

## ANALYSIS OF THE HOT FORGING OF METALS INCLUDING THE EFFECT OF THE EVOLUTION OF THE MICROSTRUCTURE

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**Abstract.** *Based on experimental evidence it is well known that the material properties of metals are strongly influenced by the characteristics of the microstructure of the material, such as the average size of grains, and their orientation in grains and sub grains. During mechanical deformation processes, materials suffer significant changes in their microstructures, which in turn affect considerably the mechanical properties of materials. Our aim, in this work is to present a thermo-mechanical model and propose a numerical scheme for the analysis of hot forging processes of metals that includes the evolution of the microstructure of the material.*

*The accounted mechanism of evolution of the microstructure of the material during a hot forging process includes the hardening of the material, the dynamic recrystallisation, which leads to the decrease of the grain sizes, and the grain growth phenomenon.*

*A total Lagrangian approach is employed in the formulation of the problem. Also, a logarithmic strain measure is employed together with the associated rotated Kirchhoff stress tensor. The discretization of the problem is done by applying the Galerkin finite element method, where plane strain and axisymmetric problems are considered. The metal is modeled by a viscoplastic material model that includes as internal variables, besides the viscoplastic strain and the accumulated effective viscoplastic strain, the material strength and the average grain size of the material. In order to investigate the adequacy of the presented theory and to attest the effectiveness of the proposed numerical scheme, one solves a set of simple problems.*

**Keywords:** *microstructure evolution, finite thermo viscoplasticity, hot forging process*

### 1. INTRODUCTION

In hot forging of metals, materials experiment changes in their microstructures that affect considerably their mechanical properties. The evolution of the microstructure under hot deformation processes generally involve strain hardening of the material, dynamical recovering, recrystallisation and grain growth, been these processes strongly dependent on temperature and rate of deformation.

The process of recrystallisation leads to the local annihilation of a large number of dislocations and is in general associated with the decrease of the average grain size. As a result, the material suffers locally a softening, giving rise to localized deformations, which may produce slip lines and nucleation of micro cracks. These localizations are also responsible for residual stresses and strains after the cooling process of the material. The prediction of such phenomena is fundamental for the control of the mechanical properties of metals undergoing hot deformation processes.

Manny different constitutive models have been proposed in the literature, with the aim of describing the evolution of the microstructure of metals, under thermo mechanical loadings, see Lin and Dean (2005), Lin and Liu (2003), Cheng (2000), Bontcheva and Petzov (2003), Molinari and Ravichandran (2005), Poliak and Jonas (2003) and Busso (1998). Since the research in this field is still recent, there is no relation that may be seen as consolidated.

The objective of this work is to extend the work of Busso (1998) and Ganapathysubramanian and Zabarar (2004) in order to account in the analysis the effect of the coupled thermomechanical deformation process. Moreover, the description of the evolution of the microstructure under hot deformation processes, following the ideas of Busso (1998), is done with the use of internal variables. This results in a consistent formulation that is able to describe the recrystallisation, the strain hardening, and the static and dynamic recovering processes of a material under hot finite deformation modeled by a thermal viscoplastic constitutive relation.

Macroscopically, the grain refinement due to dynamic recrystallisation causes a softening of the material as shown in Fig. 1, for different deformation rates.

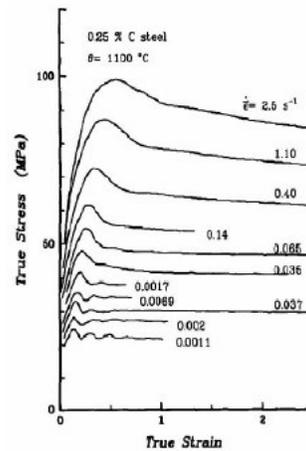


Figure 1. Stress-strain curves for different deformation rates

From experimental observations, the extension of the process of material softening during recrystallisation is related to the reduction of the mean density of dislocations and the average grain size of the material. A typical evolution of the mean density of dislocations in a material during recrystallisation is depicted in Fig. 2, in which  $\rho_0$  and  $\rho_f$  denote the initial and final mean densities of the dislocations, after recrystallisation, and  $L_0$  and  $L^*$  denote the initial and final mean grain sizes, where  $L^*$  corresponds to the stabilized average grain size obtained after the end of recrystallisation.

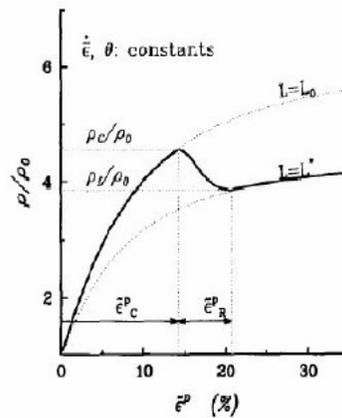


Figure 2. Evolution of the average grain size for a given temperature and deformation rate

Under isothermal processes and constants deformation rates, shown in Fig. 2, the extension of the recovery of dislocations, i.e. dislocation annihilation, during recrystallisation is given by the difference between the initial and final dislocation densities of the recrystallisation process  $\rho_c$  and  $\rho_f$  respectively. As in the macroscopic behavior, illustrated in Fig. 1, one may identify a mean critical density of the accumulated viscoplastic deformation,  $\bar{\rho}_{efc}^p$ , from which the recrystallisation takes initializes. The required deformation for the termination of the softening of the material leaving the material partially or totally recrystallised is given by  $\bar{\epsilon}_{efc}^p + \bar{\epsilon}_{efR}^p$ .

## 2. THEORETICAL DEVELOPMENT

The proposed model is implemented in a Total Lagrangian framework that considers: a multiplicative decomposition of the deformation gradient, into a plastic and an elastic part and a constitutive relation given in terms of the logarithmic, or Hencky, strain measure and the rotated Kirchhoff stress.

### 2.1. Kinetics of deformation

The viscoplasticity model presented in this paper considers the multiplicative decomposition of the deformation gradient  $\mathbf{F}$  into an elastic deformation,  $\mathbf{F}^e$ , and a plastic deformation,  $\mathbf{F}^p$ . Thus,

$$\mathbf{F} = \mathbf{F}^e \mathbf{F}^p \quad (1)$$

Moreover, since  $\mathbf{F}^e = \mathbf{R}^e \mathbf{U}^e$ , one computes the elastic strain as  $\mathbf{E}^e = \ln(\mathbf{U}^e)$  and employ the conjugate stress measure  $\bar{\boldsymbol{\tau}} = (\mathbf{R}^e)^T \boldsymbol{\tau} (\mathbf{R}^e)$ , where  $\boldsymbol{\tau}$  is the Kirchhoff stress,  $\boldsymbol{\tau} = \det(\mathbf{F}) \boldsymbol{\sigma}$ , with  $\boldsymbol{\sigma}$  denoting the Cauchy stress

## 2.2. Constitutive modelling and hyperelastic response

Here, one introduces the deviatoric rotated Kirchhoff stress, given by  $\bar{\boldsymbol{\tau}}^D = \bar{\boldsymbol{\tau}} - \frac{1}{3} \text{tr}(\bar{\boldsymbol{\tau}}) \mathbf{I}$ , The von Mises effective rotated Kirchhoff stress, given by  $q = \sqrt{\frac{3}{2} \bar{\boldsymbol{\tau}}^D : \bar{\boldsymbol{\tau}}^D}$  and the hydrostatic pressure stress, given by  $p = -\frac{1}{3} \text{tr}(\bar{\boldsymbol{\tau}})$ . Moreover, one may decompose the elastic strain as  $\mathbf{E}^e = \mathbf{E}^{eD} + e_H^e \mathbf{I}$ , with  $e_H^e = \frac{1}{3} \text{tr}[\mathbf{E}^e]$ . As a result, one may define the thermal hyperelastic response as

$$p = (2\mu + 3\bar{\lambda}) e_H^e - \alpha(2\mu + 3\bar{\lambda}) \Delta T \quad (2)$$

$$\bar{\boldsymbol{\tau}}^D = \frac{E}{(1+\nu)} \mathbf{E}^{eD} = 2\mu \mathbf{E}^{eD} \quad (3)$$

where  $\Delta T = (T - T_o)$ , with  $T_o$  denoting the initial temperature of the body. Also, one has the Fourier Law, given for the thermal problem by  $\bar{\mathbf{q}}_o = -k \nabla T$ , in which  $\bar{\mathbf{q}}_o$  is the heat flux per unit area.

## 2.3. Viscoplastic flow rule

The viscoplastic model with no yield function is given by

$$\bar{\mathbf{D}}^p = \frac{3}{2} \frac{\bar{\boldsymbol{\tau}}^D}{q} \dot{e}_{ef}^p \quad (4)$$

where

$$\dot{e}_{ef}^p = \dot{e}_{e_0}^p \exp\left(-\frac{Q_{ac}}{RT}\right) \left(\frac{L_0}{L}\right)^w \left[\sinh\left(\frac{\bar{\boldsymbol{\tau}}^D}{\xi S}\right)\right]^{\frac{1}{n_1}} \quad (5)$$

in which  $L_0$  is the initial average grain size of the material,  $\dot{e}_{e_0}^p$  is the effective rate of deformation reference loading condition, considered known,  $\xi$ ,  $n_1$  and  $w$  are material constants. The temperature dependence was introduced through the classical Arrhenius laws in terms of the activation energy  $Q_{ac}$ , considered known and the material constant  $R$ . This prescribed flow rule is complemented by postulating a null plastic spin, compatible with plastic isotropy, i.e.,  $\bar{\mathbf{W}}^p = 0$ . The evolution of the plastic deformation is given by  $\dot{\mathbf{F}}^p = \check{\mathbf{L}}^p \mathbf{F}^p$  where  $\check{\mathbf{L}}^p = (\mathbf{U}^e)^{-1} \bar{\mathbf{L}}^p (\mathbf{U}^e)$ ,

$$\bar{\mathbf{D}}^p = \frac{1}{2} \left( \bar{\mathbf{L}}^p + [\bar{\mathbf{L}}^p]^T \right) \text{ and } \bar{\mathbf{W}}^p = (\mathbf{U}^e)^{-1} \bar{\mathbf{W}}^p (\mathbf{U}^e) = 0.$$

## 2.4. Evolution law of the state variables

The evolution law for the deformation resistance variable  $S$

$$\dot{S} = \dot{H}_{hard} - \dot{R}_{reccr} \quad (6)$$

in which the hardening function is given by  $\dot{H}_{hard} = \dot{H}_{hard}(\bar{\boldsymbol{\tau}}^D, S, L, T)$ . Here,  $\dot{H}_{hard}$  is the strain hardening function of the material, which may be seen as the rate in which the mean dislocation evolve in the material, before or after recrystallisation. The dynamic recovering is given by  $\dot{R}_{reccr} = \dot{R}_{reccr}(\bar{\boldsymbol{\tau}}^D, S, L, T)$ . Here,  $\dot{R}_{reccr}$  is the function related to

the dynamic recovery process which describes the rate of annihilation of dislocations due to the nucleation of new grains during the recrystallisation process. The hardening rule, proposed by Busso (1998) and also used by Ganapathysubramanian and Zabaras (2004), employed in this work, is

$$\dot{H}_{hard} = h_s \left| 1 - \frac{S}{S^*} \right|^{a_s} \text{sign} \left( 1 - \frac{S}{S^*} \right) \dot{e}_{ef}^p \quad (7)$$

in which  $h_s$  and  $a_s$  are material parameters and  $S^*$ , is the asymptotic stabilized value of  $S$  that represents the saturation condition of the evolution of  $S$ , obtained in experimental tests, for prescribed values of  $\left( \frac{L_0}{L}, \dot{e}_{ef}^p, T \right)$ .

The function  $\text{sign}(\circ)$  is introduced in a way to permit the softening of the material, that may occur in situations of rapid increase of local temperature, in which the value of  $S$  is larger than  $S^*$ . Thus,  $S^* = S^* \left( \frac{L_0}{L}, \dot{e}_{ef}^p, T \right)$ , which may be approximated as

$$S^* = \bar{\chi} \left\{ \frac{\dot{e}_{ef}^p}{\dot{e}_{ef_0}^p} \left( \frac{L^*}{L_0} \right)^w \exp \left( \frac{Q_{ac}}{RT} \right) \right\}^{n_2} \quad (8)$$

where  $\bar{\chi}$  and  $n_2$  are material parameters. This equation may be written as  $S^* = S_0^* \left( \frac{L}{L_0} \right)^{n_2 \cdot w}$  in which

$$S_0^* = \bar{\chi} \left\{ \frac{\dot{e}_{ef}^p}{\dot{e}_{ef_0}^p} \exp \left( \frac{Q_{ac}}{RT} \right) \right\}^{n_2} . \quad (9)$$

The description of the dynamic recovery, that consists in the reduction of the density of dislocation in the material, which occurs during the recrystallisation process, requires an expression for the change of state between the initial and final stages of the recrystallisation process. In this work, the “driving force” that governs the recrystallisation process is the energy stored in the hardening process of the material, i.e., stored in the form of an increase of the density of dislocations, been this energy responsible for the nucleation of new grains in the material, which may occur in the boundary or even in the interior of the grains. In this way, the initial and final states of recrystallisation are given by the densities of dislocations  $\rho_c$  and  $\rho_f$  described in Fig. 2. Thus,

$$\dot{R}_{recr} = f_R X_R \left[ S - S_f \right] \dot{e}_{ef}^p, \quad (10)$$

where  $X_R = X_R \left( \bar{\tau}_{ef}, S, L, T \right)$  is the function that represents the volume fraction of the recrystallised material associated with the actual state, given in terms of  $(S, L)$ . Here,  $f_R$  is a material parameter that defines the magnitude of the dynamic recovery and  $S_f$  is the deformation resistance, in the end of the recrystallisation process. Here, one considers  $X_R = 1 - \exp \left\{ -A_L \dot{L}_{rec} t_R \right\}$ , in which  $t_R$  is the time or duration of the process of recrystallisation,  $A_L$  is the average size of the area of the grain and sub grain boundaries per unit of volume and  $\dot{L}_{rec}$  is the mean rate of grain refinement during a recrystallisation. For a given rate of effective deformation  $\dot{e}_{ef}^p$ , the recrystallisation time can be expressed in terms of the accumulated effective plastic deformation  $e_{ef}^p$  as

$$t_R = \left\langle \frac{e_{ef}^p - e_{ef_c}^p}{\dot{e}_{ef}^p} \right\rangle . \quad (11)$$

Notice that  $\langle \circ \rangle$  represents the positive part operator of  $(\circ)$ . With the aim of avoiding the excessive nucleation of grains and subgrains during the deformation process one considers  $A_L = \frac{f_X}{L}$ . Moreover, one may show that

$$\frac{\dot{L}_{rec}}{\dot{e}_{ef}^p} = \frac{\Delta L_{rec}}{e_{ef_R}^p} \quad (12)$$

where  $\Delta L_{rec}$  is the change of the mean grain size during the recrystallisation process. This relation is approximated by

$$\frac{\dot{L}_{rec}}{\dot{e}_{ef}^p} \approx \frac{L_0}{e_{efR}^p} \quad (13)$$

which gives

$$X_R = 1 - \exp \left\{ -f_X \left( \frac{L_0}{L^*} \right) \left\langle \frac{e_{ef}^p - e_{efc}^p}{e_{efR}^p} \right\rangle \right\} . \quad (14)$$

An expression for  $S_f$ , representing the deformation resistance in the end of the recrystallisation, is given, for  $a_s \neq 1$ , by

$$S_f = S_0^* \left( \frac{L^*}{L_0} \right)^{n_2 \cdot w} \left[ 1 - \left\langle \frac{h_s}{S_0^*} \left( \frac{L_0}{L^*} \right)^{n_2 \cdot w} (a_s - 1) [e_{efc}^p + e_{efR}^p] + \left( 1 - \frac{S_0}{S_0^*} \left( \frac{L_0}{L^*} \right)^{n_2 \cdot w} \right)^{1-a_s} \right\rangle^{\frac{1}{1-a_s}} \right] . \quad (15)$$

In order to avoid new grain refinement processes, after the primary recrystallisation, one may impose that  $S_f = S$  whenever  $e_{ef}^p > e_{efc}^p + e_{efR}^p$ . Substituting the above relations one derives

$$\dot{S} = \left\{ h_s \left| 1 - \left( \frac{L_0}{L^*} \right)^{n_2 \cdot w} \frac{S}{S_0^*} \right|^{a_s} \text{sign} \left( 1 - \frac{S}{S_0^*} \left( \frac{L_0}{L^*} \right)^{n_2 \cdot w} \right) - f_R X_R (S - S_f) \right\} \dot{e}_{ef}^p . \quad (16)$$

Figure 3 depicts the evolution of the deformation resistance and of the average grain size in terms of the accumulated effective viscoplastic strain  $e_{ef}^p$  for a typical steel, for a prescribed constant values of  $\dot{e}_{ef}^p$  and  $T$ .

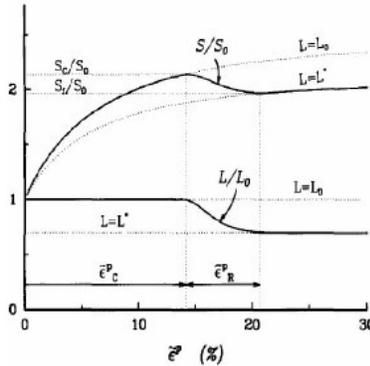


Figure 3. Typical evolution of the mean grain size for constant temperature and strain rate

A phenomenological relation largely employed in the literature relating the stabilized effective stress state after the recrystallisation  $\bar{\tau}_{ef}^*$  and the average grain size after the recrystallisation  $L^*$  is given by

$$L^* = \left( \frac{\bar{q}}{\bar{\tau}_{ef}^*} \right)^{\frac{1}{p}} \quad (17)$$

where  $\bar{p}$  and  $\bar{q} = \bar{q}(T)$  are material constants. An expression for the stabilized effective stress  $\bar{\tau}_{ef}^*$  in terms of  $\dot{e}_{ef}^p$ ,  $T$ , and  $L^*$  is given, see Busso (1998), by

$$\bar{\tau}_{ef}^* = \xi S^* \sinh^{-1} \left[ \frac{\dot{e}_{ef}^p}{\dot{e}_{fo}^p} \left( \frac{L^*}{L_0} \right)^w \exp \left( \frac{Q_{ac}}{R T} \right) \right]^{n_1} \quad (18)$$

Combining the above results with the expression for  $S^*$  gives an implicit relation, in terms of  $\dot{e}_{ef}^p$ ,  $T$  and  $L_0$

$$L^* = \left( \frac{\bar{q}}{\xi \bar{\chi} A^{n_2}} \right)^{\frac{1}{p}} \left\{ \ln \left( A^{n_1} + \sqrt{A^{2n_1} + 1} \right) \right\}^{-\frac{1}{p}}, \text{ with } A = \left[ \sinh \left( \frac{\bar{\tau}_{ef}}{\xi S} \right) \right]^{n_1} \left( \frac{L^*}{L} \right)^w. \quad (19)$$

In this way, for a given value of  $(\dot{e}_{ef}^p, T, L_0)$ , one may compute the stabilized average grain size  $L^*$ , after recrystallisation. The solution of this implicit problem is done by applying Newton's method.

The Evolution law for the mean grain size during and after the recrystallisation is given by

$$\dot{L} = \dot{L}_{ref} + \dot{L}_{grow}, \quad (20)$$

Where  $\dot{L}_{ref} = \dot{L}_{ref}(\bar{\tau}_{ef}, S, L, T, X_R)$ , is a function which describes the grain refinement process during recrystallisation and  $\dot{L}_{grow} = \dot{L}_{grow}(\bar{\tau}_{ef}, S, L, T, X_R)$ , is the function that represents the kinetics of grain increase governed by the energies stored in the grain boundaries during secondary recrystallisation – cooling process. Consider now that  $\langle L - L^* \rangle$  represents the magnitude of the loss of dynamic equilibrium occurring in the material softening process. Therefore, the rate of grain refinement may be expressed as

$$\dot{L}_{ref} = -f_R X_R \langle L - L^* \rangle \dot{e}_{ef}^p, \quad (21)$$

where  $f_R$  is a material constant. The inclusion of  $X_R$  makes  $\dot{L}_{ref} = 0$  in the critical condition of recrystallisation assuring a smooth process as observed experimentally.

If the magnitude of the loss of dynamic equilibrium is expressed in terms of the fraction of recrystallised material, at the end of recrystallisation,  $X_{R_c}$ , as  $\langle X_R - X_{R_c} \rangle$  then the rate of grain growth may be expressed as

$$\dot{L}_{grow} = \dot{L}_0 \left( 1 - \exp \{ - \langle X_R - X_{R_c} \rangle \} \right) \left( \frac{L_0}{L} \right) \exp \left( - \frac{Q_{ac}}{RT} \right) \quad (22)$$

in which  $\dot{L}_0$  is also a material constant. Also, one may obtain an expression for  $X_{R_c}$  from  $X_R$  by considering that

$L = L^*$  when  $e_{ef}^p = e_{efc}^p + e_{efR}^p$ , giving  $X_{R_c} = 1 - \exp \left\{ -fx \left( \frac{L_0}{L^*} \right) \right\}$ . Replacing these expressions in the previous relations yields

$$\dot{L} = -f_R X_R \langle L - L^* \rangle \dot{e}_{ef}^p + \dot{L}_0 \left( 1 - \exp \{ - \langle X_R - X_{R_c} \rangle \} \right) \left( \frac{L_0}{L} \right) \exp \left( - \frac{Q_{ac}}{RT} \right). \quad (23)$$

The effective accumulated viscoplastic strain from which the recrystallisation process begins, given by  $e_{efc}^p$  and depicted in Fig. 2, may be estimated as

$$e_{efc}^p = \frac{2}{\sqrt{3}} \frac{C_c}{\mu} (f_c S^* - S_0) \quad (24)$$

Where  $S_0 = S(0)$  is the initial deformation resistance,  $f_c$  is a prescribed fraction,  $C_c$  is a material constant and  $\mu = G$  is the shear modulus. The effective viscoplastic strain necessary to complete the material softening process making it partially or totally recrystallised is given by

$$e_{efR}^p = \frac{2}{\sqrt{3}} \frac{C_R}{\mu} S_0^* \left\langle \left( \frac{L}{L_0} \right)^{w,n_2} - \left( \frac{L^*}{L_0} \right)^{w,n_2} \right\rangle \quad (25)$$

## 2.5. Weak formulation of the coupled thermomechanical problem

The problem consists in determining  $(\bar{u}(X,t), T(X,t)) \in \mathbf{K}$ , for each  $t \in [0, t_f]$ , so that

$$\int_{\Omega_o} \mathbf{P} \cdot \nabla_{\bar{x}} \delta \bar{u} d\Omega_o = \int_{\Omega_o} \rho_o \bar{b}_o \cdot \delta \bar{u} d\Omega_o + \int_{\Gamma_o} \bar{t}_o \cdot \delta \bar{u} d\Gamma_o \quad \forall \delta \bar{u} \in \mathbf{V}_u \quad (26)$$

and

$$\begin{aligned} \int_{\Omega_o} \rho_o c_s \hat{T} d\Omega_o + \int_{\Omega_o} k \nabla_{\bar{x}} T \cdot \nabla \hat{T} d\Omega_o - \int_{\Omega_o} \varpi \bar{\tau} \cdot \bar{\mathbf{D}}^p \hat{T} d\Omega_o = - \int_{\Gamma_{qq}} \bar{q}_{on} \hat{T} d\Gamma \\ + \int_{\Gamma_{qh}} h(T - T_f) \hat{T} d\Gamma \quad \forall \hat{T} \in \mathbf{V}_T \end{aligned} \quad (27)$$

where  $\bar{q}_o = \mathbf{JF}^{-1} \bar{q}$ , with  $\bar{q}$  denoting the heat flux per unit of area of the deformed configuration.

In order to solve the above problem, one applies an incremental procedure which leads to an incremental weak form. A consistent tangent operator is then derived and the weak incremental form is discretized and solved by Newton's method.

## 3. EXAMPLES AND CONCLUSION

### 3.1. Uniaxial compression

Here, one considers a uniaxial compression described in Fig. 4, consist. The dimensions are  $h=1.0m$  and diameter  $d=0.2m$ . The problem is considered axisymmetric and is subjected to a linearly varying displacement with a total upset of  $\bar{u} = -0.4m$  applied in the interval  $t \in [0, 1]$  in seconds.

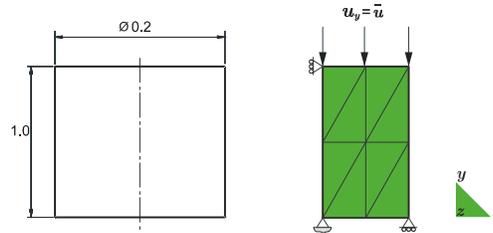


Figure 4. Uniaxial test

The material parameters used in the analysis are given by:  $L_0 = 91.0d - 06m$ ,  $\xi = 0.308$ ,  $w = 2.9$ ,  $f_c = 0.728$ ,  $R = 8.314J/mol K$ ,  $a_s = 1.55$ ,  $n_2 = 0.069$ ,  $fx = 1.0$ ,  $\bar{p} = 0.8$ ,  $\dot{L}_0 = 6.0 * 10^6 ms^{-1}$ ,  $C_c = 59$ ,  $\dot{e}_{ef0}^p = 2.618d = 11s^{-1}$ ,  $n_1 = 0.11$ ,  $Q_{ac} = 283kJmol^{-1}$ ,  $h_s = 7800.0MPa$ ,  $\bar{\chi} = 431.0MPa (N/mm^2)$ ,  $f_R = 120.0$ ,  $S_0 = 150.0MPa (N/mm^2)$ ,  $\bar{q} = 5Nmm\bar{p}^{-2} = 19905.358527Nm\bar{p}^{-2}$ ,  $f_c = 0.728$ ,  $C_R = 59$ .

Also,  $\mu = G = 57.69GPa$  is the shear modulus,  $K = 125.0GPa$  is the volumetric modulus,  $\alpha = 12.0d - 06(^{\circ}C)^{-1}$  is the thermal expansion coefficient,  $k = 43.0W/m^{\circ}C$  heat conductivity,  $\varpi = 0.9$ ,  $\rho c_p = 3.689d + 06N/m^2(^{\circ}C)$  is the density times the heat capacity,  $h = 4.5W/m^2(^{\circ}C)$  is the convection film coefficient.

The evolution of the mean grain size is depicted in Fig. 5.

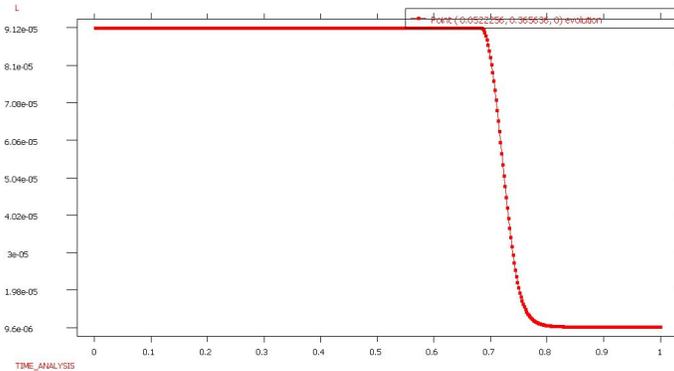


Figure 5. Evolution of the mean grain size ( $L$ ) with load parameter  $t$

The evolution of the deformation resistance is depicted in Figure 6.

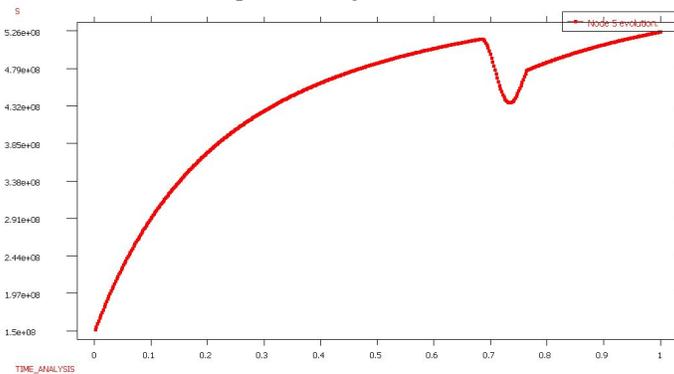


Figure 6. Evolution of the deformation resistance ( $S$ ) with load parameter  $t$

The evolution of the effective von Mises stress is depicted in Fig. 7

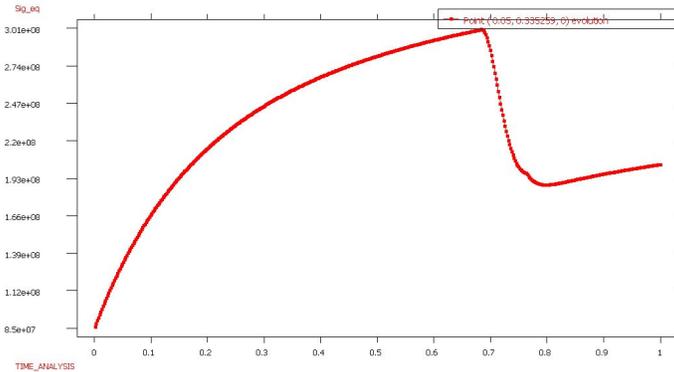


Figure 7. Evolution of the effective von Mises ( $q$ ) with load parameter  $t$

### 3.2. Compression of a Cone

Here, one considers a compression of a conical bar described in Fig. 8. The dimensions are  $h=0.1$ m and diameters  $d_s=0.09$ m and  $d_l=0.18$ m. The problem is considered axisymmetric and is subjected to a linearly varying displacement with a total upset of  $\bar{u} = -0.04$  m applied in the interval  $t \in [0, 1]$  in seconds. Here, one makes use of the same material constants given in example 3.1.

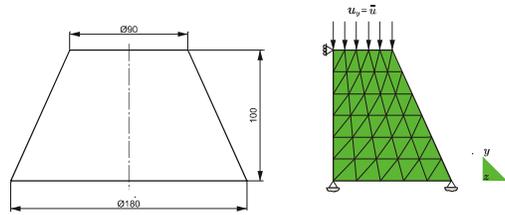


Figure 8. Compression of a conical bar test

The distribution of the deformation resistance ( $S$ ) is depicted in Fig. 9.

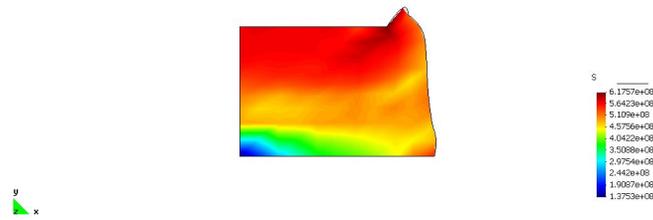


Figure 9. Distribution of the deformation resistance at the end of the loading

The distribution of the mean grain size ( $L$ ) is depicted in Fig. 10.

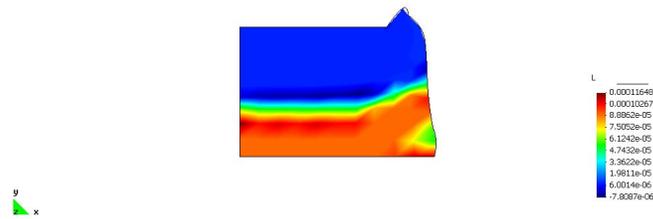


Figure 10. Distribution of the mean grain size ( $L$ ) at the end of the loading

The distribution of the von Mises stress is depicted in Fig. 11.

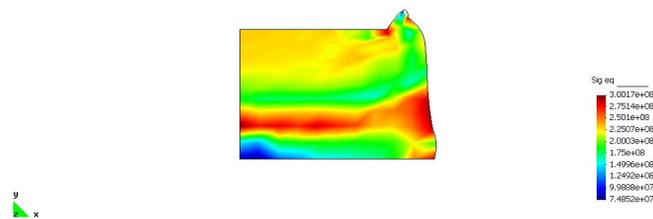


Figure 11. Distribution of the von Mises stress ( $q$ ) at the end of the loading

#### 4. CONCLUSION

The above procedure has shown to be able to describe the softening of the material once a recrystallisation process starts. Also, the modeling of the strain hardening is adequate and describe by the evolution of the deformation resistance. However, the model has failed for the application of very high strain rate, as seen in Fig. 10, where one sees negative values for the mean grain size. This problem is not present for moderately fast strain rates. To overcome this problem, of non physical solutions, one must impose some internal locking constraint assuring that the mean grain size must be larger or equal to zero. Also, a more smooth solution for the mean grain size could be obtained by considering a nonlocal theory in terms of the mean grain size,  $L$ . Another difficulty in the application of the local state method for modeling the evolution of the microstructure of the material is the large number of parameters that must be identified.

However, the approach has shown to be promising in order to be able to describe the evolution of the microstructure of metals, which is fundamental for the control of the mechanical properties of metals undergoing hot deformation processes.

## 5. ACKNOWLEDGEMENTS

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## 5. RESPONSIBILITY NOTICE

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