SUBSTRUCTURING AND MODEL REDUCTION OF THE ACOUSTIC FIELD INSIDE DUCTS

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Abstract. *This paper presents a model reduction and substructure technique for reduced dynamical models of acoustic duct systems. The behavior of the acoustic waves inside ducts is easier to understand using numerical simulations, because it is possible to observe the propagation of the wave front along the system. From the numeric point of view, acoustics simulations in ducts are accomplished accurately using the Finite Elements Method (FEM). The disadvantage in using FEM technique to simulate HVAC (air-handling and conditioning system) and exhausts systems is the computational cost demanded due to the great dimensions of the ducts since they are constituted by joints, unions, curves and obstacles that have to be modeled, thus leading to a limited frequency analysis. An alternative to obtain the reduced model of this kind of systems is the use of Component Mode Synthesis (CMS) technique. Due to the presented aspects, in this work, the use of CMS is proposed in order to obtain the bi-dimensional acoustic modal model of fluid-filled ducts of large dimensions. With the resulting modal model, the acoustic field inside the duct due to a noise source, being previously identified, is obtained. A substructure coupling method employing free-interface theoretical normal modes supplemented by Residual Flexibility Attachment Modes was used. The frequency dependent sound fields inside the ducts were obtained using the proposed technique and good agreements were obtained when they were compared to the results from the full analytical and finite element model.*

Keywords*: acoustic ducts, reduced model, substructuring, finite elements,* component *mode synthesis.*

1. INTRODUCTION

Noise emissions from HVAC or duct systems are one of the main topics related to acoustic comfort problems inside buildings, as many national and local regulations outline. The prediction of the noise emission of an HVAC system depends mainly on the acoustic performance of different HVAC components, given by manufacturers or estimated with empirical relationships (Semprini et al, 2003).

Other manner to predict the acoustic behavior of a HVAC is by analytical or numerical methods. This later can be the finite element (FE) method or the boundary element (BE) method which both are commonly used in acoustic analysis (Maess and Gaul, 2006).

Numerical techniques, involving for example a FE model or a BE model, do not have the restriction of simple system geometries, but for computational and accuracy reasons are applicable primarily to low frequency predictions. Similarly such an approach is inefficient for simulations involving a large number of geometric variations (Magalhães and Ferguson, 2005).

Duct systems are often built up by recurrent standard components, as for example duct segments, joints and flanges. These components are furthermore assembled in different combinations to complex duct systems and large dimensions. A proper analysis of the acoustical dynamic behavior of sufficiently discretised duct systems in three dimensions is, however, challenged by the large number of degrees of freedom. Thus this modeling of complex structures leads to finite elements models of very large sizes.

It is thus important to reduce the size of the system for mainly several reasons: the cost of computation when one wants to extract the eigensolutions or to predict the behavior of the full structure, the optimization or updating procedures which require fast iterative techniques and finally nonlinear mechanics.

The substructuring or also known as Component Mode Synthesis (CMS) method currently plays a considerable role in the analysis of the complex structures. This method consists in subdividing a structure in components called substructure or superelement, which are analyzed and condensed separately, while preserving the junctions DOFs between substructures. The substructures, or components, are represented by their modes. The latter include the normal modes, the rigid body modes, the static modes, the interface modes, etc. The advantages of such a method are to be able to analyze problems of quasi-unlimited size with the current data-processing resources (Masson *et al*, 2006).

The geometric modification of a substructure makes it possible to reanalyze the full structure with a low cost of computation and it makes it possible to prepare each model of substructures independently of the others. This advantage is significant for the study of the complex substructures whose designs are entrusted to various equipment suppliers or in the context of parallel processing.

The aim in this paper is to develop a reduced model of the acoustic field inside a duct provided by a noise source. The Component Mode Synthesis (CMS) method was used for the development of this alternative model. Subsequently, the CMS model implemented was then compared with an analytical and numerical model, such as finite element model.

The CMS method requires the user to model separate components of a problem in terms of a summation over constraint modes and component normal modes. The main contributors and particular techniques of the CMS method are mainly found in the field of structural dynamics, where it was originated as a substructuring technique dynamics (Magalhães and Ferguson, 2005).

Component mode synthesis involves three basic steps: division of a structure into components, definition of sets of component modes, and coupling of the component mode models to form a reduced-order system model (Craig Jr., 2000).

In this work the implementation of the CMS method for acoustic problem is demonstrated and the obtained results shows the potentially of the technique beyond the structural dynamic area. The implemented method is applied to a bi-dimensional model.

2. METHODOLOGY

The acoustic field inside a duct can be estimated using the modal parameters of the system (eigenvalues and eigenvectors values). The relation between input and output sound pressure in terms of the modal parameters can be written by Eq. (1) (Maia and Silva, 1997):

$$\alpha_{jk}(\omega) = \frac{p_j}{p_k} = \sum_{r=1}^{N} \frac{\Psi_{jr}\Psi_{kr}}{\omega_r^2 - \omega^2} \tag{1}$$

where p_j is the output sound pressure in the node j; p_k is the input sound pressure (harmonic excitation) in the node k; Ψ_{jr} is the eigenvector value in the node j relative to the mode r; Ψ_{kr} is the eigenvector in the node k (correspond to the excitation node) relative to the mode r; ω_r is the natural frequency relative to the mode r in rad/s; ω is the angular frequency under analysis in rad/s.

Although, the output sound pressure level (SPL) in the node j due to an input harmonic sound pressure in the node k is:

$$SPL_j(\omega) = 20\log_{10}\left(\frac{\sum_{r=1}^{N} \frac{\Psi_{jr}\Psi_{kr}\,p_k}{\omega_r^2 - \omega^2}}{p_o}\right) \tag{2}$$

where p_o is the reference sound pressure (0,00002 Pa).

For use the Eq. (1) and (2) the model is considered linear and without damping. Then, theses equations can be used for estimate the sound pressure due to many harmonics input sound pressure.

The eigenvalues and eigenvectors of the model cited in Eq. (1) and (2) must be estimated using the CMS technique, which is the purpose of the work and will be describe yet in this topic.

The complete acoustic model considered in this work has large dimensions and geometric obstacles inside it, then, no trivial solution is described in the available literature. This fact let us to use the CMS technique, which uses simple substructures to get a complex model. The most general type of component, or substructure, is one that is connected to one or more adjacent components by redundant interfaces.

The substructure coupling method used in the present paper is constituted of free-interface substructure normal modes supplemented by "reduced flexibility" (Residual Flexibility Attachment Modes). This method is an approximation of the method developed by Craig and Chang (1977), and the basic ideas for representing the substructures are contained in the works of MacNeal (1971) and Rubin (1975).

This CMS method is large used in the experimental field, once the eigenvalues and eigenvectors of each substructure are determined using experimental modal analysis.

In this work, the normal modes of each substructure are obtained from analytical modal models. So, the complex structure to be modeling using the CMS is divided in simple substructures that have analytical equations for obtain the eigenvalues and eigenvectors theoretical.

Component normal modes are eigenvectors, and may be classified according to the boundary conditions specified for the component – fixed-interface normal modes, free-interface normal modes, hybrid-interface normal modes, or loaded-interface normal modes (Craig Jr, 2000).

Using the Eq. (3) and Eq. (4) the normal natural frequencies and normal modal shapes, respectively, of a rectangular duct with closed ends can be estimated considering only the longitudinal axes (bidimensional model):

$$f_{ij} = \frac{c_o}{2}\left(\frac{i^2}{L_x^2} + \frac{j^2}{L_y^2}\right)^{1/2} \quad i = 0,1,2,...;\ j = 0,1,2,... \tag{3}$$

$$\theta_{ij} = cos\left(\frac{i\pi x}{L_x}\right)cos\left(\frac{j\pi y}{L_y}\right) \quad i = 0,1,2,...;\ j = 0,1,2,... \tag{4}$$

where f_{ij} is the natural frequency (Hz) of the duct respect to index *ij*, c_o is the sound speed (m/s), L_x is the length of the duct, L_y is the height of the duct, θ_{ij} is the modal shape due to mode *ij*, *x* is the coordinate in the x direction (along the length) and *y* is the coordinate in the y direction (along the height).

Since model reduction is one of the major objectives in CMS, the normal mode set is usually reduced to a smaller set of *kept normal modes* (θ_k). The upper frequency of interest in the complete model analysis defines the number of normal modes that it will be used in the CMS technique. So, it is suggest using the normal natural frequencies below 2.5 times the upper frequency of interest to have accuracy results after to do the synthesis of the substructures. The normal modes above this limit frequency (called *deleted normal modes,* θ_d) are used to estimate the residual flexibility attachment modes. It represents the flexibility of the higher-frequency modes.

Residual-flexibility attachment modes (δ_f) may be defined for the inertial forces applied at the interface coordinates of the substructures (boundary nodes of the substructure 1= boundary nodes of the substructure 2), by the following equation:

$$\delta_f = \theta_d \cdot \Lambda_{dd}^{-1} \cdot \theta_{db}^{'} \tag{5}$$

$$\Lambda_{dd} = \left(f_d \cdot 2 \cdot \pi\right)^2 \tag{6}$$

where the subscript *d* refers to deleted modes and *b* refers to boundary nodes of the substructure, the superscript ´ refers to transpose matrix and *-1* refers to inverse matrix, Λ_{dd} are the eigenvalues not considered in the normal modes set and f_d are the deleted natural frequencies obtained from Eq. (3).

The order of magnitude of the residual flexibility is smaller than that of the flexibility of the kept modes. Incorporation of residual-flexibility attachment modes into the component mode set ensures complete representation of static deflection of the component due to the inertial forces applied at interface DOFs.

The next step is to use a Component Mode Synthesis Methods to do the union of the substructures. In this work was used the attachment-mode method approach.

If a reduced set of component normal modes is used without including a complete set of interface attachment modes, the component mode set is not statically complete. However, methods that employ free-interface normal modes together with attachment modes (including residual flexibility attachment modes and/or inertia-relief attachment modes) are widely used especially in context of experimental verification of finite element models (Craig Jr, 2000).

For the considered synthesis method and considering two substructures, α and β, the motion equation can be written in terms of generalized coordinates:

$$\overline{M}\ddot{q} + \overline{K}q = 0 \tag{7}$$

where *q* is the kept modal coordinates and can be defined as:

$$u = S \cdot q \tag{8}$$

and *u* is the modal coordinates, *S* is a compatibility matrix (it will be defined latter) and $q = \begin{Bmatrix} u_k^{\alpha} \\ u_k^{\beta} \end{Bmatrix}$ where the subscript *k* refers to kept modes.

The coupled-system mass and stiffness matrices have the forms:

$$\overline{M} = \begin{bmatrix} M_u^{\alpha\alpha} & M_u^{\alpha\beta} \\ M_u^{\beta\alpha} & M_u^{\beta\beta} \end{bmatrix}, \quad \overline{K} = \begin{bmatrix} K_u^{\alpha\alpha} & K_u^{\alpha\beta} \\ K_u^{\beta\alpha} & K_u^{\beta\beta} \end{bmatrix} \tag{9}$$

The matrices in the Eq. (9) can be re-written in the form:

$$\overline{M} = \begin{bmatrix} I_{kk}^{\alpha} + \theta_{kb}^{\alpha'} V \theta_{kb}^{\alpha} & -\theta_{kb}^{\alpha'} V \theta_{kb}^{\beta} \\ -\theta_{kb}^{\beta'} V \theta_{kb}^{\alpha} & I_{kk}^{\beta} + \theta_{kb}^{\beta'} V \theta_{kb}^{\beta} \end{bmatrix}, \quad \overline{K} = \begin{bmatrix} \Lambda_{kk}^{\alpha} + \theta_{kb}^{\alpha'} \hat{A} \theta_{kb}^{\alpha} & -\theta_{kb}^{\alpha'} \hat{A} \theta_{kb}^{b} \\ -\theta_{kb}^{\beta'} \hat{A} \theta_{kb}^{\alpha} & \Lambda_{kk}^{\beta} + \theta_{kb}^{\beta'} \hat{A} \theta_{kb}^{\beta} \end{bmatrix} \tag{10}$$

where,

$$V = \hat{A}'(\beta_{ff}^{\alpha} + \beta_{ff}^{\beta})\hat{A} \tag{11}$$

$$\hat{A} = \left(\delta_{fb}^{\alpha} + \delta_{fb}^{\beta}\right)^{-1} \tag{12}$$

$$\beta_{ff} = \theta_{db}\left(\Lambda_{dd}^{-1}\right)^{2}\theta_{db}' \tag{13}$$

Solving the reduced problem (Eq. 7), the eigenvalues and eigenvectors (Π) of the connected system (substructures α and β) are obtained in the modal coordinates q. To return to the original modal base (Ψ), it is used the following linear transformation:

$$\Psi = \zeta \cdot S \cdot \Pi \tag{14}$$

where:

$$\zeta = \begin{bmatrix} \delta_{fb}^{\alpha} & 0 & \theta_{kb}^{\alpha} & 0 \\ 0 & \delta_{fb}^{\beta} & 0 & \theta_{kb}^{\beta} \\ \delta_{fi}^{\alpha} & 0 & \theta_{ki}^{\alpha} & 0 \\ 0 & \delta_{fi}^{\beta} & 0 & \theta_{ki}^{\beta} \end{bmatrix} \tag{15}$$

$$S = \begin{bmatrix} -\hat{A}\theta_{kb}^{\alpha} & \hat{A}\theta_{kb}^{\beta} \\ \hat{A}\theta_{kb}^{\alpha} & -\hat{A}\theta_{kb}^{\beta} \\ I_{kk}^{\alpha} & 0 \\ 0 & I_{kk}^{\beta} \end{bmatrix} \tag{16}$$

where the subscript I refers to internal nodes of the substructure.

Complementing the methodology used, there are two conditions that must be obeyed:

- The number of boundary coordinates of each substructure must be equal.
- The number of flexibility attachment modes must be equal to the number of boundary coordinates of the connected substructures.

The results obtained with the methodology proposed are showed in the next item.

3. NUMERICAL RESULTS

The methodology described was used to obtain the acoustic field inside a duct. For this work, two models of acoustic ducts were used. Both models don't consider the fluid-structure interaction between the internal fluid and the ducts walls (rigid walls).

The first model has simple geometry (Fig. 1) and its analytical solution is available in the literature, so it was modeled using CMS technique only to validate the methodology.

The second model (Fig. 2) has a complex geometry due to a plate inserted in the middle of the duct in the longitudinal axes. The results obtained with the CMS technique for this model were validated using the finite element method (FEM). Both models were excited by a point noise source.

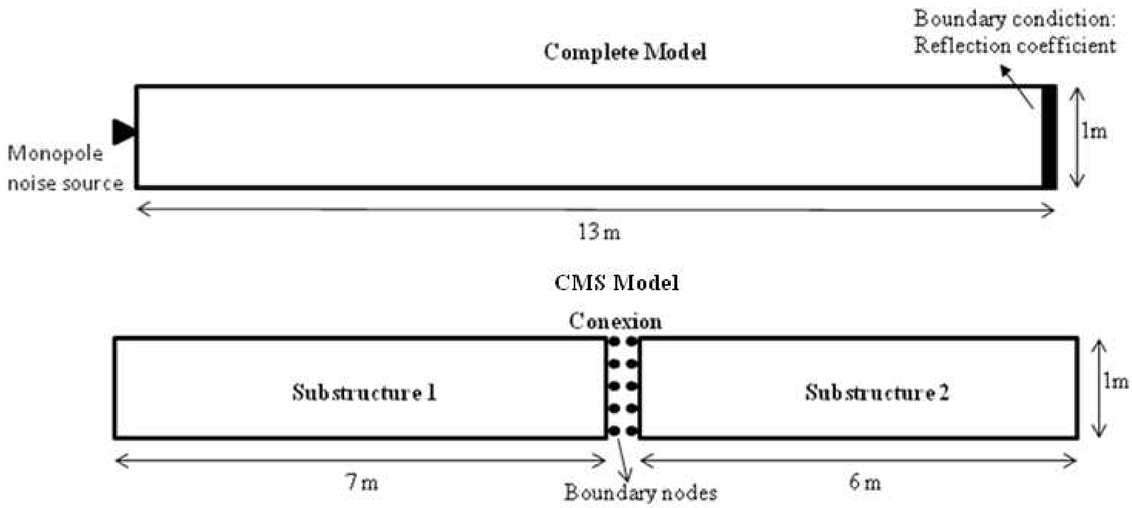

Figure 1 – Duct acoustic of simple geometry

Each model (CMS model of the figures 1 and 2) was discretised in internal nodes and boundary nodes. We can see in the figures only the boundary nodes, although the substructures 1 and 2 have internal nodes spaced of 0,02m in longitudinal direction and 0,25m in the vertical direction. In these nodes (internal and boundary) the Eq. (3) and (4) were computed.

The frequency of analysis of interest is 500 Hz, so the kept normal modes have the upper limit frequency of 1250 Hz. Above 1250 Hz the normal modes (called "deleted") are used to estimate the flexibility attachment modes.

In the acoustic models were assumed that:

- The fluid is inviscid and the averaged pressure is uniform.
- The pressure fluctuations in the acoustic field are lower than the mean pressure of the fluid.
- Reflection coefficient simulating an open duct in the end termination.

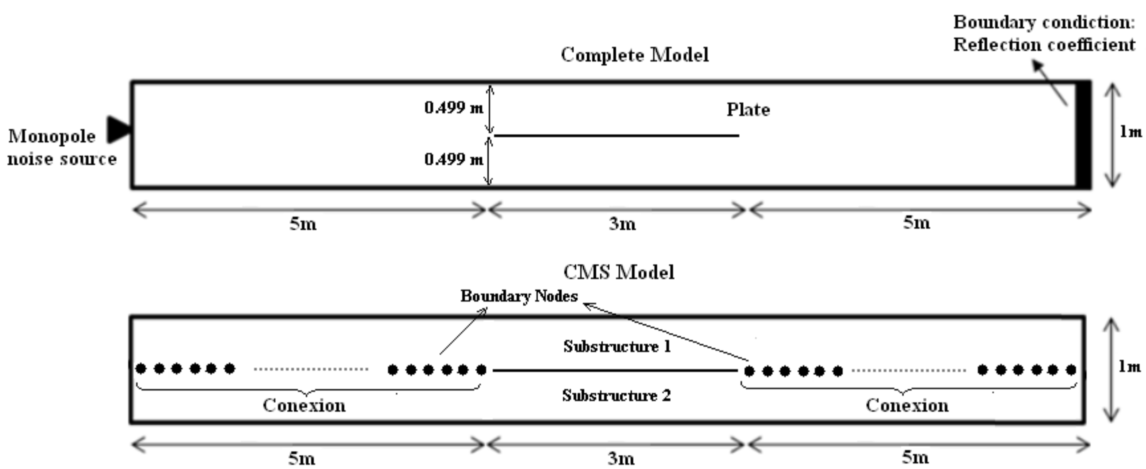

Figure 2 –Duct with a thinner plate inside it.

The modal models obtained with the CMS technique had good agreements with the analytical and finite element (FE) models. For the first model it was obtained MAC index near the unity for the frequency range of interest, and for the second model, the perceptual error between the CMS and FE model didn't have value major than 3% for the first twenty modes (Nunes and Duarte, 2009).

In the figures (3) and (4) it is shown the acoustic pressure inside of the simple duct (Fig. (1)) estimated from the CMS results for the frequencies: 400 and 500 Hz, respectively. It was simulated a harmonic noise source and the sound pressure level was computed using the Eq.(2). The software Matlab® was used to estimate and plot the results from modal model (CMS technique) and the finite element method (Ansys®) was used to validate and to compare the results.

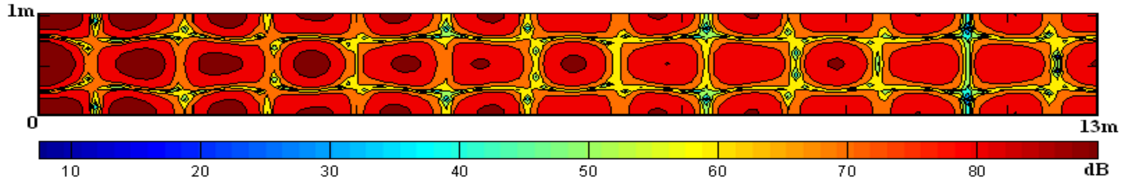

Figure 3a – Sound pressure level inside the simple duct at 400Hz (CMS technique).

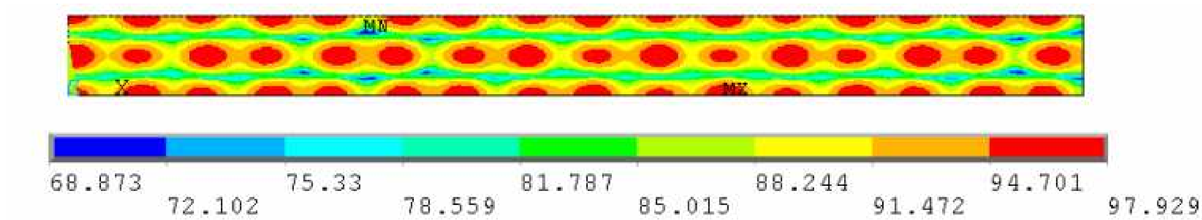

Figure 3b – Sound pressure level inside the simple duct at 400Hz (FEM method).

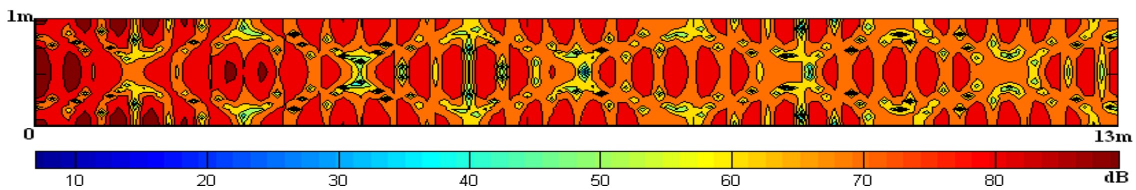

Figure 4a – Sound pressure level inside the simple duct at 500Hz (CMS technique).

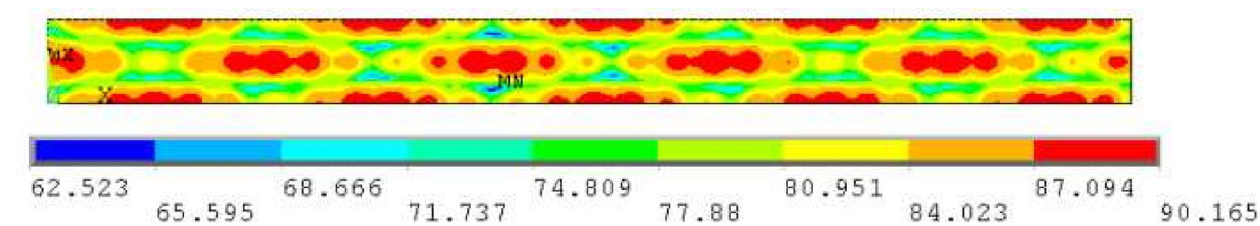

Figure 4b – Sound pressure level inside the simple duct at 500Hz (FEM method).

We can see in Fig. (3) and Fig. (4) that the acoustic field for both models, CMS and FEM, produced similar results. The vantage of the CMS model in relation of the FE model is the reduced model that decreases the processing time for an optimization process, for example.

The sound pressure level in the duct with a thinner plate inside it is showed in the Fig. (5) and Fig. (6) for the tonal frequencies: 300 and 400 Hz, respectively.

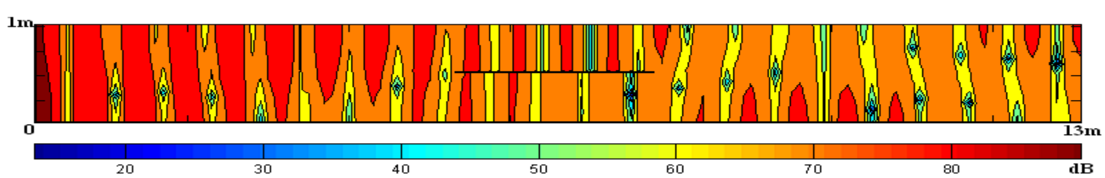

Figure 5a – Sound pressure level inside the duct with a plate at 300 Hz (CMS technique).

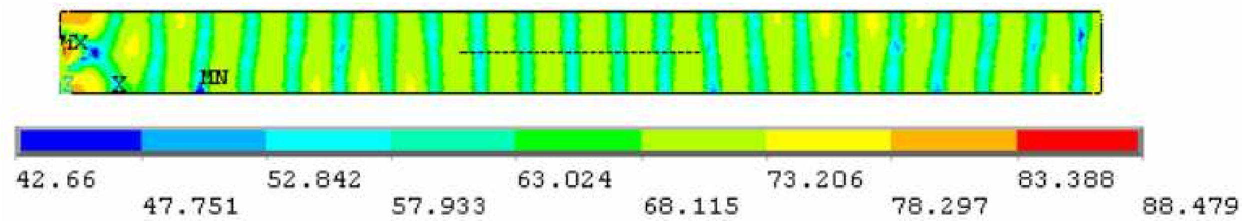

Figure 5b – Sound pressure level inside the duct with a plate at 300 Hz (FEM method).

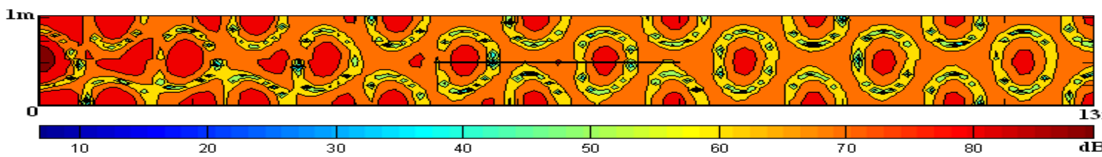

Figure 6a – Sound pressure level inside the duct with a plate at 400 Hz (CMS technique).

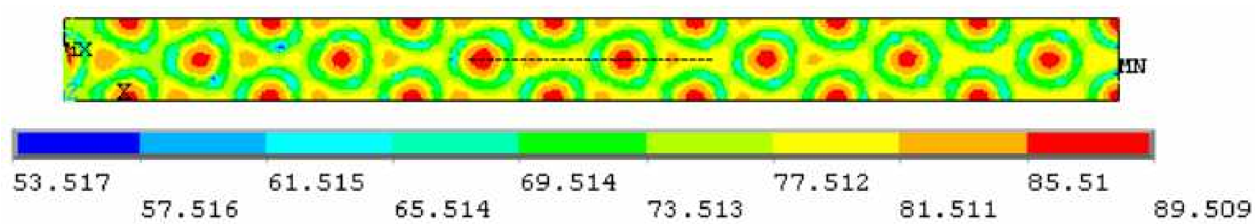

Figure 6b – Sound pressure level inside the duct with a plate at 400 Hz (FEM method).

Although the color of the scale used is different, the results are agreement between both techniques. In the Fig. (5a) and (5b) we can see that along the cavities formed for the plate and duct the sound wave is plan, once the cut-off frequency (343 Hz) in these cavities have a value major than the excitation frequency (300 Hz). Outside the cavities, or along the original duct, the cut-off frequency is 171.5 Hz, so the sound waves have high-order modes.

We can see in Fig. (6a) and (6b) that the sound field is complex along the entire duct. It's notable that for this excitation frequency the partition inside the duct doesn't have effect under the sound field. This fact is due to the cut-off frequency inside the partition being lower than 400 Hz (excitation frequency).

It was verified that the computational time processing of the SMC model is about three times less than the FEM model.

The number of d.o.f for the SMC and FEM model without partition inside the duct are the same for both, but for the model with partition, the FEM model has much more d.o.f. when compared with the SMC model because the geometry of the first became complex inside the duct and the mesh need to be refined near the partition. This refinement is not necessary for the SMC model, so the number of d.o.f. for this model is decreased.

The component modal synthesis technique showed to be a good tool to obtain the sound field inside a duct with geometric restriction, what difficult the numeric model increasing the complexity of the mesh (for example in FEM) and consequently in the solver. So, this acoustic model obtained from SMC technique may be used in optimization routine, like heuristic methods, where the computational processing time is an important parameter and may be reduced using the SMC model if compared with the FEM model.

4. CONCLUSION

The objective of this paper was to develop a reduced model of the acoustic field inside a duct provided by a noise source using the Component Mode Synthesis (CMS) method which was implemented for an acoustic problem. Then, the technique applied to acoustic was demonstrated and the obtained results shows the potentially of the technique beyond the structural dynamic area. The implemented method was applied to a bi-dimensional model.

In previous work was shown the good agreements between the modal models obtained with the CMS technique and the analytical and finite element (FE) models. For a simple duct without obstacles inside it the MAC index was near the unity for the frequency range of interest, and for the duct model with a plate inside it, the perceptual error between the CMS and FE model didn't have value major than 3% for the twenty firstly modes (Nunes and Duarte, 2009).

In this work, we could see that the acoustic field obtained from the modal models, using CMS technique, and the analytical and FEM model produced similar results. The vantage of the CMS model in relation of the FE model is the reduced model that decreases the processing time for an optimization process, for example.

With the reduced acoustic model obtained from CMS technique is possible to see that the complex acoustic field along the duct, although between the cavities formed for the plates inside it where the sound waves are plan.

The component modal synthesis technique showed to be a good tool to obtain the sound field inside a duct with geometric restriction, what difficult the numeric model increasing the complexity of the mesh (for example in FEM) and the solver.

5. ACKNOWLEDGEMENTS

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6. REFERENCES

Craig Jr., R. R., Chang, C. J., On the attachment modes in substructure coupling for dynamic analysis, AIAA J., paper 77-405, March 1977.

Craig Jr., R. R., Coupling of substructures for dynamic analyses - An overview, AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference and Exhibit, 41st, Atlanta, GA, Apr. 3-6, 2000.

Craig Jr., R. R., Brief Tutorial on Substructure Analysis and Testing, The University of Texas at Austin, 2000.

MacNeal, R. H., “A Hybrid Method of Component Mode Synthesis,” J. Computers & Structures, Vol. 1, No. 4, pp. 581-601, Dec. 1971.

Maess, M., Gaul, L., Substructuring and model reduction of pipe components interacting with acoustic fluids, Mechanical Systems and Signal Processing 20 (2006) 45–64.

Magalhães, M. D. C., Ferguson, N. S., The development of a Component Mode Synthesis (CMS) model for three-dimensional fluid–structure interaction, J. Acoust. Soc. Am. 118 (6), December 2005.

Maia, N. M. M., Silva, J. M. M., Theoretical and Experimental Modal Analysis, 1 Ed., John Wiley & Sons Inc, 468 p., 1997.

Masson, G., Ait Brik, B., Cogan, S., Bouhaddi, N., Component mode synthesis (CMS) based on an enriched ritz approach for efficient structural optimization, Journal of Sound and Vibration 296 (2006) 845–860.

Nunes, M. A. A., Duarte, M. A.V., Teodoro, E. B., Component Mode Synthesis Methodology in Ducts Acoustic Modal Analysis, Proceedings of the XIII International Symposium on Dynamic Problems of Mechanics (DINAME 2009), Angra dos Reis, RJ , Brazil, March, 2009.

Rubin, S., “Improved Component-Mode Representation for Structural Dynamic Analysis,” AIAA Journal, Vol. 13, No. 8, pp. 995-1006, Aug. 1975.

Semprini, G., Guidorzi, P. and Garai, M., Experimental Evaluation of Noise Propagation through Rectangular Ducts in HVAC Systems, EuroNoise 2003, Naples, Paper ID: 242.

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