VARIABLE STRUCTURE CASCADE CONTROL OF A PNEUMATIC POSITIONING SYSTEM

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Abstract. This paper proposes a variable structure cascade approach to be applied in pneumatic positioning systems control. The cascade methodology consists of dividing the whole pneumatic positioning system model into two subsystems: mechanical and pneumatic. The mechanical subsystem model is employed to determine the amount of pressure required to drive a specific load to its desired position. With that information, the control input acts upon the pneumatic subsystem, so that the required pressure is provided to the mechanical subsystem. In order to ensure the robustness of the controlled system with respect to parameter uncertainties and payload mass variations, we propose the use of a variable structure technique applied to the pneumatic subsystem control. When the effects of friction and external applied forces are present, the controlled system is proved to converge to a limited region in state space.

Keywords: robotics, pneumatic control systems, slide mode control, cascade control

1. Introduction

Pneumatic positioning systems are very attractive for many applications because they are cheap, lightweight, clean, easy to assemble and present a good force/weight ratio. In spite of these advantages, pneumatic positioning systems present some undesirable characteristics which limit their use in applications that require a fast and precise response. These undesirable characteristics derive from the high compressibility of the air and from the nonlinearities present in pneumatic systems.

Due to its highly nonlinear nature, a pneumatic positioning system has performance characteristics that strongly depend on the degree of uncertainty of the parameters employed in its mathematical model. This problem is particularly serious when it regards friction effects, which are very difficult to model accurately. Besides, there are operation parameters that could be unknown or change with time, such as the magnitude of the payload mass, or the presence of external forces. Any effective control strategy must be able to deal accordingly with such difficulties.

To overcome such problems, a cascade control strategy has been developed in which the pneumatic positioning system is interpreted as an interconnected system: a mechanical subsystem driven by the force generated by a pneumatic subsystem (Guenther and Perondi, 2003). In this work, in order to increase the system robustness with respect to parameter uncertainties and variations in operating conditions, the force control in the pneumatic subsystem is performed by means of a variable structure technique. Such a control scheme proves to be very useful, because variable structure control is an inherently nonlinear technique. Therefore, the nonlinear behaviour of the pneumatic positioning system can be directly addressed, which results in good performance characteristics over a wide range of operating conditions.

The convergence of the tracking errors to a limited region in the state space is demonstrated by means of the Lyapunov direct method. Such a convergence is shown to hold even in presence of system parameter uncertainties, external forces, and friction effects. Simulation results illustrate the main properties of the proposed controller.

This paper is organized as follows. Section 2 is dedicated to present the theoretical model, while, in Section 3, the proposed controller is described. The controller stability properties are stated in Section 4. In Section 5, the simulation results are presented. Finally, the main conclusions are outlined in Section 6.

2. The Dynamic Model

The dynamic model used in this work is developed based on: (i) the description of the relationship between the air mass flow rate and pressure changes in the cylinder chambers, and (ii) the equilibrium of the forces acting at the piston, including the friction force. For a schematic view of the system to be modeled, refer to Fig. 1.

The relationship between the air mass flow rate and the pressure changes in the chambers is obtained using energy conservation laws, and the force equilibrium is given by Newton’s second law. The friction force and the external forces are modeled in a unified way.
2.1. Conservation of Energy

The internal energy of the mass flowing into chamber 1 is \( C_p q_{m1} T \), where \( C_p \) is the constant pressure specific heat of the air, \( T \) is the air supply temperature, and \( q_{m1} = (dm_1 / dt) \) is the air mass flow rate into chamber 1. The rate at which work is done by the moving piston is \( p_1 \dot{V}_1 \), where \( p_1 \) is the absolute pressure in chamber 1 and \( \dot{V}_1 = (dV_1 / dt) \) is the volumetric flow rate. The time air internal energy change rate in the cylinder is \( d(C_r p_1 V_1 T) / dt \), where \( C_r \) is the constant volume specific heat of the air and \( \rho_1 \) is the air density. We consider the ratio between the specific heat values as \( r = C_v / C_r \), and that \( \rho_1 = C_v / (R T) \) for an ideal gas, where \( R \) is the universal gas constant. An energy balance yields

\[
q_{m1} T - \frac{p_1}{C_p} \frac{dV_1}{dt} = \frac{1}{r R} \frac{d}{dt} (p_1 V_1) \tag{1}
\]

where the rate of heat transfer through the cylinder walls (\( \dot{Q} \)) is considered negligible. The total volume of chamber 1 is given by \( V_1 = Ay + V_{10} \), where \( A \) is the cylinder cross-sectional area, \( y \) is the piston position and \( V_{10} \) is the dead volume of air in the line and at the chamber 1 extremity. The change rate for this volume is \( \dot{V}_1 = Ay \), where \( \dot{y} = dy / dt \) is the piston velocity. After calculating the derivative term in the right hand side of (1) we can solve this equation to obtain

\[
\dot{p}_1 = -\frac{Ay}{Ay + V_{10}} p_1 + \frac{RrT}{Ay + V_{10}} q_{m1} \tag{2}
\]

where \( C_p = (rR) / (r - 1) \). Similarly, and considering that the pressure variations in each chamber are opposite, for chamber 2 of the cylinder we obtain

\[
\dot{p}_2 = \frac{Ay}{A(L - y) + V_{20}} p_2 + \frac{RrT}{A(L - y) + V_{20}} q_{m2} \tag{3}
\]

where \( L \) is the cylinder stroke. Assuming that the mass flow rates are nonlinear functions of the servovalve control voltage (\( u \)) and of the cylinder pressures, that is, \( q_{m1} = q_{m1}(p_1, u) \) and \( q_{m2} = q_{m2}(p_2, u) \), expressions (2) and (3) result in

\[
\dot{p}_1 = -\frac{Ay}{Ay + V_{10}} p_1 + \frac{RrT}{Ay + V_{10}} q_{m1}(p_1, u) \tag{4}
\]

\[
\dot{p}_2 = \frac{Ay}{A(L - y) + V_{20}} p_2 + \frac{RrT}{A(L - y) + V_{20}} q_{m2}(p_2, u) \tag{5}
\]

2.2. Piston Dynamics

Applying Newton’s second law to the piston-load assembly yields

\[
M \ddot{y} + F = A(p_1 - p_2) \tag{6}
\]
where \( M \) is the mass of the piston-load assembly, \( F \) represents the effects of friction and external forces, and \( A(p_1 - p_2) \) is the force related to the pressure difference between the two sides of the piston (see Fig. 1).

### 2.3. The Interconnected Model

Equations (4), (5) and (6) compose a fourth order nonlinear dynamic model of the pneumatic positioning system, without direct friction modeling. To rewrite this model in an interconnected form, appropriate for our cascade controller design, we define

\[
M\ddot{y} + F = A p_\Delta
\]

(7)

The pressure difference change rate, calculated using expressions (4) and (5), is given by

\[
\dot{p}_\Delta = R_T r \left[ \frac{q_{m1}(p_1,u)}{A_y + V_{10}} - \frac{q_{m2}(p_2,u)}{A(L-y) + V_{20}} \right] - r A_y \left[ \frac{p_1}{A_y + V_{10}} + \frac{p_2}{A(L-y) + V_{20}} \right]
\]

(8)

Separating \( \dot{p}_\Delta \) into the terms affected by the servovalve control voltage \( u \) and the terms which are functions only of piston position and velocity, we obtain the functions \( \dot{u} = \dot{u}(p_1, p_2, y, u) \) and \( \dot{h} = \dot{h}(p_1, p_2, y, \dot{y}) \), given, respectively, by

\[
\dot{u}(p_1, p_2, y, u) = R_T r \left[ \frac{q_{m1}(p_1,u)}{A_y + V_{10}} - \frac{q_{m2}(p_2,u)}{A(L-y) + V_{20}} \right]
\]

(9)

\[
\dot{h}(p_1, p_2, y, \dot{y}) = -r A_y \left[ \frac{p_1}{A_y + V_{10}} + \frac{p_2}{A(L-y) + V_{20}} \right]
\]

(10)

This allows us to rewrite expression (8) as

\[
\dot{p}_\Delta = \dot{h}(p_1, p_2, y, \dot{y}) + \dot{u}(p_1, p_2, y, u)
\]

(11)

Equations (7) and (11) describe the pneumatic positioning system dynamics. Equation (7) represents the mechanical subsystem driven by a pneumatic force \( g = A p_\Delta \). Equation (11) describes the dynamics of the pneumatic subsystem, in which this pneumatic force is generated by commanding the control voltage \( u \) appropriately. This interpretation reinforces the interconnected model description (Fig.2).

![Figure 2. Pneumatic system described as two interconnected subsystems](image)

### 3. The Control Strategy

We present here the cascade control strategy, based on the methodology of order reduction described in Utkin (1987), associated to a variable structure approach. The cascade control strategy has been used successfully in the control of robot manipulators with electric actuators (Guenter and Hsu, 1993), to control flexible joint manipulators (Hsu and Guenter, 1993), hydraulic actuators (Guenter and De Pieri, 1997, Cunha et al., 2002) and pneumatic servodrives with friction compensation (Perondi, 2002). Examples of use of the variable structure control strategy can be found in the control of electric actuators (Furtunato, 1997, Silva, 1998), hydraulic actuators (Guenter et al., 2000), and pneumatic actuators (Pandian et al., 1997).
According to the cascade strategy, the pneumatic positioning system is interpreted as an interconnected system like that presented in Fig. 2 and its equations can thus be rewritten in a convenient form. To perform this task we initially define the pressure difference tracking error as

$$\Delta p = p - p_{ad}$$

(12)

where \( p_{ad} \) is the desired pressure difference to be defined based on the desired force \( g_d = Ap_{ad} \). This is the desired force required on the piston-load assembly mass to obtain a desired tracking performance. Using the definition (12), equations (7) and (11) may be rewritten as

$$M\dot{y} = Ap_{ad} + d(t)$$

(13)

$$\dot{p}_a = \hat{h}(p_1, p_2, y, \dot{y}) + \hat{u}(p_1, p_2, y, u)$$

(14)

where \( d(t) \) is an input disturbance given by

$$d(t) = A\Delta p - F$$

(15)

The system (13), (14) is in the cascade form. Equation (13) can be interpreted as a mechanical subsystem driven by a desired force \( g_d \) and subjected to an input disturbance \( d(t) \). Equation (14) represents the pneumatic subsystem.

The design of the cascade controller for the system (13), (14) can be summarized as follows:

(i) – Compute a control law \( g_d(t) = Ap_{ad}(t) \) for the mechanical subsystem (13) such that the piston displacement \( y(t) \) achieves a desired trajectory \( y_d(t) \) taking into account the presence of the disturbance \( d(t) \); and then

(ii) – Compute a control law \( u(t) \) such that the pneumatic subsystem (14) applies a pneumatic force \( g(t) = Ap_{ad}(t) \) to the mechanical subsystem that tracks the desired force \( g_d(t) = Ap_{ad}(t) \).

In this paper the design of the mechanical subsystem control law \( g_d(t) \) is based on the controller proposed by Slotine and Li (1988). The control law \( \hat{u}(t) \) is synthesized accordingly to the variable structure control technique in the slide mode approach and the control signal \( u(t) \) is obtained from \( \hat{u}(t) \) through a diffeomorphism. So, once the desired control law \( \hat{u}(t) \) is computed, its inversion yields the corresponding signal \( u(p_1, p_2, y, \hat{u}) \) to be applied to the servovalve. Figure 3 illustrates the complete process.

![Figure 3. Complete Control Strategy](image)

### 3.1. Tracking Control of the Mechanical Subsystem

Based on Slotine and Li (1988), the following control law to obtain trajectory tracking in the mechanical subsystem is proposed:

$$g_d = M\dot{y} - KD_s$$

(16)

where \( K_D \) is a positive constant, \( \dot{y}_r \) is the reference velocity and \( s \) is a measure of the velocity tracking error. In fact, \( \dot{y}_r \) can be obtained by modifying the desired velocity \( \dot{y}_d \) as follows
\[
y = \dot{y}_s - \lambda \dot{y}; \quad \ddot{y} = y - y_\lambda; \quad s = \ddot{y} - \dot{y}_s = \dot{y} + \lambda \ddot{y}
\]  
(17)

where \( \lambda \) is a positive constant. The substitution of Eq. (16) in the mechanic subsystem model (Eq. (13)) yields:

\[
M\ddot{s} + K_D s = A\dot{p}_\lambda - F
\]  
(18)

Consider the non-negative function

\[
2V_1 = Ms^2 + P\ddot{y}^2
\]  
(19)

where \( P \) is a positive constant. Using (17) and (18), the time derivative of (19) is given by

\[
\dot{V}_1 = -K_D s^2 + A\dot{p}_\lambda s + P\dddot{y}^2 - Fs
\]  
(20)

Expression (20) will be used in the stability analysis.

### 3.2. Tracking Control of the Pneumatic Subsystem

In order to obtain force tracking in the pneumatic subsystem (14) we propose the following control law

\[
\dot{u} = \dot{p}_{ld} - \hat{h}(p_1, p_2, y, \dot{y}) - k \text{sgn}(\hat{p}_\lambda) - As
\]  
(21)

where \( \dot{p}_{ld} \) is the time derivative of the desired differential pressure, \( \hat{h}(p_1, p_2, y, \dot{y}) \) is the part of the pneumatic subsystem that is independent of \( \dot{u}(t) \) in Eq. (14), \( k \) is a positive constant, \( A \) is the cylinder cross-sectional area, and \( s \) is defined in (17). The function \( \text{sgn}(\hat{p}_\lambda) \) is defined as \( \text{sgn}(\hat{p}_\lambda) = -1, \hat{p}_\lambda < 0 \) and \( \text{sgn}(\hat{p}_\lambda) = -1, \hat{p}_\lambda > 0 \). The control law described in Eq. (21) is subject to parameter errors in its \( \hat{h}(p_1, p_2, y, \dot{y}) \) term. For that reason, we define the parameter estimation error:

\[
\Delta \dot{h} = \hat{h}(p_1, p_2, y, \dot{y}) - \hat{h}^*(p_1, p_2, y, \dot{y})
\]  
(22)

where \( \hat{h}^*(p_1, p_2, y, \dot{y}) \) is the ideal term corresponding to exact parameter estimation. Substituting (21) in (14), and including the parameter estimation term \( \Delta \dot{h} \), the resulting pneumatic subsystem closed loop dynamics gives

\[
\hat{p}_\lambda = \Delta \ddot{h} - k \text{sgn}(\hat{p}_\lambda) - As
\]  
(23)

The design of the constant \( k \) is based on the non-negative function \( V_2 \) defined as

\[
2V_2 = \hat{p}_\lambda^2
\]  
(24)

The time derivative of (24) is given by

\[
\dot{V}_2 = \hat{p}_\lambda \Delta \ddot{h} - k \text{sgn}(\hat{p}_\lambda) - As
\]  
(25)

In the ideal case, \( \Delta \ddot{h} = 0 \), and expression (25) results in the following expression that will also be used in the stability analysis:

\[
\dot{V}_2 = -k |\hat{p}_\lambda| - As\hat{p}_\lambda
\]  
(26)
4. Stability Analysis

Consider the controlled pneumatic positioning system, given by $\Omega = \{13\}, \{14\}, \{16\}, \{21\}$. We assume that the desired piston position $y_d(t)$ and its derivatives, up to 3rd order, are uniformly bounded. For this system, the tracking errors convergence properties are stated below.

**Main result** – For the case in which all the system parameters are known, and there is the combined effect of external and/or friction forces $F$, given an initial condition, the controller gains can be chosen in order to obtain the convergence of the tracking errors, given by the system errors vector $\mu = \begin{bmatrix} \dot{y} \\ \ddot{y} \sqrt{P_{\lambda}} \end{bmatrix}$, to a residual set $R$ as $t \to \infty$. The set $R$ depends on the upper bound for $F$ and on the controller gains.

**Proof:** Consider non-negative function

$$2V = 2V_1 + 2V_2 = Ms^2 + P\dot{y}^2 + \dot{P}_{\lambda}^2$$

where the functions $V_1$ and $V_2$ are defined in (19) and (24), respectively. It is easy to show that $V(t)$ is null in the state space origin, only. Furthermore, for any other state combination, $V(t)$ is a positive quantity. Then, $V(t)$ is positive definite. According to expressions (20) and (26), and choosing $P = 2\lambda K_D$, the time derivative of Eq. (27) is given by

$$\dot{V}(t) = -\mu^T N \mu - F \mu$$

(28)

where $N = diag[\lambda K_D, K_D, k]$, and $F = [F \lambda, F]$. From (28), even if $V(t)$ is definite positive, $\dot{V}(t)$ is not definite negative for all $\mu \neq 0$. From the Rayleigh-Ritz theorem, it can be written that

$$\dot{V} \leq -\lambda_{\min}(N) \|\mu\|^2 + \|\mu\| \|F\|$$

(29)

where $\lambda_{\min}(N)$ is the minimum eigenvalue of the matrix $N$.

Considering that there exists an upper bound $\bar{F} \geq \|F\|$ for the norm of the vector $F$, that is, the external and friction forces are limited, it can be seen that

$$\dot{V} \leq -\lambda_{\min}(N) \|\mu\|^2 + \bar{F} \|\mu\|$$

(30)

From the Lyapunov condition $\dot{V}(t) < 0$, the convergence condition is given by

$$\|\mu\| > \frac{\bar{F}}{\lambda_{\min}(N)}$$

(31)

If the expression (31) is satisfied, $\dot{V}(t)$ is negative, and $V(t)$ decreases with time. That means that $\|\mu(t)\|$ also decreases with time, until inequation (31) is not satisfied anymore. Then, according to Lyapunov theory, it is possible – but not guaranteed – that $\|\mu(t)\|$ starts to increase again. However, the possible growth of $\|\mu(t)\|$ occurs only until inequation (31) is once more satisfied, and the whole process is repeated. So, as $t \to \infty$, $\|\mu(t)\|$ converges to a limited region given by:

$$\|\mu\| \leq \frac{\bar{F}}{\lambda_{\min}(N)}$$

(32)

This completes the proof.

**Observation:** It is possible to show, by means of a similar analysis, that $\|\mu(t)\|$ is confined to a limited region, even if there are parameters errors present in the model of the system. In this case, the value of $\|\mu(t)\|$ depends not only on the gains of the controller and the upper bounds of friction and external forces. It depends also on the upper bounds of the parameter estimation errors.
5. Simulation Results

Most parameters used in the simulations are assumed to correspond to their exactly known values in the real system: $A=4.19 \times 10^{-4}$ [m$^2$], $r=1.4$, $R=286.9$ [J/Kg/K], $T=293.15$ [K], $L=1$ [m], $V_{ss}=1.96 \times 10^{-6}$ [m$^3$], and $V_{sl}=4.91 \times 10^{-6}$ [m$^3$]. The payload mass $M$ assumed by the control law is always 0.3 [kg]. The mass used in the open loop model of the system, however, is defined in the simulation starting process. So, the payload mass value can be different of the controller estimative for the mass.

All the simulation tests were performed without applying external forces, but the friction effects are present in the open loop model. Once the proposed control scheme does not address friction directly, such friction effects represent an unmodelled dynamics to be dealt with by the controller. In this paper, the friction force is described according to the LuGre friction model proposed by Canudas et al (1995). This model can describe complex friction behaviors, such as stick-slip motion, presliding displacement, Dahl and Stribeck effects and frictional lag. The LuGre parameters used in the servopneumatic open loop model were identified and presented by Perondi and Guenther (2003).

The mass flow functions $q_{\phi}(p, u)$ used in equations (4) and (5) are identified according to the methodology presented in Perondi and Guenther (2003). This allows the calculation of the servovalve control voltage using the inverse of the control law defined in Eq.(21) (see Perondi and Guenther, 2003, for details).

The simulation tests are performed using a sinusoidal desired trajectory. The desired trajectory is given by $y_{d}(t) = y_{\text{max}} \sin(\omega t)$, $y_{\text{max}}=0.45$ [m] and $\omega=2$ [rad/s]. This trajectory can be seen as the curve in black, in Fig. 4.

![Figure 4. Simulation results – position trajectories](image1)

![Figure 5 - Simulation results – position errors](image2)
In order to obtain a precise, fast and oscillations-free position response, the control gains used are \( K_\alpha = 1500, \lambda = 50, k = 5000 \). These values are determined by means of pole placement considerations, when the transfer functions for the closed loop system (with the cascade controller) are estimated (see Perondi, 2002, for more details). The results presented in figures 4 and 5 show the efficiency of the proposed controller in guaranteeing system convergence to the desired trajectory in presence of the unmodelled friction effects, and for different mass values.

Figure 4 shows the position trajectories of the system, with the desired trajectory as reference. Figure 5 shows the trajectory errors for each mass value. These simulation results confirm the tracking error convergence theoretically established for the controller operating in the presence of external friction forces. Also, the small variation in the system performance over a fairly wide range of mass values indicates that the proposed controller has good robustness properties with respect to payload mass.

6. Conclusions

In this work a cascade controller with variable structure approach for a pneumatic positioning system was proposed, and the convergence of its tracking errors in the presence of friction forces to a limited region in state space was demonstrated.

The proposed control strategy is able to compensate friction and external forces without need of explicit models for them. It is also possible to show that such compensation is achievable even when the system parameters are not exactly known.

The simulation results confirm the theoretical conclusion. In particular, it can be seen that such results hold over a fairly wide variation range of payload mass values.

Future work will include experimental results under real operating conditions. Similar control laws, with more straightforward structure, without need of the acceleration signal, and minor demanding level for the servovalve performance, are currently under research.

7. References


8. Responsibility notice

The authors are the only responsible for the printed material included in this paper.