A STUDY ON THE SOLIDIFICATION OF PCM AROUND CURVED TUBE

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Abstract. *This paper presents the results of an experimental and numerical study on the solidification of PCM around a curved tube through which a cold fluid flows. The objectives of the study include among others, investigation of the effects of the Dean number, cooling fluid flow rate and its temperature on the interface velocity end time for complete solidification. A pure conduction model is adopted for the solidification process and the immobilization technique based upon the Landau transform is used. The energy equation and the associated boundary conditions are discretized by the control volumes method. The numerical code is optimized and validated .The numerical predictions of the effects of the Dean number, the working fluid flow rate, and its temperature, and the tube material are presented and discussed. An experimental rig is constructed and instrumented. The effects of the same influencing parameters are obtained and compared with the numerical prediction. Although the model is simple the comparisons show reasonably good agreement validating in this manner the numerical model, method of solution and the numerical predictions. This indicates that the numerical code can be used to predict the performance of curved tubes in latent heat storage systems..*

Keywords *PCM, Solidification, Phase Change, Curved Tube, Coiled Tubes.*

1. INTRODUCTION

Heat transfer with phase change is an important subject in the analysis and design of heat transfer equipments such as evaporators, condensers, heat pipes and latent heat storage system. For these and other reasons why this subject received and still receiving much attention from researches of the area who devoted efforts and time to theoretical, numerical and experimental investigations.

There is a great deal of work handling the analytical, numerical and experimental aspects of heat transfer with phase change, such as Sparrow and Chuck (1984), Yao and Prusa (1989), Ismail (1998), Ismail and Gonçalves (1999) and Gonçalves (1996). Dincer and Rosen (2002), Ismail and Jesus (2001), Padilha (1990), Jesus (1998), Sparrow and Hsu (1981), Cao e Faghri (1991) and Sinha and Gupta (1982).

Thomson (1876) and later Eustice (1910, 1911) and Morales (2000) realized the first experimental studies on the flow inside curved tubes and identified experimentally the presence of a pair of vortex in the flow field in the curved tube. Dean (1927, 1928) developed a theoretical model to describe the flow in a curved tube and was able to obtain from his model the flow in a straight pipe as a limiting case when the Dean number is very high. Jitchote and Robertson (2000) realized a theoretical study on the flow in curved tubes and were able to the demonstrate the presence of the pair of vortex in the flows field.

The study of heat transfer in a curved tube was developed by Hauwes (1932). He showed that the temperature profiles are different from that of a straight pipe, that the local heat transfer coefficients on the inner and outer curvatures are different and that the coefficient for the external side is more than that for the internal side.

Yang and Chang (1993) realized a numerical study of the heat transfer in laminar flow in curved tubes for different Dean numbers, analyzed the effects of the curvature of the tube, Reynolds number, Prandtl number and obtained conclusions similar to those of Ozisik and Topakoglu (1968). Oiwake and Inaba (1986) studied experimentally the solidification process inside a “U” tube to identify the parameters which lead to the blockage tube due to complete internal solidification of the flowing water leading ultimately to the tube fracture. Braun and Beer (1995) studied experimentally and theoretically the solidification of water inside a channel of square cross section with a water turbulent flow. They found that the solidified mass is not symmetrical around the curved part of the duct.

Benta (2001) studied the process of solidification around a curved tube submersed in a liquid PCM both numerically and experimentally. Benta found experimentally that the layers of solidified PCM are thicker on the outer side than on the inner side, that the increase of the Dean number leads to increase the solidified mass. The numerical predictions showed a reasonably good agreement with the experimental results.

This paper presents a numerical model based upon the immobilization technique which enables tracking the solidification front precisely and predict the influence of the Dean number, Reynolds number, Stefan number, the material of the tube, and their effects on the solidified mass, the velocity of the interface and the time for complete solidification. The validation of the numerical predictions is done by running an extensive number of experiments where the temperature of the circulating fluid, the curvature of the tube and the mass flow of the circulating are varied. The numerical predictions are compared with the experimental results and a good agreement is found.

2. EXPERIMENTAL SETUP

Figure 1 shows the experimental rig, which is composed of a refrigeration unit working with R-22. The machine works in a closed circuit with a secondary fluid tank containing ethanol whose temperature can achieve -25°C. The test tank is full with PCM and has lateral walls and top, made of transparent acrylic to permit observation and photographing. The test tank is 1,53x0,52x0,5 m and a volume of 0,4m³, is very well insulated. The geometry under test is inserted into the test tank and is maintained in the horizontal position. The cold secondary fluid flowing in the curved tube is fed from the secondary fluid circuit. The mass flow rate, the temperature at entry and exit are measured by a calibrated orifice plate and thermocouples. Also measurements of temperature are made along the straight part of the curved tube, also in diverses positions in the test tank to determine the mean temperature of the PCM, Fig 2.

The thermocouples leads are connected to data acquisition system and fed directly to a dedicated PC. Photographs of the solidification mass around the straight and curved part of the tube are taken vertically from the top. The photographing intervals initially are very small during the first 2 hours to be able to determine the interface velocity with good precision. The total test period usually lasted more than 18 hours to be able to determine precisely the time for complete solidification. The experiments were realized for secondary fluid temperatures varying from -8 to -25°C and flow rates from 0,0617 to 0,1426 l/s. The photographs of the interface positions are digitalized and then used to calculate the interface velocity, the solidified mass fraction and energy stored.

Analysis of error propagation for the experiments is realized and included in the results. A typical photograph of the interface is shows in Fig 2.

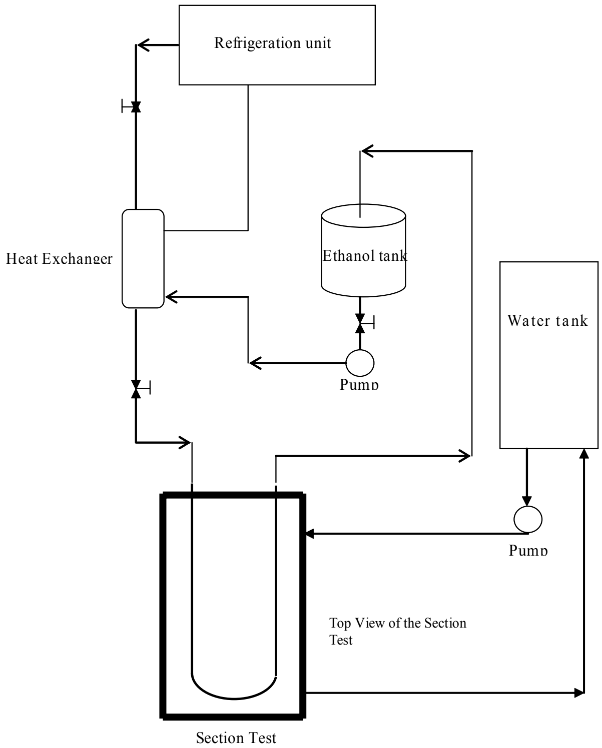

Figure 1 – Scheme diagram of the experimental setup

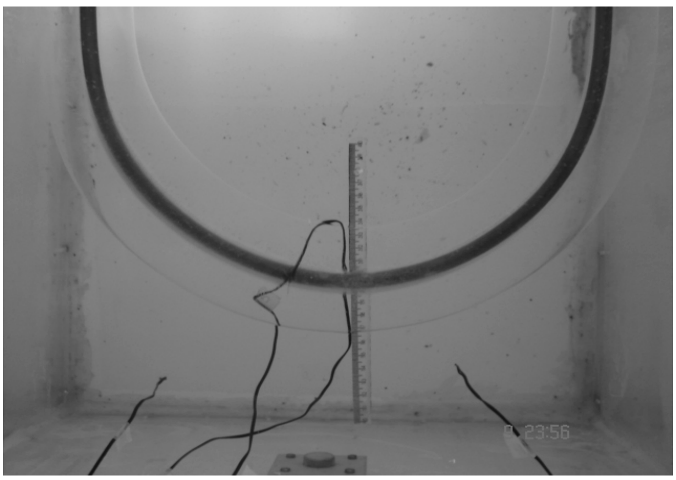

Figure 2 Photograph of the interface.

3. FORMULATION OF THE PROBLEM

The problem of phase change around a curved tube submersed in the PCM is illustrated in Fig 3. It is assumed that the heat transfer between the PCM and the fluid is by pure conduction, ignoring the natural convection effects in the liquid phase. Although the internal hydrodynamic flow problem is not modeled in the present case, the effects due curvature are accounted for by using different the heat transfer coefficients on the internal and external sides fine curved tube. In order to formulate the problem, one considers also the traditional assumptions that the curvature ratio R/r>>1, Dean (1927, 1928) and that the PCM is initially in the liquid phase and at its saturation temperature, incompressible, pure and its physical properties are constant and independent of its temperature.

Based upon these assumptions one can write the energy equation for the solid phase as:

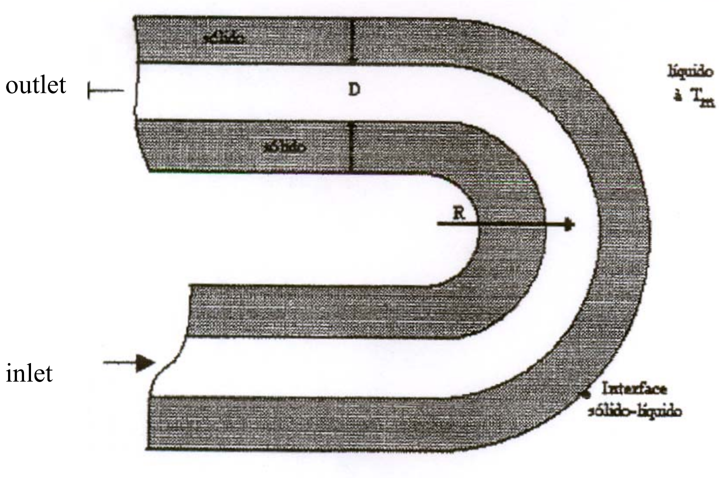

Figure 3 Layout of the problem.

$$\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T_s}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 T_s}{\partial \theta^2}\right]=\frac{1}{\alpha_s}\frac{\partial T_s}{\partial t} \tag{1}$$

where the subscript **s** denotes the solid phase. The boundary condition can be written ignoring the tube thermal resistence since the tube is made of cupper, and, this has very a high thermal conductivity.

$$k_s\left.\frac{\partial T}{\partial r}\right|_{r=r_i}=h\left(T_w-T_f\right) \qquad r=r_i \tag{2}$$

The boundary conditions at the interface, are:

$$T_s\left(r,\theta,t\right)=T_m \rightarrow r=r_i(t) \tag{3}$$

$$\left[1+\frac{1}{r_i^2}\left(\frac{\partial r_i}{\partial \theta}\right)^2\right]\left(k\frac{\partial T_s}{\partial r}\right)=\rho L\frac{dr_i}{dt} \qquad\qquad r=r_i(t) \tag{4}$$

equation (4) satisfy the energy balance at the interface.

The moving interface problem is treated here by the immobilization technique using the Landau Transform (1949). In this case a new radial variable η_s is defined such that:

$$\eta_s=\frac{r-r_w}{r_i-r_w}=\frac{r-r_w}{\delta_s(t)} \tag{5}$$

where $\delta_s(t)$ is the thickness of the solid layer. The annular space is described by the following domain

$$0 \le \eta_s \le 1 \tag{6}$$

The equation for the solid domain can be written in dimensionless form by using the following new variables

$$\phi = \frac{T - T_m}{T_m - T_f}\,;\; R = \frac{r}{r_w}$$

$$R = \frac{r}{r_w}\,;\; \Delta = R_i - 1$$

$$\tau = F_o Ste\,;\; Fo = \frac{\alpha t}{r_w^2}$$

$$Ste = \frac{c_{ps}\left(T_m - T_f\right)}{L} \tag{7}$$

where T_m, T_f, F_o and Ste are the phase change temperature, the working fluid temperature, the Fourier number and the Stefan number, respectively.

The equation of the solid domain in the dimensionless form is solved numerically by the method of control volumes and formulation totally implicit resulting in a system of compled equations.

For the interface an explicit formulation is used for the transient terms as in Benta (2001) and Jesus (1998), resulting in the following equation, more details can be found in Benta (2001)

$$\int_{\tau}^{\tau+\Delta\tau}\int_{w}^{e}\int_{s}^{n} ste\frac{\partial\phi}{\partial\tau}Rd\theta d\eta d\tau = \int_{\tau}^{\tau+\Delta\tau}\int_{w}^{e}\int_{s}^{n}\frac{1}{R\Delta^2}\frac{\partial}{\partial\eta}\left(R\frac{\partial\phi}{\partial\eta}\right)Rd\theta d\eta d\tau + \\ + \int_{\tau}^{\tau+\Delta\tau}\int_{w}^{e}\int_{s}^{n}\frac{1}{R}\frac{\partial}{\partial\theta}\left(\frac{1}{R}\frac{\partial\phi}{\partial\theta}\right)Rd\theta d\eta d\tau + \int_{\tau}^{\tau+\Delta\tau}\int_{w}^{e}\int_{s}^{n} bRd\theta d\eta d\tau \tag{8}$$

The integration of equation (8) results in

$$\left[\frac{Ste\forall_p}{\Delta} + \frac{\Delta\theta}{\Delta_p^2}\frac{R_n}{\Delta\eta} + \frac{\Delta\theta}{\Delta_p^2}\frac{R_s}{\Delta\eta} + \frac{\Delta\eta}{R_e\Delta\theta} + \frac{\Delta\eta}{R_w\Delta\theta}\right]\phi_p = \\ = \frac{\Delta\theta}{\Delta_p^2}\frac{R_n}{\Delta\eta}\phi_N + \frac{\Delta\theta}{\Delta_p^2}\frac{R_s}{\Delta\eta}\phi_S + \frac{\Delta\eta}{R_e\Delta\theta}\phi_E + \frac{\Delta\eta}{R_w\Delta\theta}\phi_W + \frac{Ste\forall_p}{\Delta\tau}\phi_p^o b^*\forall_p \tag{9}$$

The coefficients ϕ_N, ϕ_S, ϕ_E, ϕ_W and ϕ_p will be dominated by a_N, a_S, a_E, a_W and a_p . Consequently one can write equation (9) in the compact form:

$$a_p\phi_p = a_N\phi_N + a_S\phi_S + a_E\phi_E + a_W\phi_W + B \tag{10}$$

where the coefficients are given by:

$$a_N = \frac{\Delta\theta}{\Delta_p^2}\frac{R_N}{\Delta\eta} \tag{10a}$$

$$a_S = \frac{\Delta\theta}{\Delta_p^2}\frac{R_S}{\Delta\eta} \tag{10b}$$

$$a_E = \frac{\Delta\eta}{R_e \Delta\theta} \tag{10c}$$

$$a_W = \frac{\Delta\eta}{R_w \Delta\theta} \tag{10d}$$

$$a_p = a_N + a_S + a_E + a_W + a_p^o \tag{10e}$$

$$B = a_p^o \phi_p^o + b^* \forall_p \tag{10f}$$

and the coefficient of point **P** for the preceding time interval is:

$$a_p^o = \frac{Ste \forall_p}{\Delta\tau} \tag{10g}$$

The discretization of the boundary condition at the wall can be written as:

$$\phi_p = \frac{\phi_i \dfrac{k_{sl}}{\Delta_S \Delta\eta k_f} - \dfrac{Nu}{2}}{\dfrac{Nu}{2} + \dfrac{k_{sl}}{\Delta_S \Delta\eta k_f}} \tag{11}$$

where the Nusselt number, Nu, is obtained from a constant heat flux correlation.

The explicit treatment of the moving interface developed by Sparrow and Chuck (1990) has the advantage of eliminating the possibility of coupling the energy equation. This explicit treatment is based upon the determination of the time derivative of equation (12) in an intermediate time interval $\tau^o + \frac{1}{2}\Delta\tau$ since the value in the precedent interval τ^o is known.

$$\left[1 + \frac{1}{(\Delta+1)^2}\left(\frac{\partial\Delta}{\partial\theta}\right)^2\right]\left(\frac{1}{\Delta}\frac{\partial\phi}{\partial\eta}\right) = \frac{d\Delta}{d\tau} \tag{12}$$

After knowing the temperature distribution in the preceding time τ^o and the initial solidified mass Δs, the expression at the interface in this specific time interval can be given by:

$$\Delta_s^{\tau_o + \Delta\tau/2} = \Delta_s^o + \left(\frac{\partial\Delta_s}{\partial\tau}\right)^o \frac{\Delta\tau}{2} \tag{13}$$

The discretized forms of equation (12) in the time τ^o and $\tau^o + \Delta\tau/2$ are respectively:

$$\left[1 + \frac{1}{(\Delta_s^o+1)^2}\left(\frac{\partial\Delta_s^o}{\partial\theta}\right)^2\right]\left(\frac{1}{\Delta_s^o}\frac{(\phi-\phi^o)}{\Delta\eta}\right) = \left(\frac{d\Delta}{d\tau}\right)^o \tag{14}$$

$$\left[1 + \frac{1}{\left(\Delta_s^{(\tau^o+\Delta\tau/2)}+1\right)^2}\left(\frac{\partial\Delta_s^{(\tau^o+\Delta\tau/2)}}{\partial\theta}\right)^2\right]\left(\frac{1}{\Delta_s^{(\tau^o+\Delta\tau/2)}}\frac{(\phi-\phi^o)}{\Delta\eta}\right) = \left(\frac{d\Delta}{d\tau}\right)^{(\tau^o+\Delta\tau/2)} \tag{15}$$

Using equation (15) it is possible to determine the interface position in the time interval $\tau^o + \Delta\tau/2$, knowing the temperature gradient is the same as in τ^o. To calculate the position of the interface in the time interval $\tau^o + \Delta\tau$, one can use the expression:

$$\Delta_s^{\tau^o+\Delta\tau} = \Delta_s^o + \left(\frac{\partial \Delta_s}{\partial \tau}\right)^{\tau+\frac{1}{2}\Delta\tau} \Delta\tau \tag{16}$$

The resulting system of equations is solved by the line-by-line method. The solidified volume can be determined by the equation:

$$V_s = \int_0^1\int_0^\pi R d\theta \Delta_s d\eta \tag{17}$$

and the solidified mass fraction can be obtained by multiplying V_s by the density of the solid phase:

$$M_s = V_s \rho_s \tag{18}$$

The numerical code was tested and optimized to be able to produce results which are independent of the size of the computational grid. The final grids used for the calculations are: the size of the spacial grid is 200 control volumes in the radial direction and 15 control volumes in the angular direction, the time step is 10^{-4}, and the convergence limit adopted is 10^{-3}. The relative error is found to be 0,0199.

4. RESULTS AND DISCUSSION

Figures 4 and 5 show the variations of the interface position and velocity as functions of time. The curves also show the interface positions of the inner and outer sides of the curved tube, and indicate the time for the complete solidification at the end of the process.

From the temperatures indicated on the graphs one can observe that the lower the temperature, the thicker the solidified mass layer and hence more energy stored.

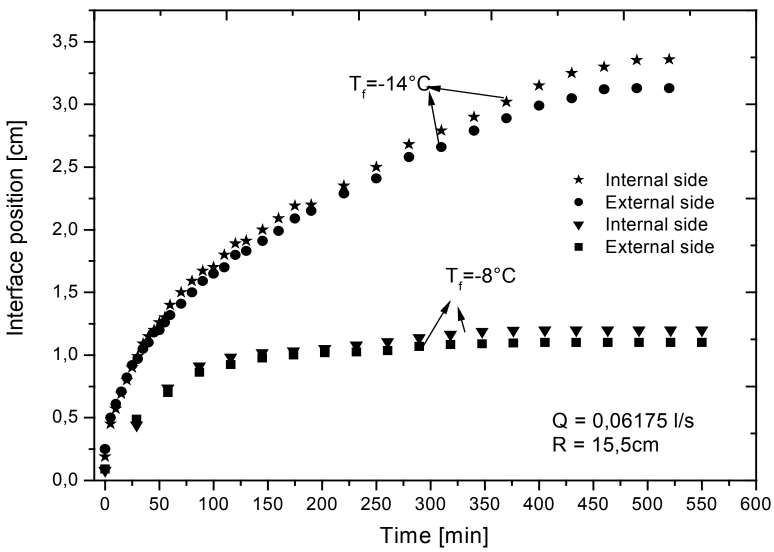

Figure 4 Variation of the interface position with time.

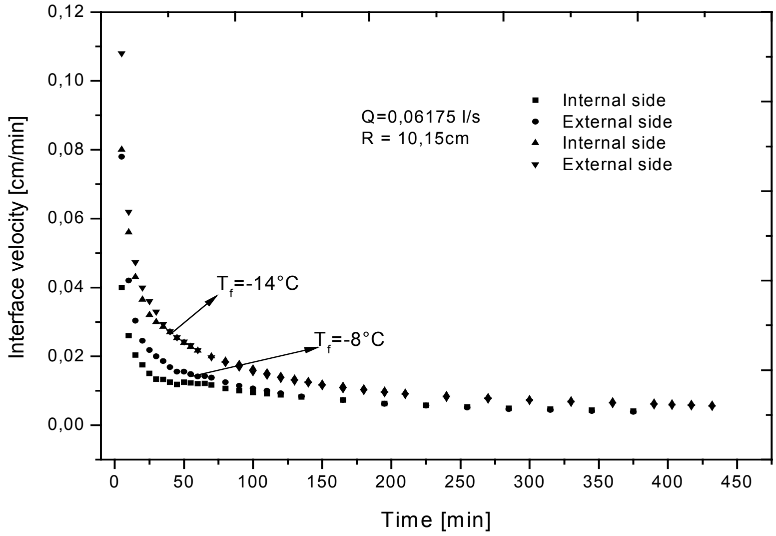

Figure 5 Variation of the interface velocity as a function of time.

Figures 6 show comparisons between the numerical predictions and the experimental results under the same conditions, showing reasonably good agreement.

It is important to mention here that different convective heat transfer coefficients for the internal e external side of the curved tube were obtained from correlations available on curved tubes.

Figure 7 shows comparisons between the numerical predictions and the experiments indicating the effects of the Dean number on the interface velocity for both the internal and external sides of the curved tubes. As can be seen increasing the Dean number leads to increase the interface velocity. Also the interface velocity is in creased by decreasing the temperature of the working fluid

Figure 8 shows the effects of changing the volumetric flow rate on the interface velocity at the external and internal sides of the curved tube. As can be seen increasing the volumetric flow rate increases the interface velocity. The numerical predictions show similar trends as can be verified from figure 8.

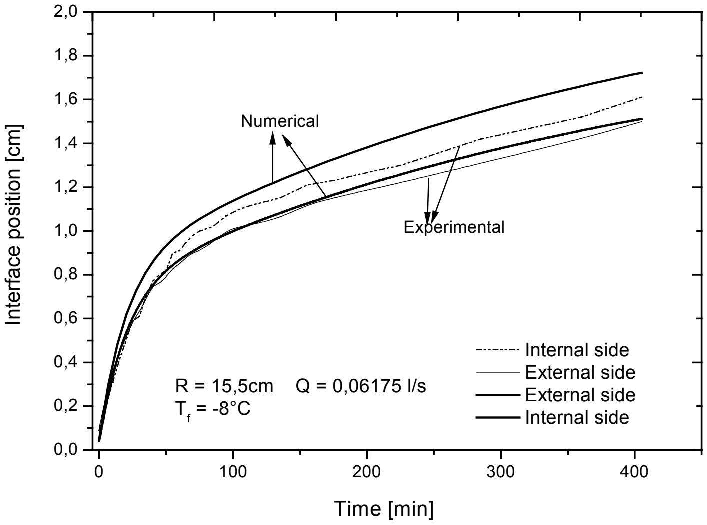

Figure 6 Comparison between the numerical predictions and experimental results.

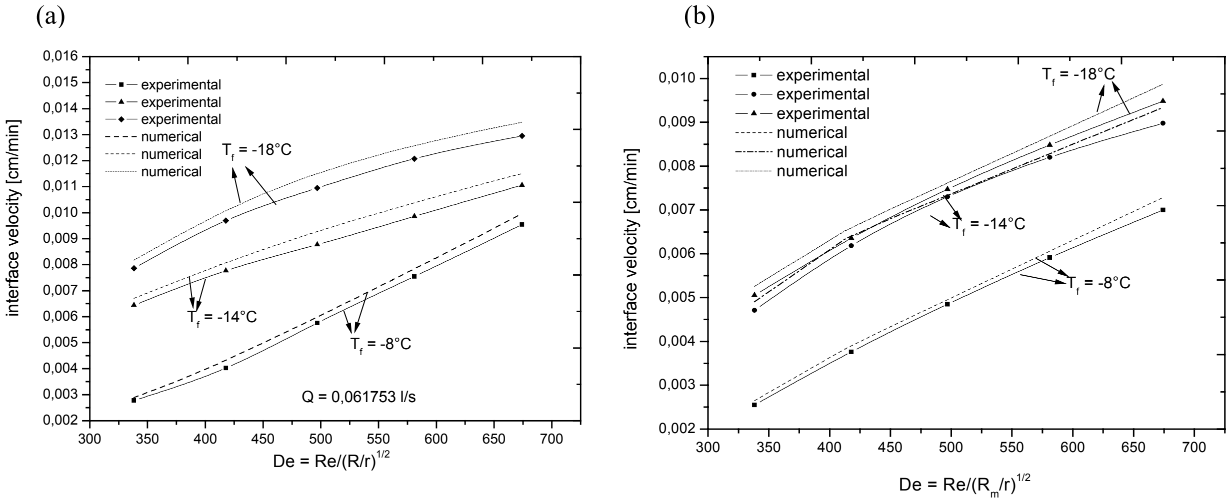

Figure 7 (a) Effects of the Dean number on the interface velocity on the internal side of the curved tube. (b) Effects of the Dean number on the interface velocity on the external side of the curved tube

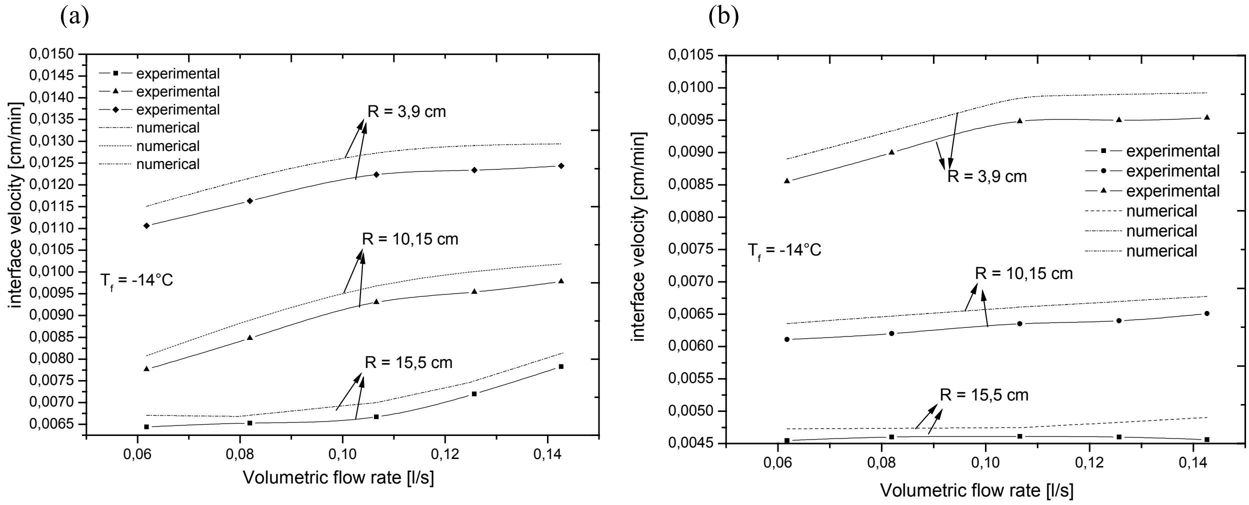

Figure 8 (a) Effects of the volumetric flow rate on the interface velocity on the internal side of the curved tube. (b) Effects of the volumetric flow rate on the interface velocity on the external side of the curved tube.

Figure 9 shows comparisons between the numerical predictions and the experimental results for the effects of the Dean number on the time for complete solidification and one can observe that the time for complete solidification decreases with the increase of the Dean number.

Figure 10 shows the effect of the refrigerant volumetric flow rate on the time for complete solidification. As can be seen increasing the flow rate tends to reduce, the time for complete solidification. Also it is clear that the increase of the curvature radius leads to increasing the time for complete solidification. The comparison between the numerical predictions and experiments is satisfactory.

Figure 11 shows the effect of the working fluid temperature on the time for complete solidification. It can be seen that lowering the working temperature leads to reducing the time for complete solidification. Again one can observe that the increase of the curvature radius increases the time for complete solidification due to the fact that more mass is included the within curved region. Also one can observe reasonably good agreement between the experiments and the numerical predictions. The effect of the tube material on the interface velocity is shown in figure 12, indicating that thermally good conducting materials lead to high interface velocity.

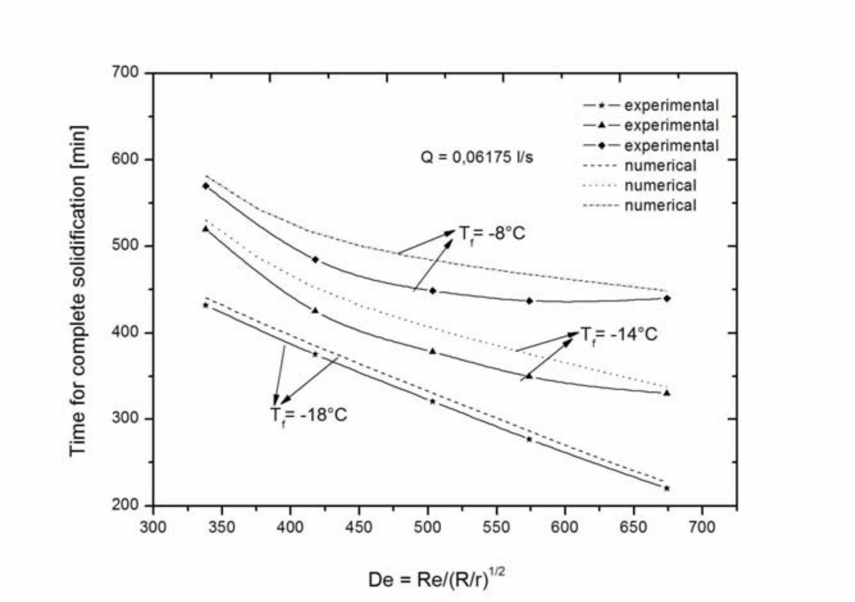

Figure 9 Effects of the Dean number on the time for complete solidification.

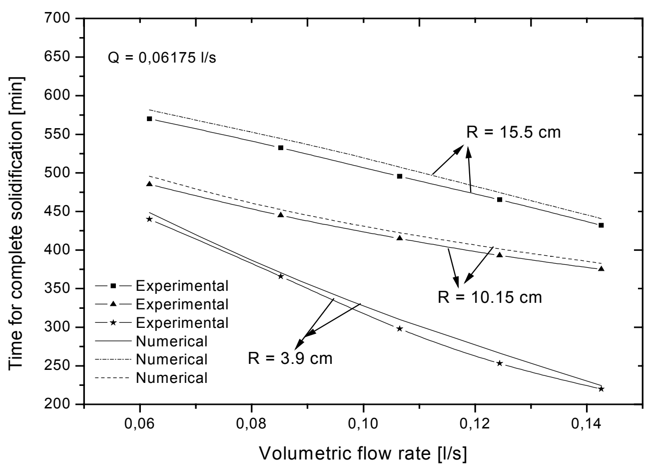

Figure 10 Effects of the volumetric flow rate on the time for complete solidification.

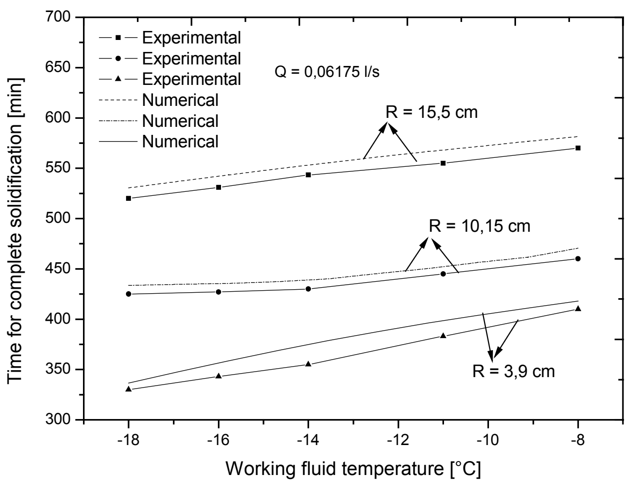

Figure 11 Effects of the working fluid temperature on the time for complete solidification

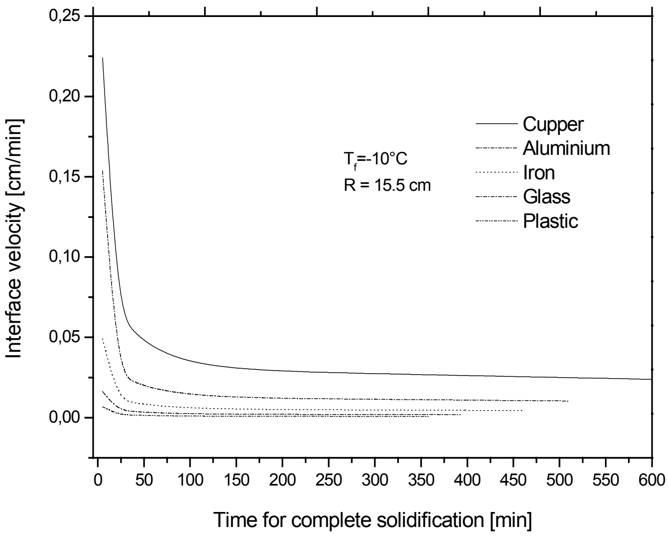

Figure 12 Effect of the curved tube material on the time for complete solidification.

5. CONCLUSION

The most important conclusions of the present work are that the developed model showed reasonably agreement when compared with the experimental results. The fact that the model did not include the solution of the velocity field inside the curved tube coupled with the external temperature field led to the discrepancies between the numerical predictions and the experimental results. The velocity of the interface, the time for complete solidification, are affected by varying the velocity of the working fluid, the temperature of the working fluid and the Dean number predicted by the model seem to agree well enough with the experiments.

6. ACKNOWLEDGEMENTS

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