

## MODELLING A TILTING THREE-WHEELED NARROW VEHICLE WITH SIX DEGREES OF FREEDOM

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**Abstract.** *The increasing of urban population has been taken a high influence in environmental management. Pollution, irrational space usage and mineral resources collapses expectatives are the most important challenges that science and governments will face from now to next centuries. Contributing to this scenario, the personal transportation has a strong participation, because they are underused most of the time! Our research group has proposing some alternatives to XXIst century society, as presented here. We have been establishing some concepts regard to narrow commuter vehicles, specially a three wheeled ones. This paper presents a first mathematical model of our concept, using Lagrange's Equation approach. We present the analytical model for 6 degrees of freedom, and a numerical model as well. Some numerical simulations are showed, and results are supplied, in the way that we can understand and test the vehicle behavior. The innovative question that we propose is the tilting capability. Our model simulates the vehicle leaning to inside turns, in the way that stability increases for any speed and radius turn situation. A simple controller has been built to stabilizes the vehicle in several different speed and steering angle conditions, that are, in fact, the main problem variables. We conclude our work showing the next steps of our project, that goals to prototype 1:1 scale production.*

**Keywords:** *tilting vehicle; dynamic model; Lagrange Equation of Movement.*

### 1. INTRODUCTION

The XXIst Century has some challenges for the society, and some of them could be crucial for our surviving in the Earth. Actually, we are voracious fossil fuel consumers, and its ending is unavoidably. The increasing of urban population has some advantages but, on the other hand, produces a series of disturbances that have a great impact in our life.

Pollution is one of the most important issues that urban life has been facing. Air and water pollution have been made a great environmental disaster, especially in urban centers, where population concentration and growing has high rates, comparing to other places. Internal combustion engines have a great participation in this chaotic scenario, essentially in Greenhouse Gas emissions, like carbon monoxide. A lower rate of carbon monoxide is the main key for new vehicles, and a short term solution is the small engine productions, replacing the big ones. Additionally, small engines have another advantage; they consume less fuel, another challenge for this century.

On the other hand, comparing to big engines, the small ones have less power, and consequently, they are specified to small vehicles, with few mass. This drives us to small vehicles like a feasible solution for urban personal transportation, because they could solve some of related problems listed above.

In this direction a lot of effort has been taken by research centers at universities and industries, like CLEVER project, acronym for Compact Low Emission Vehicle for Urban Transport (Ashmore, 2004),(Johannsen *et al*, 2003), (Johannsen *et al*, 2006). This vehicle has two occupants capacity, in tandem position, is a very small car – 1 meter width – and has an incredibly consume, 66 km/l. There are also new projects based on electrical power train, like SAM, manufactured by Cree Ltd. in Switzerland (Cree, 2006),(SAM, 2007), a commercial vehicle that costs less then € 7.000,00. There is also hybrid concepts vehicle, like Aptera (Stewart, 2007), that uses a small internal combustion engine (ICE) to produces energy for an electrical one that moves the vehicle.

These concepts are complete different regarding to power train, but they have in common the small vehicle size, designed for urban use, and three wheels base.

Looking for some relevant contributions, our research lab is working on a conceptual commuter vehicle, called Flue: a three wheeled car, two wheels on the front, for two passengers, with low emission compromise, for urban uses. We started our dynamical study in Vieira *et al* (2007), and nowadays we are working on control system definitions.

Here we present our six degrees of freedom dynamical model, which allows us to simulate any kind of the vehicle behavior. In the second section our analytical model is shown, basically the equations of movement definitions that we made from lagrangean approach. On the third section we show the numerical model that ran under Matlab Software for simulation under four different scenarios at the forth section: accelerating in a straight line, to evaluate pitch angle; curving for both sides in max speed; curving for both sides in slow speed and accelerating and curving at the same time. These three models target to rolling angle behavior evaluation. At the end, we present some conclusions and result discussions.

## 2. THE ANALYTICAL MODEL

To proceed to Lagrangean Equations of Movement, we start our approach defining the velocity space of vehicle. The velocity space is representing by velocity equation definitions, based on each mass of vehicle, following a multi body approach that is the first step of our modeling.

Following Fig. 1 one can see that we have four different mass:

1. RW - Rear wheel, considered mass 1;
2. MC – Main Chassis, considered mass 2;
3. RFW – Right Front Wheel, considered mass 3;
4. LFW – Left Front Wheel, considered mass 4.

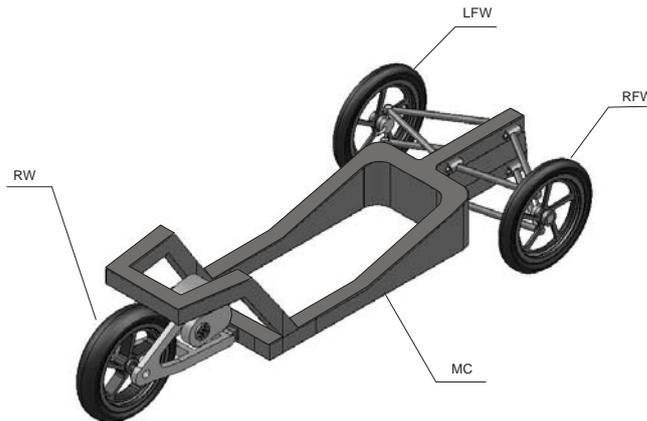


Figure 1. Model mass definition in chassis representation.

In the same way, we defined our coordinate system, following Fig. 2 schema: that is  $X$  axle along longitudinal vehicle plane,  $Y$  axle along transversal vehicle plane, and  $Z$  as vertical one.

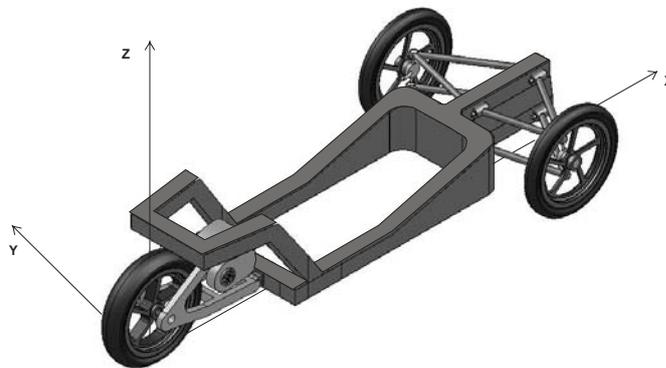


Figure 2. Inertial coordinate system regards to chassis vehicle.

We also have to define the six degrees of freedom of our model:

1.  $x$  – displacement in  $X$  direction;
2.  $y$  – displacement in  $Y$  direction;
3.  $z$  – displacement in  $Z$  direction;
4.  $\psi$  – rotation in  $Z$  direction (yaw);
5.  $\varphi$  – rotation in  $X$  direction (rolling);
6.  $\theta$  – rotation in  $Y$  direction (pitch).

We will deduce the motion equations for the first vehicle mass to show the method application, replicating it for all other three mass. According to Fig. 3, we can define linear speeds of first body as following:

$$\begin{aligned}
 u_1 &= u \\
 v_1 &= v - h_1 \dot{\varphi} \cos(\varphi) \\
 w_1 &= w_{limi} - h_1 \dot{\varphi} \sin(\varphi)
 \end{aligned}
 \tag{1}$$

Where:  $u$  is the vehicle linear speed in  $X$  direction,  $v$  the vehicle linear speed in  $Y$  direction,  $w_{limi}$  the linear speed of contact point tire/ground in  $Z$  direction,  $h_1$  the mass center distance to the origin in  $Z$  direction,  $\varphi$  and  $\dot{\varphi}$  the vehicle rotation around  $X$  axis, and its variation at time. One notes that all equations are given on positive directions, for angles and distances.

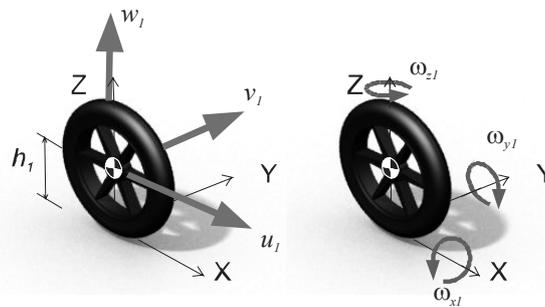


Figure 3. Linear speeds (a) and angular speeds (b) of rear wheel.

On the same way, angular speeds of rear wheel mass will be given by:

$$\begin{aligned}
 \omega_{x1} &= \dot{\varphi} \\
 \omega_{y1} &= 0 \\
 \omega_{z1} &= \dot{\psi}
 \end{aligned}
 \tag{2}$$

Where  $\dot{\psi}$  is the yaw ( $\psi$ ) rate. We also denote  $\omega_{ij}$  to angular mass velocity, where  $i$  is the rotation axis direction and  $j$  the mass number.

Important to notice that even the pitch angle is presented, it will not be considered in 1<sup>st</sup>, 3<sup>rd</sup> and 4<sup>th</sup> mass, because the pitch effect on wheels is the rolling action, in the way that no pitch will occur at all. So, the 2<sup>nd</sup> mass will be the only one that will pitch, what could be reinforced by mass values, and corresponding effects regarding to Vehicle Inertial Momentum.

In this approach, to 2<sup>nd</sup> mass, the linear speeds will be given by:

$$\begin{aligned}
 u_2 &= u + h_2 \dot{\theta} \cos(\theta) \\
 v_2 &= v + a_2 \dot{\psi} - h_2 \dot{\varphi} \cos(\varphi) \\
 w_2 &= w - h_2 \dot{\varphi} \sin(\varphi) - a_2 \dot{\theta} \sin(\theta)
 \end{aligned}
 \tag{3}$$

Where:  $h_2$  is the 2<sup>nd</sup> mass center of gravity distance to  $XY$  plane,  $\theta$  and  $\dot{\theta}$  is the pitch and its variation at time, and  $a_2$  2<sup>nd</sup> mass center of gravity distance to  $YZ$  plane.

The angular velocities for this mass are:

$$\begin{aligned}
 \omega_{x2} &= \dot{\varphi} \\
 \omega_{y2} &= \dot{\theta} \\
 \omega_{z2} &= \dot{\psi}
 \end{aligned}
 \tag{4}$$

Replicating the same reasoning for all bodies, at the end we will have 12 linear speed equations and 12 for angular ones that will be useful for Kinetic Energy equation of the vehicle definition, what will be given by:

$$\begin{aligned}
 T = & \frac{1}{2} m_1 (u^2 + (v - h_1 \dot{\phi} \cos(\varphi))^2 + (w_{1ini} - h_1 \dot{\phi} \sin(\varphi))^2) + \\
 & \frac{1}{2} m_2 ((u + h_2 \dot{\theta} \cos(\theta))^2 + (v + a_2 \dot{\psi} - h_2 \dot{\phi} \cos(\varphi))^2 + (w - h_2 \dot{\phi} \sin(\varphi) - a_2 \dot{\theta} \sin(\theta))^2) + \\
 & \frac{1}{2} m_3 ((u + b_3 \dot{\psi})^2 + (v + a_3 \dot{\psi} - h_3 \dot{\phi})^2 + (w_{3ini} - b_3 \dot{\phi})^2) + \\
 & \frac{1}{2} m_4 ((u - b_4 \dot{\psi})^2 + (v + a_4 \dot{\psi} - h_4 \dot{\phi})^2 + (w_{4ini} + b_4 \dot{\phi})^2) + \\
 & \frac{1}{2} \dot{\psi}^2 [I_{z1} + I_{z2} + I_{z3} + I_{z4}] + \frac{1}{2} \dot{\phi}^2 [I_{x1} + I_{x2} + I_{x3} + I_{x4}] + \frac{1}{2} \dot{\theta}^2 I_{y2} \\
 & \dot{\psi} \dot{\phi} [I_{xz1} + I_{xz2} + I_{xz3} + I_{xz4}] + \dot{\theta} [\dot{\psi} I_{yz} + \dot{\phi} I_{xy2}]
 \end{aligned} \tag{5}$$

To Potential Energy we have to consider that we will have a load transfer between rear and front axles, in the way that one spring is under compression, at the same time that other one is under traction. Assuming  $\delta_r$  like the rear spring deformation caused by pitch,  $\delta_f$  as the front one, and that vehicle is absolutely symmetrical regarding to longitudinal vehicle axle, the pitch potential energy is:

$$U_{arf} = \frac{1}{2} (k_1 \delta_r^2 + (k_3 + k_4) \delta_f^2) \tag{6}$$

Where  $k_1$ ,  $k_3$  and  $k_4$  are the rear, right front and right left spring stiffness. The vehicle potential energy will be given by:

$$U = \frac{1}{2} (k_1 + k_3 + k_4) z^2 + mgh_2 (1 + \cos(\varphi)) + \frac{1}{2} (k_1 \delta_r^2 + (k_3 + k_4) \delta_f^2) \tag{7}$$

Where  $z$ , is the vertical displacement, positive in  $Z$  axle positive direction. One can identify three different elements in potential energy equation. The first one regards to  $Z$  axle displacement, the second one given by pendulum effect, and the last one regards to pitch effect.

Spend some time analyzing the potential energy of our vehicle; we concluded that in tilting models, a pendulum potential energy has to be evaluated, so that the original pendulum equation has to be attached in Eq. (7).

Following the approach presented by (Lalanne, Berthier, and Der Hagopian, 1986), (Hand, 1988), (Lowndes, 1998), (Rajamani, 2005), (Pacejka, 2005, and (Leal, Rosa and Nicolazzi, 200X) the Lagrangian Equation for a vehicle movement is given by:

$$\frac{d}{dt} \frac{\partial T}{\partial \dot{q}_i} - \frac{\partial T}{\partial q_i} + \frac{\partial U}{\partial q_i} + \frac{\partial \mathfrak{S}}{\partial \dot{q}_i} = Q_i \tag{8}$$

Where  $Q_i$  is the external generalized forces,  $T$  the kinetic energy,  $U$  potential one,  $\mathfrak{S}$  the dissipation function, and  $q_i$  the generalized coordinates.

An approach to solve the Lagrangian equation is its representation in matrix form (Leal, Rosa, Nicolazzi, 200X), so that we can solve Eq. (8), solving:

$$\mathbf{M}\ddot{\mathbf{q}}(t) + \mathbf{C}\dot{\mathbf{q}}(t) + \mathbf{K}\mathbf{q}(t) = \mathbf{f}(t) \tag{9}$$

Where:  $\mathbf{M}$  is the inertial matrix,  $\mathbf{C}$  the damping matrix,  $\mathbf{K}$  the stiffness matrix,  $\mathbf{q}$  the generalized coordinator vector, and  $\mathbf{f}$  the external forces vector.

In this way, we can generate the  $\mathbf{M}$  matrix members by:

$$m_{ij} = \frac{\partial^2 T}{\partial \dot{q}_i \partial \dot{q}_j} \tag{10}$$

The  $\mathbf{K}$  matrix, on the same hand will be given by:

$$k_{ij} = \frac{\partial^2 U}{\partial q_i \partial q_j} \tag{11}$$

And, **C** matrix by:

$$c_{ij} = \frac{\partial^2 \mathfrak{F}}{\partial \dot{q}_i \partial \dot{q}_j} \quad (12)$$

Where  $\mathfrak{F}$  is the Rayleigh dissipative function, given by:

$$\mathfrak{F} = \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n c_{ij} \dot{q}_i \dot{q}_j \quad (13)$$

Where  $c_{ij}$  is the damping constant for  $q_{ij}$  generalized coordinate.

The damping interacts with three degrees of freedom of our model: in  $z$  displacement, rolling angle ( $\varphi$ ), and pitch angle ( $\theta$ ), that are respective 3<sup>rd</sup>, 5<sup>th</sup>, and 6<sup>th</sup> elements of our generalized coordinate vector (**q**). So, the Rayleigh dissipative function for our model will be given by:

$$\mathfrak{F} = \frac{1}{2} c_{33} w^2 + \frac{1}{2} c_{55} (b\dot{\varphi})^2 + \frac{1}{2} c_{66} (d_{CG}\dot{\theta})^2 \quad (14)$$

Where:

$$\begin{aligned} c_{33} &= c_{z1} + c_{z3} + c_{z4} \\ c_{55} &= c_{z3} + c_{z4} \\ c_{66} &= c_{z1} + c_{z3} + c_{z4} \end{aligned} \quad (15)$$

Where  $c_{zi}$  is damping constant of  $i$ -th wheel and  $d_{CG}$  is the longitudinal distance between spring and vehicle center of gravity.

The external forces vector has a generic form given by:

$$\mathbf{f}(t) = \begin{Bmatrix} F_x \\ F_y \\ F_z \\ M_\psi \\ M_\varphi \\ M_\theta \end{Bmatrix} \quad (16)$$

Where each element is responsible for movements on each vehicle's degrees of freedom, that are represented by generalized coordinator vector, and its temporal derivatives:

$$\mathbf{q} = \begin{Bmatrix} x(t) \\ y(t) \\ z(t) \\ \psi(t) \\ \varphi(t) \\ \theta(t) \end{Bmatrix} \quad \dot{\mathbf{q}} = \begin{Bmatrix} u(t) \\ v(t) \\ w(t) \\ \dot{\psi}(t) \\ \dot{\varphi}(t) \\ \dot{\theta}(t) \end{Bmatrix} \quad \ddot{\mathbf{q}} = \begin{Bmatrix} \dot{u}(t) \\ \dot{v}(t) \\ \dot{w}(t) \\ \ddot{\psi}(t) \\ \ddot{\varphi}(t) \\ \ddot{\theta}(t) \end{Bmatrix} \quad (17)$$

### 3. THE NUMERICAL MODEL

After analytical effort, we produced a numerical model to run inside Matlab<sup>1</sup> software analysis, based on Lagrangian equation given by Eq. (9). The aim of the numerical model is to allow an easier behavior test, running under commercial numerical simulation software.

<sup>1</sup> Matlab is a Trade Mark of MathWorks

We created a numerical model using Matlab software, in the way that future controller system models could be implemented and evaluated faster.

To adapt our analytical model to numerical one, we transformed it in a state variable version problem, increasing the number of variables and equations to 12, in the way that we could solve it under Matlab.

We defined also the inputs of our numerical model, to allow a solution finding, so that schematic representation became like showing in Fig. 4.

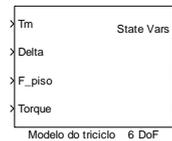


Figure 4. Schematic representation of s-function numerical model in Simulink.

The numerical model has four inputs, represented by:

1. Engine Torque ( $T_m$ ): that represents actual torque supplied to wheels at any simulation step.
2. Steering angle ( $\Delta$ ): it is the angle between vertical wheel plan of front wheels and longitudinal vehicle plane. This angle defines lateral vehicle movements.
3. Ground force ( $F_{\text{piso}}$ ): this value represents ground/vehicle interface, and is not null when ground profile changes, in the way that the vehicle moves.
4. Tilting torque ( $T_{\text{torque}}$ ): the vehicle has to tilt toward inside turn, what is not a natural effect, because inertial forces push outside. This input has this capability, to lean vehicle, and represents an actual actuator.

In output port, we have state variables (Stat Vars), that are all six degrees of freedom of analytical model (Eq. 17a) plus respective rates (Eq. 17b), that became twelve.

#### 4. RUNNING THE NUMERICAL MODEL

The numerical model was simulated under some constrains, and the vehicle properties as presented in (Vieira, Roqueiro and Nicolazzi, 2009) that are listed below:

1. We considered three spring/dumper sets, one for each wheel, placed above respective center of mass;
2. The total vehicle mass is 415 kg, included one passenger;
3. Longitudinal speed is limited to 15 m/s;
4. In our model, the wheels are considered as thin as possible so that contact point is fixed;
5. The vehicle has 60% of its mass over rear axle;
6. There is no slip between tire and ground, and every torque is transferred to soil;
7. There are no limits to the tilt angle values, so that the soil constrain is not present in this model;
8. The main dimensions are given by fig. 5;
9. The soil is a flat surface that means that  $w_{1ini}$ ,  $w_{3ini}$ , and  $w_{4ini}$ , and  $F_{\text{piso}}$  are zero;
10. The engine has a maximum torque of 73 Nm.

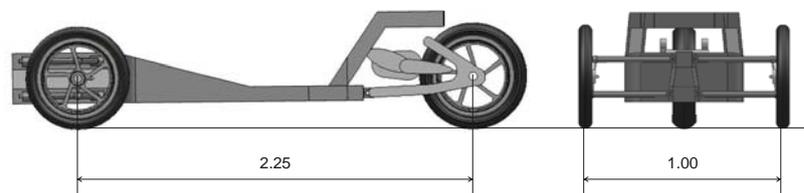


Figure 5. Vehicle main dimensions in meters.

We created four different scenarios to simulate our model:

1. Acceleration from 0 m/s to max speed (15m/s).
2. A consecutive steering maneuvering, turning to left, to center and to right at max speed (15m/s).
3. A consecutive steering maneuvering, turning to left, to center and to right at low speed (5 m/s).
4. A consecutive steering maneuvering, turning to left, to center and to right, accelerating the vehicle from 0 m/s to max speed (15 m/s).

For the first scenario the vehicle is running in straight path, and only longitudinal forces are presented. We have no lateral disturbance at all, and the pitch angle will be tested regarding inertial to effects.

The second, third and fourth scenarios have several external forces. The most important input is the steering angle, which values are given by the graphical representation in Fig. 6.

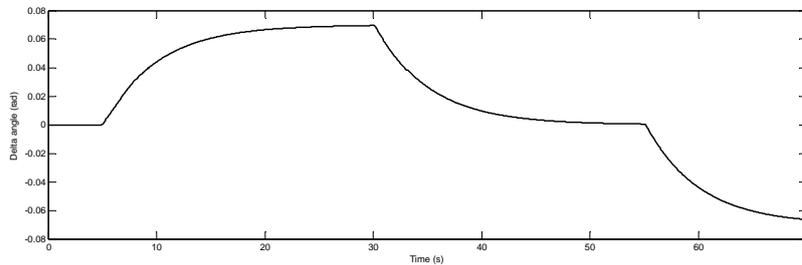


Figure 6. Steering angle input for 2<sup>nd</sup>, 3<sup>rd</sup> and 4<sup>th</sup> scenarios.

One can see in Fig. 6 that we have the steering wheel turned to left at the beginning (positive region in the graphic), after that it remains in the center (null region) and, finally to right (negative region in the graphic). This distribution is important to verify the inertial effects of side-to-side steering, an important issue for our model, because it has no restrictions for rolling angle, so that equilibrium points are maintained exclusively by the control system.

The control system that we use was based on STC – Steering Tilt Control model, proposed by (Gohl *et al*, 2004), (Kidane *et al*, 2008), that has already given some good results for our 5 DoF model (Vieira, 2008). STC is the control method used by bicycle and motorcycle riders to balance their vehicles, based in equilibrium transversal force created by steering action. Using STC controller, we can create the counter-steering action that could reduce the necessary energy to lean the vehicle. However, the controller method is out of scope of this paper, and some good references of its definition is given by (Hibbard and Karnopp, 1996), (Gohl *et al*, 2004), (Kidane *et al*, 2008).

The objective of first scenario was the pitch behavior analysis, like shown in Fig. 7. We had adopted some suspension attributes considering the vehicle mass (Vieira, 2009) that were critical to pitch behavior. The simulation considers that from zero to max speed, the engine transfers 100% of its torque to the wheels, and no slip is allowed.

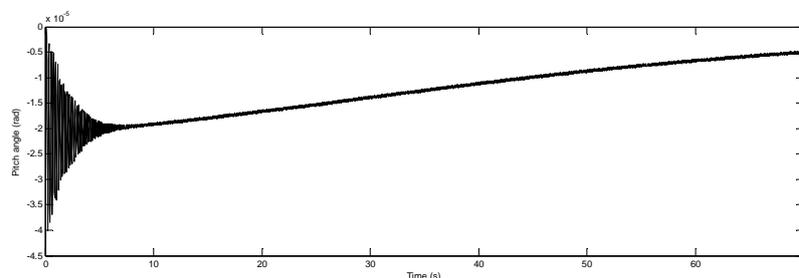


Figure 7. Pitch behavior for 1<sup>st</sup> simulation.

One can see from the 1<sup>st</sup> simulation that pitch behavior is as it was expected. We have a negative angle value that leans to zero while the acceleration decreases, like showing in Fig. 8. The speed derivative leans to zero at the same time that maximum speed is going to reach.

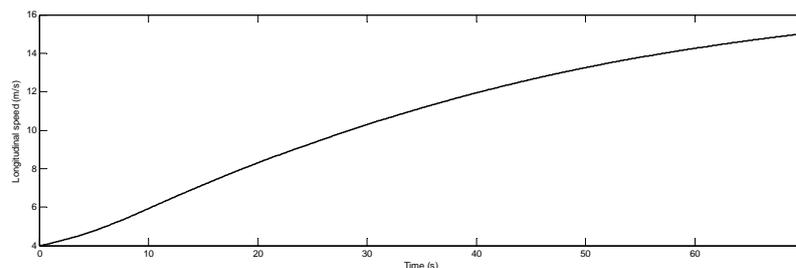


Figure 8. Longitudinal speed for the 1<sup>st</sup> simulation.

The second and third simulations have the stability evaluation goal. We have found the vehicle in the same disturbance, but in two different speed values, max speed and low speed. Important to notice that we are not evaluating the controller response, of course, the controller response is not in question but only the vehicle model behavior, which represented a good accuracy. One can see that the rolling angles are both acceptable, like shown in Fig. 9a and Fig. 9b.

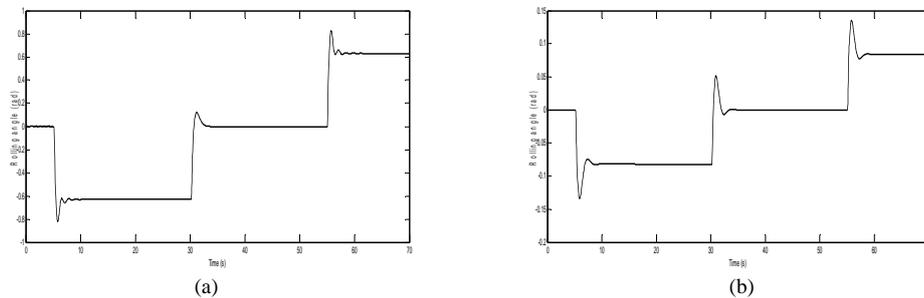


Figure 9. Rolling angle for (a) 2<sup>nd</sup> simulation and (b) 3<sup>rd</sup> simulation.

An analytical evaluation of max desirable rolling angle in both situations produces 0.6304 rad and 0.0809 rad respectively for max speed and low speed that is closer enough to numerical model solution (0.6307 rad and 0.0819 rad).

The 4<sup>th</sup> simulation is an application of all disturbances presented in 1<sup>st</sup>, 2<sup>nd</sup> and 3<sup>rd</sup> ones, and at this point, rolling angle and pitch angle have an important analysis, like shown in Fig. 11.

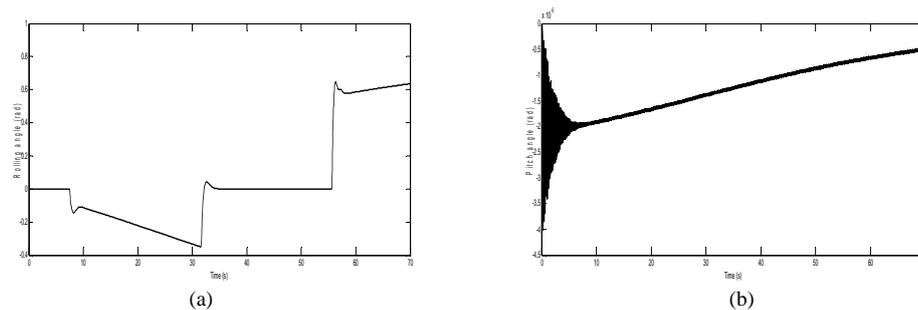


Figure 11. (a) Rolling angle and (b) pitch angle for 4<sup>th</sup> simulation.

One can see that we have a strong relation between rolling angle and speed that varies like showed in Fig. 8. On the first section of Fig. 11 (negative values of rolling angle) we have a high increasing angle, because the speed derivative is higher than on the third section, where the rolling angle is positive. So the controller leans vehicle at the same time that the speed increases, what shows an expected behavior from stability point of view.

## 5. CONCLUSIONS

The results achieved by four simulation processes could show us that our analytical model has a behavior very close to our expectations. We evaluate all six degrees of freedom of our model, but only rolling and pitch were showed because they are more significant for our evaluation. They are the best response to inputs that we used, engine torque variation and steering angle.

For the 1<sup>st</sup> simulation we could verify that there is a relation between pitch angle and longitudinal acceleration. As lower acceleration becomes, lower is the pitch angle. The small values were expected because the max acceleration of the vehicle is very low, about 0.2 m/s<sup>2</sup>. Increasing this value, we could verify more pitch in our vehicle.

The numerical values compared to analytical one testify that our model has a very close reality representation. For max speed, the rolling angle error was less than 0.1%, and for small one a value of 1.2% was reached. Both are acceptable which demonstrate a very accurate model.

From 4<sup>th</sup> simulation we could verify that for small values, have no impact between the steering angles and pitch angle of the vehicle. In our model, we have a variation of center of gravity height when steering, but in both cases, 0.07 rad had not affected this value in the way that it could change the pitch vehicle.

Following our research study, the next step will be the controller system definition. We are working on some different approaches, like robust controller, based on artificial intelligence, and predictive ones. Some simulations will show us which will be the most accurate, and appropriated, regarding to the PID controller that we have used in this

paper. After that, we will build a 1:1 scale model to implement the better solution, objectifying a drive by wire vehicle prototype for urban uses, to be used as research platform for mobility solutions studies.

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