APPLICATION OF THE TWO-LOAD METHOD ON THE EVALUATION OF THE ACOUSTICAL EFFICIENCY OF HELMHOLTZ RESONATORS

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Abstract. Helmholtz resonators are often used in noise control applications such as exhaust noise systems and HVAC-ducts. Acting as a band-pass filter, the resonator is inserted in the main piping and adjusted to attenuate specific frequency components by reflecting the energy back to the source. An important parameter in order to quantify the acoustical efficiency of an Helmholtz resonator is the Transmission Loss. This loss can be experimentally measured using a set-up similar to an Impedance Tube. The work shows the evaluation of the Transmission Loss of Helmholtz resonators using the technique known as the Two-Load Method, which can reliably assess the Transmission Loss even without an anechoic termination on the tube and is of easy implementation. Different configurations of the resonator are used and the experimental results are compared to the theoretical ones.

Keywords: Acoustical Efficiency, Noise Control, Two-Load Method, Helmholtz Resonator

1. Introduction

In many applications and technical systems one is faced with the noise propagation along pipes and ducts. This problem may arise for instance in exhaust systems of internal combustion engines. The exhaust pipe used to guide the combustion gases to the exterior of the vehicle also transmits the combustion generated noise to the outside. In heating, ventilation or air-conditioning systems (HVAC) the aerodynamic noise generated by rotating fans is transmitted through the air distribution ducts. Also the structural noise of the vibrating walls of the duct itself is transmitted.

It is clear that noise control measures must be taken to reduce the sound radiation of the terminations of the ducts while keeping energy losses for the flow inside at a minimum. To this intent mufflers and resonators inserted in the ducts are a common choice. Helmholtz resonators act as a band-pass filter, reflecting part of the sound energy back to the source in a well-known way.

The paper presents the evaluation of the acoustical efficiency of a resonator using as parameter the Transmission Loss (TL), relating the incident energy to the transmitted energy in a duct section after the resonator. The Transmission Loss is determined experimentally utilizing the technique known as the Two-Load method. In order to control the propagated noise in a duct Pota and Kelkar (2001) use a feedback controller to drive a loudspeaker directly interfering with the acoustical energy flow. The final goal of this research is to use different configurations for the resonator as the heart of a noise control system with automated tuning of the filter frequency instead.

2. Transmission Loss

There are many possibilities to describe the influence of noise control measures. It may be quantified, for instance, through the difference in sound pressure levels measured at a specified distance of the radiating source before and after the introduction of a noise control barrier. This is known as the Insertion Loss (IL) for it relates the levels with and without the barrier. Since the sound pressure levels are the actual quantity being measured they are also influenced by environmental aspects, though the IL cannot be a measure of the efficiency of the chosen control method alone.

The Transmission Loss is better suited to characterize the acoustical efficiency for it relates the incident and transmitted sound energies in and through the noise abatement device. Inside a duct or pipe we may consider the energy from the source that flowing into a transversal section where the noise control is placed and the energy flowing out of this section after passing it. Nevertheless the reflected sound waves inside the tube must be taken into account when
looking for the energies, or equivalently the sound power (Kinsler et al., 1982 and Gerges, 2000). Figure 1 shows schematically the incident and reflected waves in the pipe, equation (1) is the formal definition of the TL:

\[
TL = 10 \log \left( \frac{W_i}{W_t} \right)
\]

where \( TL \) is the Transmission Loss \( W_i \) is the incident sound power and \( W_t \) is the transmitted sound power.

![Figure 1. Energy flow inside the duct adapted from Tao and Seybert (2003)](image)

As can be inferred from Fig. 1 the amount of sound power reflected by the end of the tube, opposed to the source, depends on the nature of the termination. An anechoic termination would absorb entirely the energy, causing the wave reflected back to the resonator section to be negligible.

Since the direct measurement of the sound powers \( W_i \) and \( W_t \), inside the tube, would not be an easy task we must look at ways to assess them from sound pressure measurements alone.

2.1. Two-Load Method

Relating the acoustic pressures and particle velocities on a section right behind the resonator and a section right after it, in order to be able to calculate the sound power, we get four parameters to characterize, acoustically, the path between the chosen sections, Eq.(2).

\[
\begin{bmatrix}
  p_1 \\
  v_1 
\end{bmatrix} =
\begin{bmatrix}
  A_{12} & B_{12} \\
  C_{12} & D_{12}
\end{bmatrix}
\begin{bmatrix}
  p_2 \\
  v_2
\end{bmatrix}
\]

If only sound pressure measurements are to be done in two sections, determining \( p_1 \) and \( p_2 \), we are left with six unknowns, \( A_{12}, B_{12}, C_{12}, D_{12}, v_1 \) and \( v_2 \), and only two equations. As can be seen in Chung and Blaser (1980) and in the work from Tao and Seybert (2003) the use of four microphone positions and two different acoustic load conditions, \( Z_a \) and \( Z_b \), for the tube termination allows the determination of the unknown parameters and the TL can be calculated accordingly. The same procedure can be used mathematically from simulation results (Scientific Technical Software, 2003) to calculate the TL. The expressions are here repeated in eq. (3) to eq. (7) with the same meaning from Braga and Gerges (2004) neglecting flow inside the duct:

\[
A = \frac{(H_{31a} - H_{32a})(H_{34b} - H_{34a}) + \cos(kl_{34})(H_{32b} - H_{32a})}{H_{34b} - H_{34a}}
\]

\[
B = \frac{j \rho c \sin(kl_{34})(H_{32a} - H_{32b})}{H_{34b} - H_{34a}}
\]

\[
C = \frac{(H_{31a} - \cos(kl_{12})(H_{32a})) (H_{34b} - \cos(kl_{34})) - (H_{31b} - \cos(kl_{12})(H_{32b})) (H_{34a} - \cos(kl_{34}))}{j \rho c \sin(kl_{12})(H_{34b} - H_{34a})}
\]

\[
D = \frac{j \rho c \sin(kl_{34})(H_{31a} - H_{31b}) + \cos(kl_{12})(H_{32b} - H_{32a})}{j \rho c \sin(kl_{12})(H_{34b} - H_{34a})}
\]
\[ TL = 20 \log \left( \frac{1}{2} \left| A + \frac{B}{c \rho} + \rho c C + D \right| \right) + 10 \log \left( \frac{S_i}{S_j} \right) \]  

where \( H_{xy} \) is the measured transfer function between microphone positions \( x \) and \( y \), \( k \) is the wave number and \( l_{xy} \) is the distance between microphone positions \( x \) and \( y \). The complex unity being \( j \). The speed of sound is \( c \) and the density of the air is \( \rho \). Figure 2 shows microphone positions 1 through 4. The cross sectional areas \( S_i \) and \( S_t \) of the duct before and after the resonator are equal.

Figure 2. Microphone arrangement according to Tao and Seybert (2003)

The subscripts \( a \) and \( b \) represent quantities measured either with load configuration \( Z_a \) or load configuration \( Z_b \). These different loads should be chosen as different reflective characteristics of the termination. There is no need that one of the terminations chosen be truly anechoic, it must only be more absorbent or more reflective than the other load configuration. This is one of the main advantages of the method. Despite the type of load chosen, it is important to keep at a minimum extraneous background noise from the environment coming into the duct from the open termination.

2.2. Helmholtz Resonator

The Helmholz Resonator consists of a volume \( V \) connected to the duct by a neck of length \( L \) and with cross section \( S \). According to Kinsler et al. (1982) the resonator acts as a band-pass filter. Its characteristics can be tuned to a specific frequency range by varying its parameters.

Considering the resonant situation of the resonator the Transmission Loss can be written, Eq. (8), as:

\[ TL_H = 10 \log \left( 1 + \frac{e^2}{4 S_d^2 \left( \frac{1.5 \frac{d}{2} + L}{S} \right)^2} \right) \]

where \( S_d \) is the cross section of the duct and the length \( L \) of the neck has an empirical correction term with \( d \) representing the diameter of the neck. Equation (8) is valid if viscosity effects and absorption in the resonator are neglected. The plot of Eq. (8) as a function of frequency is shown in figure 3.

The tuning capabilities of the resonator by varying the parameters of Eq. 8 when used in a control loop can be useful for the implementation of noise control systems for ducts and is the final objective of this research.
3. Experimental Results

The behavior of the resonator was tested experimentally using adjustable resonators shown in Fig. 4. Different configurations of the resonator were used by changing the internal volume, the diameter of the cylindrical volume and diameter and length of the connecting neck. The experimental set-up follows the work from Braga and Gerges (2004).
The duct under consideration is adapted from an Impedance Tube with a loudspeaker as sound source and fitted with an extra section for the resonator and extended to include two more measurement positions. Excitation, with random white noise in the range from 0.1Hz to 3000Hz, is achieved through the signal generator of an spectral analyzer. Table 1 summarizes the equipment used in the set-up, which can be seen in Fig. 5.

Table 1. Instrumentation.

<table>
<thead>
<tr>
<th>Instrument</th>
<th>Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capacitive Microphones</td>
<td>Bruel &amp; Kjäer model 4165</td>
</tr>
<tr>
<td>Microphone Power Supply</td>
<td>Bruel &amp; Kjäer model 2807</td>
</tr>
<tr>
<td>Frequency Analyzer</td>
<td>Hewlett-Packard 3562A</td>
</tr>
<tr>
<td>Acoustic Calibrator</td>
<td>Larson Davis CAL200</td>
</tr>
<tr>
<td>Impedance Tube</td>
<td>40mm internal diameter</td>
</tr>
<tr>
<td>Source</td>
<td>Power Amplifier and 4” Loudspeaker</td>
</tr>
<tr>
<td>Microphone 1 Distance to the Source</td>
<td>1200mm</td>
</tr>
<tr>
<td>Microphone 2 Distance to the Source</td>
<td>1250mm</td>
</tr>
<tr>
<td>Microphone 3 Distance to the Source</td>
<td>1810mm</td>
</tr>
<tr>
<td>Microphone 4 Distance to the Source</td>
<td>1860mm</td>
</tr>
<tr>
<td>Distance Between Resonator and Source</td>
<td>1535mm</td>
</tr>
</tbody>
</table>

Besides the two-channel analyzer a sound card in a personal computer is also used as acquisition device. Two microphones were used at a time in three different measurements, for each load condition, to assess the necessary transfer functions from Eq. (7). The frequency response of the sound card was previously checked, either as signal generator and as acquisition device, against the analyzer. This verification has shown that the sound card was suitable to make the measurements configured to 6kHz acquisition frequency.

The microphone signals were calibrated before and after each measurement with an acoustic calibrator and no noticeable deviation was found in the microphone sensitivities. The signals were recorded in WAV-format, together with the calibration signals, and the transfer functions were calculated numerically. The following figures show results obtained using 4096 points for the FFT algorithm and averaging 100 spectra without overlap.

Different load conditions were achieved by using two different terminations with different absorbent characteristics. None of the loads were truly anechoic, which is an advantage for the practical implementation.

Figure 6 shows that the experimental and theoretical results are in good agreement. The calculated and measured frequencies of the Helmholtz resonator are the same. The Transmission Loss, for higher frequencies is influenced by higher order resonances, not modeled by Eq.(8). This can be seen at the peaks observed at 730Hz and 1400Hz. Below 30Hz the differences between theoretical values of the TL and the experimental results are related to the poor efficiency of the 4” loudspeaker used, for frequencies below 50Hz.

Different configurations of the resonator were tested. The resonator with larger volumes from a cylindrical cavity with a diameter of 96mm is connected to the main tube through a neck with 50mm length and 40mm diameter and also...
through a neck with 75mm length and the same 40mm diameter. A smaller resonator is built with an internal diameter of 40mm and neck of 34mm length and 20mm diameter. By varying the position of a piston inside the resonators their volumes can be changed. Figures 7 through 9 show the Transmission Losses for the whole range of parameters considered.

Figure 6. Experimental and Theoretical Results
(Resonator Volume=1810x10^3 mm^3 / Neck Diameter=40mm / Neck Length=50mm)

Figure 7. Experimental Results (Resonator Ø=40mm)
The resonator with smaller volume can be used to attenuate a good frequency range without having to vary the position of the piston too much. Its advantage lies in the little space occupied, but the acoustic performance is inferior to that of the resonator with larger volume, as shown in figures 7 and 8. Nevertheless, in order to achieve good values of the TL in the low frequency range the larger resonator is needed. It can be seen from Fig. 9 that the neck length has little effect on the resonant frequencies of the resonator but influences the magnitude of the TL. A broad range of attenuation can be achieved by tuning the system varying, for instance, the volume of the resonator. A combined use of different, adjustable, configurations may also use the higher modes to create a desired adaptive filter characteristics.

Since one of the possible applications of the complete system is in exhaust pipes from combustion engines, it is important to have good efficiency at low frequencies, below 100 Hz. An actual application will make use of more than only one resonator.
4. Conclusion

The two-load method for determining the Transmission Loss of noise control devices for duct propagation was implemented.

Different configurations of Helmholtz resonators were tested and the TL values measured exhibit a good agreement with theoretical ones.

The easiness of the practical implementation makes it possible to measure the TL of different arrangements of mufflers, resonators and other devices with the established procedure and used instrumentation. This is of interest for the automotive industry, for instance, which must evaluate the performance of different mufflers for exhaust pipes.

The set-up using an impedance tube serve as basis for the implementation of a control loop to tune the resonator characteristics according to the input noise conditions. This is the next step to be taken in the ongoing research.

5. References


6. Responsibility notice

The authors are the only responsible for the printed material included in this paper.