IDENTIFICATION OF THE AERODYNAMIC AND CONTROL DERIVATIVES AND FLIGHT PATH RECONSTRUCTION OF THE SZD-50-3 PUCHAZ SAILPLANE

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Abstract. This work focuses the system identification process which is a general procedure to match the observed input-output response of a dynamic system by a proper choice of an input-output model and its physical parameters. From this point of view, the aircraft system identification comprises proper choice of aerodynamic models and the development of parameter estimation techniques by minimization of the mismatch error between the predicted and the real aircraft response. Another focus of this work is the problem of Flight Path Reconstruction (FPR) which arises naturally when the main goal is an accurate identification of the aircraft parameters, because, in this case, the proper characterization of the flight sensors constitutes a fundamental preliminary step. This method is applied to obtain the aerodynamic and control derivatives of the sailplane SZD-50-3 Puchaz from flight test data with measurement noise and bias. Experimental results are reported, with flight test data acquired by probes, inertial sensors and GPS receivers.

Keywords: parametric identification, sensor calibration, flight path reconstruction

1. Introduction

One of the most important phases of the design and evaluation process in the aeronautical design nowadays concerns modeling and simulation. In order to accomplish this task, the system identification and parameter estimation constitutes a fundamental step. System Identification is a general procedure to match the observed input-output response of a dynamic system by a proper choice of an input-output model and its physical parameters. From this point of view, the aircraft system identification or inverse modelling comprises proper choice of aerodynamic models, the development of parameter estimation techniques by optimization of the mismatch error between predicted and real aircraft response and the development of proper tools for integration of the equations of motion within the system simulation and correlated activities [3].

A fundamental preliminary step for an accurate identification of the aircraft parameters is the proper characterization of the sensors. For example, if the bias of a certain sensor is not adequately estimated, the accuracy of the ensuing parameter identification may be degraded. The Flight Path Reconstruction (FPR) technique is specially useful in the validation of the acquisition instruments applied in an aircraft. The interpretation of its results can furnish important information with respect to sources of problems. Additionally, it decreases the uncertainties about the quality of data, which is one of the main causes of poor flight test results.

One of the first approaches for FPR may be found in [5]. The authors employ the kinematic model of an aircraft, with
6 degrees of freedom, and then consider an augmented state vector, incorporating the parameters to be identified. This procedure leads to a general problem of state estimation with nonlinear dynamics, solved by the extended Kalman filter approach. A more detailed investigation of the problem is conducted by [9]. Experimental results are reported for the approach based on extended Kalman filter, but the GPS readings are not used.

In this work, the FPR problem and the system identification are investigated by parametric estimation of a nonlinear model, based on the Output-error method and Gauss-Newton optimization algorithm using the software ESTIMA. The results are reported for a sailplane, the SZD-50-3 Puchaz. The Output-error with Gauss-Newton algorithm is used here to determine the stationary aerodynamic derivatives of the aircraft, using a nonlinear longitudinal model. The effectiveness of the implemented parameter estimation method was tested by matching real flight test data with the predicted response of the aircraft.

This work is structured as follows: in the first part, the parametric estimation method is described with special attention to the Gauss-Newton algorithm. In section 3, the kinematic model for FPR and the results obtained are used to model the sensor errors. The experimental results obtained in the parameter estimation are analyzed in section 4.

2. Parametric Estimation by the Output-Error Method

In this section, the parametric identification, in particular the parameter estimation applied to a nonlinear model of the longitudinal motion of an aircraft in space state formulation is presented. The output-error method is one of the most used estimation methods in aircraft identification and aerodynamic parameter estimation [6], [7], [8]. It has several desirable statistical properties, including its applicability to nonlinear dynamical systems and the proper accounting of measurements noise [8].

The structure of the model is considered to be known, and the identification process consists in determining the parameter vector \( \Theta \), which gives the best prediction of the output signal \( y(t) \), using some sort of optimization criteria. The attainment of an estimate through optimization of a cost function based on the prediction error of the plant requires, usually, the minimization of a nonlinear function. Thus, the Gauss-Newton method is used here to estimate the parameters in the model. Therefore, the cost function to be minimized involves the prediction error,

\[
e(k) = \hat{y}(k) - y(k)
\]

where \( \hat{y}(k) \) is the output prediction based on the actual estimate \( \hat{\Theta} \) of the parameter vector \( \Theta \).

2.1 Maximum Likelihood Estimation Criteria

Consider a dynamic system, identifiable, with model structure \( M(\Theta) \) defined and output \( y \). Suppose that \( p(y|\Theta) \) is the conditional probability gaussian distribution of the random variable \( y \) with dimension \( m \), mean \( f(\Theta) \) and covariance \( R \), with dimension \( m \times m \). \( p(y|\Theta) \) is known as the likelihood functional, and in [2] the authors attribute its name due to the fact that it is a measure of the probability of occurrence of the observation \( y \) for a given parameter \( \Theta \). The Maximum Likelihood Estimate is defined as the value of \( \Theta \) which maximizes this functional, in such a way that the best estimate of \( \Theta \), according to the MLE criteria is

\[
\hat{\Theta} = \text{Arg Max } p(y|\Theta)
\]

Thus, the likelihood functional is

\[
p(y|\Theta) = \frac{1}{(2\pi)^{m/2}|R|^{n/2}} \cdot \exp \left\{ -\frac{1}{2} \sum_{k=1}^{n} [e(k, \Theta)]^T [R]^{-1} [e(k, \Theta)] \right\}
\]

(3)

whose maximization is equivalent to the minimization of

\[
J(\Theta) = \sum_{k=1}^{n} \frac{1}{2} [e(k, \Theta)]^T [R]^{-1} [e(k, \Theta)] + ln|R|
\]

(4)

since, in the optimization process, \( J(\Theta) \) is equivalent to \( -ln \ p(y|\Theta) \), except for a constant term.

2.2 Minimization of the Cost Function by Gauss-Newton

The identification algorithms based on the Gauss-Newton method is of second order. This method, although complex, is suitable for a quadratic cost function, and is expected to converge quickly. First, we approximate \( J(\Theta) \) by a parabolic function \( J_L(\Theta) \) under the condition \( \Theta_L \) (retaining only the 3 first Taylor series terms),

\[
J_L(\Theta) \approx J(\Theta_L) + (\Theta - \Theta_L)^T \nabla^T \Theta J(\Theta_L) + \frac{1}{2} (\Theta - \Theta_L)^T [\nabla^2 \Theta J(\Theta_L)] (\Theta - \Theta_L)
\]

(5)
The optimization condition is obtained when,
\[ \nabla_{\Theta} J(\Theta^*) = 0 \]  
(6)

Applying (6) to equation (5), results, for \( \Theta \) close to the local minima \( \Theta^* \),
\[ \nabla_{\Theta} J_L(\Theta) \approx \nabla_{\Theta} J(\Theta_L) + (\Theta - \Theta_L)^T [\nabla^2_{\Theta} J(\Theta_L)] \]  
(7)
which can be used to find the minima of the original cost function through the recursion,
\[ \Theta_{t+1} = \Theta_t - [\nabla^2_{\Theta} J(\Theta_t)]^{-1} \nabla_{\Theta} J(\Theta_t) \]  
(8)

The complexity in the calculation of the Hessian matrix, \( \nabla^2_{\Theta} J(\Theta_L) \) in (8), is avoided through the Gauss-Newton method, which uses the approximation,
\[ \nabla^2_{\Theta} J(\Theta) \approx \sum_{k=1}^{n} [\nabla_{\Theta} \hat{y}_k(\Theta)]^T \hat{R}^{-1} [\nabla_{\Theta} \hat{y}_k(\Theta)] \]  
(9)

where the terms involving the second derivative are discarded. The gradient of the estimated output, \( \nabla_{\Theta} \hat{y}_k(\Theta) \), is called Sensibility Function.

The inversion in (8) is not performed in an explicit manner, i.e., typically the original equation
\[ [\nabla^2_{\Theta} J(\Theta)] \Delta \hat{\Theta} = \nabla^T_{\Theta} J(\Theta_t) \]  
(10)
is solved via SVD.

3. Flight Path Reconstruction

The aim of a data compatibility check, often called flight path reconstruction, is to ensure that the measurements used for aerodynamic model identification are consistent and error free. For example, the measured angle of attack must match that reconstructed from the measured linear accelerations and angular rates. Such a verification is possible in the case of flight data because the well defined kinematic equations of aircraft motion provide a convenient mean to bootstrap the information through a numerical procedure. Unlike aerodynamic modeling, since there are no uncertainties involved in the kinematic model, the compatibility check provides an accurate information about the aircraft states. In addition, it provides estimates of scale factors, zero shifts, i.e., biases, and time shifts in the recorded data.

3.1 Kinematic Model for FPR

The kinematic equations of aircraft motion consist of a set of first order ordinary differential equations which, instead of physical variables, use observed variables (accelerations and angular rates) as forcing functions. Once the aircraft mass (or any other physical properties) does not belong to the set of equations, this consist on a set of kinematic relations. The solution of these equations is obtained using the components of acceleration \((a_x, a_y, a_z)\) and angular rates \((p, q, r)\) as input variables, which are measured by the sensors installed on the aircraft. These direct measurements allow the application of the FPR technique before the parameter identification of the aircraft motion.

**State equations:**
\[
\begin{align*}
\dot{u} &= (a_x - \Delta a_x) - (q - \Delta q)w + (r - \Delta r)v - g \text{sen} \theta \\
\dot{v} &= (a_y - \Delta a_y) - (r - \Delta r)u + (p - \Delta p)w + g \text{cos} \theta \text{sen} \phi \\
\dot{w} &= (a_z - \Delta a_z) - (p - \Delta p)v + (q - \Delta q)u + g \text{cos} \theta \text{cos} \phi \\
\dot{\phi} &= (p - \Delta p) + (q - \Delta q) \text{sen} \phi \text{tan} \theta + (r - \Delta r) \text{cos} \phi \text{tan} \theta \\
\dot{\theta} &= (q - \Delta q) \text{cos} \phi \text{sec} \phi - (r - \Delta r) \text{sen} \phi \\
\dot{\psi} &= (q - \Delta q) \text{sen} \phi \text{sec} \theta + (r - \Delta r) \text{cos} \phi \text{sec} \theta \\
\dot{h} &= u \text{sen} \theta - v \text{cos} \theta \text{sen} \phi - w \text{cos} \theta \text{cos} \phi
\end{align*}
\]  
(11)

where \((u, v, w)\) are the velocity components along the \(x, y\), and \(z\) body-fixed axes respectively, \((\phi, \theta, \psi)\) are the Euler angles, \(h\) is the altitude, \((a_x, a_y, a_z)\) are the linear accelerations along the three axes, and \((p, q, r)\) are the rotational rates about the three body-fixed axes.

In eq. (11), the three linear accelerations \((a_x, a_y, a_z)\) and the angular rates \((p, q, r)\) are the input variables, which are assumed to be biased. The systematic errors, i.e. constant biases, in the measurements of the input variables are denoted by \((\Delta a_x, \Delta a_y, \Delta a_z, \Delta p, \Delta q, \Delta r)\) respectively.

The kinematic equations are derived referred to some suitable reference point. In the present case, the aircraft center of gravity is assumed to be the reference point. Accordingly, the linear accelerations must also be referred to the center of
For the sake of notation simplicity and clarity, the time delays are not shown in eq. (13).

In all data acquisition. In Fig. 2 we have some pictures of this instrumentation.

CEA/FDAS (Flight Data Acquisition System). This system is microcontroller based, and uses PDAs to control and store developing an autonomous acquisition system to be used in unmanned air vehicles and in light aircraft flight tests called 1 we show also the inertial and geometric data of the aircraft.

swept forward to place both cockpits ahead of the wing, thereby providing excellent visibility from both cockpits. In Fig.

established the Puchacz as one of the world’s favorite two-place sailplanes. It is fully aerobatic and approved for spins,
sailplane designed primarily for clubs and soaring schools. The Puchacz features beautifully finished fiberglass construc-
tion, a quiet and comfortable tandem cockpit, and exceptional handling qualities. This combination of characteristics has

The instrumentation used to perform the data acquisition consists in: i) autonomous acquisition system using micro

factors, biases and initial conditions denoted as follows:

\begin{align}
\dot{x} &= a_{x0} + (q^2 + r^2)x - (pq - r)Y - (pr + q)Z \\
\dot{y} &= a_{y0} - (pq + r)x + (r^2 + p^2)Y - (p^2 - 2pr + q^2)Z \\
\dot{z} &= a_{z0} - (p^2 - q^2)x - (pr + p)Y + (p^2 + q^2)Z
\end{align}

where \((X_a, Y_a, Z_a)\) denote the offset distances of the accelerometer from the current CG. These distances are specified as a part of the aircraft mass characteristics and geometry.

The measurement equations are given by:

\begin{align}
\dot{\phi}_m &= K_{\phi} \phi + \Delta \phi \\
\dot{\theta}_m &= K_{\theta} \theta + \Delta \theta \\
\dot{\psi}_m &= K_{\psi} \psi + \Delta \psi \\
V_{tas} &= K_{V_{tas}} \sqrt{v_{tas}^2 + w_{tas}^2 + \Delta V_{tas}} \\
\alpha_{aos} &= K_{\alpha_{aos}} \arctan \left( \frac{\alpha_{aos}}{\sqrt{\alpha_{aos}^2 + \alpha_{aos}^2 + \alpha_{aos}^2}} \right) + \Delta \alpha_{aos} \\
\beta_{aos} &= K_{\beta_{aos}} \arcsin \left( \frac{\beta_{aos}}{\sqrt{\alpha_{aos}^2 + \alpha_{aos}^2 + \alpha_{aos}^2}} \right) + \Delta \beta_{aos} \\
h &= K_h h + \Delta h \\
V_{CAS} &= 340.294 \sqrt{5 \left( \frac{p_{dyn}_{\text{p}0,\text{CAS}}}{{1 + 1}^{2/7}} \right)^2 - \frac{1}{0.9444}}
\end{align}

where the subscripts \(m\) denotes the measured values appearing on the left hand side of eq. (13). The quantities appearing

Each measurement is assumed to be erroneous, and hence provision is made to calibrate them through a linear scale

The velocity components, for example, \((u_{aos}, v_{aos}, w_{aos})\) at the angle of sideslip 1 are given by:

\begin{align}
u_{aos} &= u - (r - \Delta r)Y_{aos} + (q - \Delta q)Z_{aos} \\
v_{aos} &= v - (p - \Delta p)Z_{aos} + (r - \Delta r)X_{aos} \\
w_{aos} &= w - (q - \Delta q)X_{aos} + (p - \Delta p)Y_{aos}
\end{align}

where \((X_{aos}, Y_{aos}, Z_{aos})\) denote the offset distances from the center of gravity to the angle of sideslip.

System parameters:

In the model given by eqs. (11) and (13), the parameter vector to be estimated is formed by the 19 unknown scale

\begin{align}
\Theta &= [\Delta a_x \ \Delta a_y \ \Delta a_z \ \Delta p \ \Delta q \ \Delta r \ \Delta v_0 \ \Delta w_0 \ \Delta y_0 \ \Delta \theta(0) \ \Delta \psi(0) \ \Delta h(0)]^T
\end{align}

The other parameters not listed here, like scale factors and biases of the other output variables that appear in eq. (13),

are considered to be constant and equal to one and zero, respectively, since these sensors are very accurate.

3.2 Aircraft Information and Data Acquisition System

The SZD-50-3 PUCHACZ (pronounced poo-hots, which means owl in Polish) is a two-place training and aerobatics sailplane designed primarily for clubs and soaring schools. The Puchacz features beautifully finished fiberglass construction, a quiet and comfortable tandem cockpit, and exceptional handling qualities. This combination of characteristics has established the Puchacz as one of the world’s favorite two-place sailplanes. It is fully aerobatic and approved for spins, inverted flight, slow rolls and even snap rolls. As shown in the Fig. 1 three-view, its cantilevered taper wings are slightly swept forward to place both cockpits ahead of the wing, thereby providing excellent visibility from both cockpits. In Fig. 1 we show also the inertial and data of the aircraft.

The instrumentation used to perform the data acquisition consists in: i) autonomous acquisition system using micro

controllers; ii) solid state inertial platform; iii) pitot probe ; iv) attack and sideslip angle indicators; v) potentiometer on command system; vi) load cell on command system; vii) thermometer and ix) GPS. CEA UFMG is developing an autonomous acquisition system to be used in unmanned air vehicles and in light aircraft flight tests called CEA/FDAS (Flight Data Acquisition System). This system is microcontroller based, and uses PDAs to control and store all data acquisition. In Fig. 2 we have some pictures of this instrumentation.
3.3 FPR Results

A flight test was performed and data was gathered with sampling time \( t = 0.1 \) s. Two time segments were used to perform the FPR procedure. One with maneuvers exciting the longitudinal motion and the other exciting the lateral-directional motion.

Based on the input signals and on the measured variables according to the output equations (13), the identification algorithm was used to determine the parameter vector containing the 19 parameters indicated in eq. (15). The values obtained are:

- \( \Delta a_x = -0.0214 \) [G], \( \Delta a_y = -0.0277 \) [G], \( \Delta a_z = -0.0298 \) [G], \( \Delta p = -0.141 \) [deg/s], \( \Delta q = 0.503 \) [deg/s], \( \Delta r = -0.0739 \) [deg/s], \( K_\alpha = 0.892 \), \( \Delta \alpha = 11.811 \) [deg], \( K_\beta = 1.353 \), \( \Delta \beta = -1.1029 \) [deg], \( K_{V_T} = 0.9 \), \( \Delta V_T = -2.8889 \) [KTS].

These values of scale factors and biases are validated by the match of the observation variables, showed in Fig. 3. Then they are used as constants in the phase of parametric estimation of the longitudinal motion, where they appear in the output equations only, eq. (19).
4. Parametric Identification

4.1 Dynamic Model of the Longitudinal Aircraft Motion

The state equations pertaining to the longitudinal mode of aircraft motion are given by:

\[
\begin{align*}
\dot{q} &= \frac{1}{T_v} \left[ \dot{\theta} S_m C_m^{CG} - I_{xz} \left( p^2 - r^2 \right) + (I_z - I_x) pv \right] \\
\dot{\theta} &= q \cos \phi - r \sin \phi \\
\dot{u} &= -qw + rv - q \sin \theta + \frac{\alpha q}{m} C_X \\
\dot{v} &= -pv + qu - q \cos \theta \cos \phi + \frac{\alpha q}{m} C_Z \\
\dot{h} &= u \sin \theta - v \cos \theta \sin \phi - w \cos \theta \cos \phi
\end{align*}
\]  

where \( C_X, C_Z \) and \( C_m^{CG} \) denote the aerodynamic coefficients of longitudinal force, vertical force and pitching moment. The longitudinal and vertical force coefficients, \( C_X \) and \( C_Z \), are obtained from the lift and drag coefficients, \( C_L \) and \( C_D \) and the pitching moment coefficient \( C_m^{CG} \) referred to the CG is given by:

\[
\begin{align*}
C_X &= -C_D \cos \alpha + C_L \sin \alpha \\
C_Z &= -C_D \sin \alpha - C_L \cos \alpha \\
C_m^{CG} &= C_m^{RP} + C_X \frac{\alpha q}{m} - C_Z \frac{\alpha q}{m}
\end{align*}
\]  

where \( C_m^{RP} \) is the aerodynamic pitching moment referred to the aerodynamic reference point (usually the aerodynamic center), and \((x_{rpcg}, z_{rpcg})\) are the distances of the reference point to the CG along the \( x \) and \( z \) axes.

The kinetic pressure is given by \( \dot{q} = \frac{1}{2} \rho V^2 \) where the density of air, \( \rho \), is specified as a part of the aircraft mass characteristics. The true air speed, \( V_t \), is computed using the state variables \( u \) and \( w \); the component \( v \) is obtained from the measurements of the flow quantities and treated as an input variable.

**Aerodynamic Model:**

The unknown aerodynamic derivatives appear in the following postulated model:

\[
\begin{align*}
C_L &= C_{L_0} + C_{L_\alpha} \alpha + C_{L_q} \frac{\alpha q}{M} + C_{L_{12}} \delta E \\
C_D &= C_{D_0} + \frac{1}{2} \chi^2 C_L^2 \\
C_m^{RP} &= C_m^{mu} + C_{mu} \alpha + C_{mu} \frac{\alpha q}{M} + C_{mu} \delta E
\end{align*}
\]  

The drag is modelled using a drag-polar. The Oswald-factor \( e \) characterizes the increase in drag over the ideal conditions caused by non-elliptical lift distribution.

**Response equations:**

\[
\begin{align*}
q_m &= q + \Delta q \\
\alpha_{aoa} &= F_a \tan^{-1} \left( \frac{u_{aoa}}{w_{aoa}} \right) + \Delta \alpha_{aoa} \\
\theta_m &= \theta + \Delta \theta \\
h_m &= h \\
V_{tm} &= \sqrt{u_{tas}^2 + w_{tas}^2} \\
\dot{\alpha}_{aoa} &= x_{bascg} (\dot{q}^2 + r^2) + y_{bascg} (p \dot{r} - \dot{\alpha} q) + z_{bascg} (p r + \dot{\alpha} q) + \Delta \alpha_x \\
\dot{\alpha}_{aoa} &= x_{bascg} (\dot{q}^2 + r^2) + y_{bascg} (p \dot{r} - \dot{\alpha} q) + z_{bascg} (p r + \dot{\alpha} q) + \Delta \alpha_z \\
\end{align*}
\]

where \((x_{bascg}, y_{bascg}, z_{bascg})\) are the distances of the accelerometer from the CG along \( x \) and \( z \) axes. The accelerations \( \dot{\alpha}_{aoa} \) and \( \ddot{\alpha}_{aoa} \) at CG are given by:

\[
\begin{align*}
\dot{\alpha}_{CG} &= \frac{1}{M} (\dot{q} SC_X + X_{tw}) \\
\ddot{\alpha}_{CG} &= \frac{1}{M} (\dot{q} SC_Z + Z_{tw})
\end{align*}
\]

The velocity components \((u_{aoa}, v_{aoa}, w_{aoa})\) at the angle of attack sensor are given by:

\[
\begin{align*}
u_{aoa1} &= u - (r - \Delta r) Y_{aoa1} + (q - \Delta q) Z_{aoa1} \\
v_{aoa1} &= v - (p - \Delta p) Z_{aoa1} + (r - \Delta r) Y_{aoa1} \\
w_{aoa1} &= w - (q - \Delta q) X_{aoa1} + (p - \Delta p) Y_{aoa1}
\end{align*}
\]

where \((X_{aoa}, Y_{aoa}, Z_{aoa})\) denote the offset distances from the center of gravity to the basis angle of attack sensor.

**System parameters:**

Equation (18) has 10 unknown parameters that need to be estimated, giving \( \Theta \in \mathbb{R}^{10} \), i.e.,

\[
\Theta = [C_{L_0}, C_{L_\alpha}, C_{L_q}, C_{L_{12}}, C_{D_0}, C_{mu}, C_{mu}, C_{mu}, C_{mu}]^T
\]
4.2 Experimental Results

The aerodynamic derivatives associated with the longitudinal model, as shown in eq. (16) and (19), were estimated by matching the real flight test data with the model predicted simulation. Traditional maneuvers, like doublet and 3211, were applied to a sailplane to investigate the effectiveness of the discussed Output-error method, applied to estimate the aerodynamic parameter vector defined in eq. (22). The aircraft measured input signal is the elevator deflection, $\delta e(t)$. The time history of the aircraft input-output relationship was measured with a sampling time of 0.1s, and the measured points gives an observation time window of 40s.

Table 1 shows the final values of the non-dimensional aerodynamic derivatives obtained by the method. To initialize the algorithm we use values calculated based on the aircraft geometrical and inertial information and [12]. Since the flight data employed to generate Table 1 was obtained experimentally and no wind tunnel tests are available, an indirect measure of performance is used, based on the convergence of the parameters and the prediction error. So, the main focus of the present inverse aerodynamic modeling is to check that this local minimization procedure can provide good matching to the experimental flight data and stable input-output modeling for the aircraft. This prediction capability, as obtained by the output-error method, can be accessed from the model validation results shown fig. 4, where we can observe that all the parameters present excellent convergence properties, and in fig. 5, where the estimation error are small for most of the output variables.

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Initial values</th>
<th>GN method</th>
</tr>
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<tbody>
<tr>
<td>$C_{L_0}$</td>
<td>0.96149</td>
<td>0.40000</td>
</tr>
<tr>
<td>$C_{L_\alpha}$</td>
<td>4.97876</td>
<td>5.00001</td>
</tr>
<tr>
<td>$C_{L_q}$</td>
<td>11.64880</td>
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<td>$C_{L_e}$</td>
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<td>$C_{D_0}$</td>
<td>0.01150</td>
<td>0.01000</td>
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<td>$e$</td>
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<td>$C_{m_\alpha}$</td>
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<tr>
<td>$C_{m_{\alpha\alpha}}$</td>
<td>-0.39591</td>
<td>-3.10000</td>
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</tbody>
</table>

Figure 4. Convergence plots of the parameters and determinant of the covariance matrix, $R$. 
5. Conclusions

Based on the small prediction error and good agreement between the calibrated values, we can conclude that the proposed procedure for FPR, based on parametric identification via optimization using the Gauss-Newton method, exhibited satisfactory performance. Thus, the proposed procedure constitutes a relevant alternative for practical applications.

This work also presented the estimation of the aircraft aerodynamic derivatives of the longitudinal motion, which presented good convergence properties and good matching to the experimental flight data. The results obtained with the Output-Error method and Gauss-Newton algorithm demonstrate the feasibility of the identification procedure.

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7. References


Roskam, J., 2001, "Airplane flight dynamics and automatic flight controls", DARcorporation, Lawrence, KS.

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