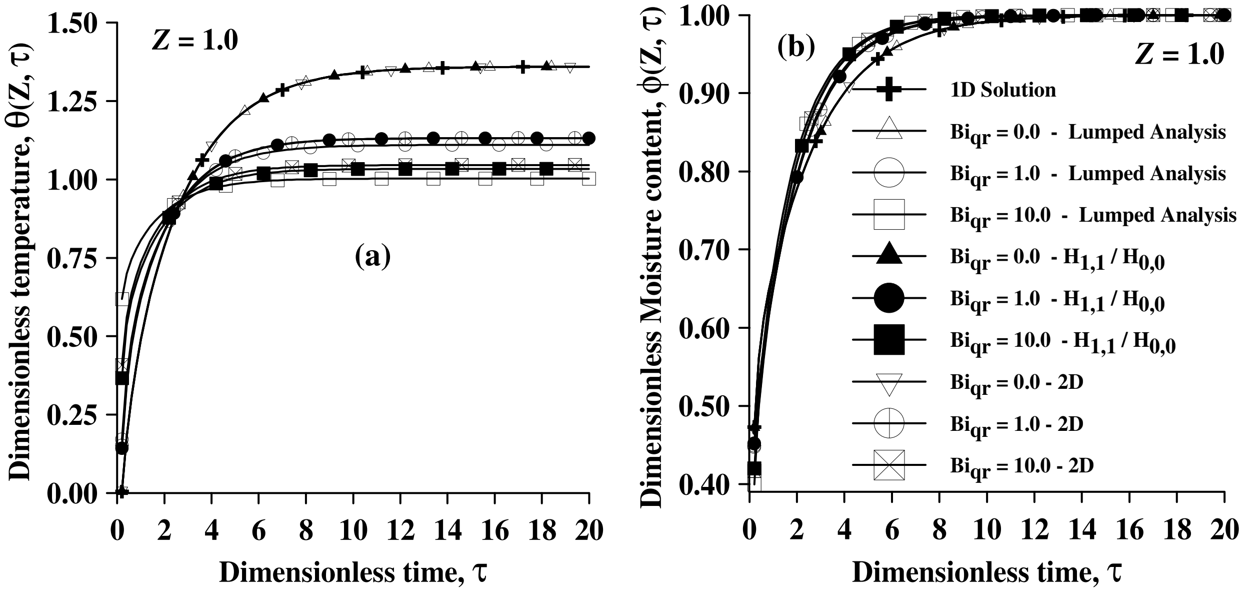

Figure 3 – Comparison of lumped, improved lumped and two-dimensional solutions for temperature and moisture potential profiles with different thermal Biot numbers.

5. CONCLUSIONS

The use of hybrid tools in formulation, solution and computation of thermal problems has been discussed and illustrated. The hybrid nature present in these research fronts has been allowing for exciting findings on improved characteristics and for continuous progress in comparison with conventional approaches. While much has already been achieved, as demonstrated by the ample literature available, research needs, at the same pace, become more evident. The Coupled Integral Equations Approach has been recently employed to provide *a priori* error analysis, with encouraging results, and should be progressively extended to more complex nonlinear formulations. The Generalized Integral Transform Technique enters now a phase of algorithm refinement and optimization, which includes advanced filtering and reordering schemes, enhanced approaches for ODE systems, and automatic implementation for arbitrarily irregular geometries. The *Mathematica* system has been intensively employed in conjunction with the approaches above described, aimed at further facilitating the analytical development task.

6. ACKNOWLEDGEMENTS

The authors would like to acknowledge the partial financial support provided by CNPq, PRONEX, and FAPERJ, along the developments here reported.

7. REFERENCES

Cheroto, S., Silva-Guigon, S.M., Ribeiro, J.W., and Cotta, R.M., "Lumped-Differential Formulations for Drying in Capillary Porous Media"*, Drying Technology*, V.15, nos. 3&4, 811-835, 1997.

Cotta, R. M. and Orlande, H.R.B., "Hybrid Approaches in Heat and Mass Transfer:- A Brazilian Experience with Applications in National Strategic Projects", *Heat Transfer Eng.,* Invited Editorial, V.24, no.4, pp.1-5, 2003.

Cotta, R. M., Luz Neto, H., Alves, L.S. de B., and Quaresma, J.N.N., "Integral Transforms for Natural Convection in Cavities Filled with Porous Media", in: *Transport Phenomena in Porous Media*, vol.3, Eds. D. Ingham and I. Pop, Elsevier, UK, 2004.

Cotta, R.M. (ed.), "*The Integral Transform Method in Thermal and Fluids Sciences and Engineering"*, Begell House, New York, 1998.

Cotta, R.M. and Mikhailov. M.D., *"Heat Conduction:- Lumped Analysis, Integral Transforms, Symbolic Computation"*, John Wiley, New York, 1997.

Cotta, R.M., "Benchmark Results in Computational Heat and Fluid Flow: - The Integral Transform Method", *Int J. Heat & Mass Transfer* (Invited Paper), **37**, Suppl. 1, 381-394, 1994.

Cotta, R.M., "Hybrid Numerical-Analytical Approach to Nonlinear Diffusion Problems", *Num. Heat Transfer, Part B - Fundamentals*, V.127, 217-226, 1990.

Cotta, R.M., "Improved Lumped-Differential Formulations in Heat Transfer", Invited Chapter**,** *Volume 3 - Modeling of Engineering Heat Transfer Phenomena, Chapter 10***,** Eds. B.Sunden & M. Faghri, Heat Transfer Series, Computational Mechanics Publications, UK, 1998.

Cotta, R.M., "*Integral Transforms in Computational Heat and Fluid Flow"*, CRC Press, Boca Raton, FL, 1993.

Cotta, R.M., and Mikhailov, M.D., 2004, "Hybrid Methods and Symbolic Computations", in: *Handbook of Numerical Heat Transfer,* 2nd edition, Chapter 16, Eds. W.J. Minkowycz, E.M. Sparrow, and J.Y. Murthy, John Wiley, New York.

Cotta, R.M., Santos, C.A.C., Quaresma, J.N.N., and Perez-Guerrero, J.S., 2005, " Hybrid Integral Transforms in Convection-Diffusion: Recent Applications in Internal Flow Simulation", Invited Lecture, Proc. of the *4th Int. Conf. Computational Heat and Mass Transfer, 4th ICCHMT*, vol.1, pp.153-164, Paris-Cachan, France, May.

Dantas, L. B., Orlande, H. B., and Cotta, R. M., 2000, "A Coupled Integral Equations Approach for the Analysis of Drying in Two-Dimensional Capillary Porous Media", *8th Brazilian Congress of Thermal Sciences and Engineering, ENCIT 2000*, Porto Alegre, Brazil, October .

Dantas, L. B., Orlande, H. R. B., Cotta, R. M., 2007, "Improved lumped-differential formulations and hybrid solution methods for drying in porous media". International Journal Of Thermal Sciences. França. (46) 878–889.

Guigon, S.M., Dantas, L.B., Scofano Neto, F., and Cotta, R.M., "Exact Solution of Luikov's Equations for Drying in Capillary Porous Media", *Hybrid Meth. Eng.,* V.1, no.4, pp.365-387, 1999.

IMSL Library, MATH/LIB, Houston, TX, 1987.

Lobo, P.D.C., Mikhailov, M.D., and Özisik, M.N., 1987, "On the Complex Eigenvalues of Luikov System of Equations", *Drying Technology*, v.5, n.2, pp.273-286.

Luikov, A.V., 1980 *"Heat and Mass Transfer"* , Mir Publishers, Moscow.

Luikov, A.V., 1975, "System of Differential Equations of Heat and Mass Transfer in Capillary Porous Bodies", *Int. J. Heat & Mass Transfer*, V. 18, 1-1.

LUIKOV, A. V., 1966, *Heat and Mass Transfer in Capillary-Porous Bodies*, Pergamon Press, Oxford.

Mikhailov, M.D. and Özisik, M.N., 1994,"*Unified Analysis and Solutions of Heat and Mass Diffusion"*, John Wiley, New York (1984); also, Dover Publications.

Ribeiro, J.W. and Cotta, R.M., 1995, "On the Solution of Nonlinear Drying Problems in Capillary Porous Media Through Integral Transformation of Luikov Equations", *Int. J. Num. Meth. Eng.,* V. 38, 1001-1020.

Ribeiro, J.W., Cotta, R.M., and Mikhailov, M.D., 1993, "Integral Transform Solution of Luikov's Equations for Heat and Mass Transfer in Capillary Porous Media", *Int. J. Heat & Mass Transfer*, V. 36, no. 18, 4467-4475, 1993.

Ribeiro, J.W., Cotta, R.M., and Mikhailov, M.D., "Integral Transform Solution of Luikov's Equations for Heat and Mass Transfer in Capillary Porous Media", *Int. J. Heat & Mass Transfer*, V. 36, no. 18, 4467-4475.

Thum, R.L., Barichello, L.B., Vilhena, M.T., and Cotta, R.M., 2001, "An Analytical Approach to the Solution of Multidimensional Drying Problems", *2nd International Conference on Computational Heat and Mass Transfer, CHMT-2001***,** Rio de Janeiro, Brazil, October 2001(CD-Rom); also, *Hybrid Methods Eng.,* Special Issue, Vol.3, no.2&3, pp. 239-255.

Younsi, R., Kocaefe, D., Kocaefe, Y., 2006, Three-dimensional simulation of heat and moisture transfer in wood. Applied Thermal Engineering 26 (11-12), 1274–1285

8. RESPONSIBILITY NOTICE

The authors are the only responsible for the printed material included in this paper.