SENSITIVITY ANALYSIS OF A RANKINE CYCLE

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Abstract. *The Monte Carlo Method is a useful tool to estimate uncertainties associated to results from thermal simulation codes and the sensitivity analysis of some important parameters. This method can provide some good advantages vis-à-vis the Differential Method, where small biases are applied to single parameters in order to observe its sensitivity in respect to a set of unknowns. In the Monte Carlo Method, combined uncertainties are calculated in respect to the complete system (global uncertainty), and the sensitivity of each single parameter is not always obtained (local uncertainty). The present work describes an application of the Monte Carlo Method to obtain the sensitivity of every relevant parameter individually. Besides this two methods, an analysis based on the Fourier Transform is proposed. A complete Rankine cycle for electric power generation, based on a real plant, is modeled and analyzed. Results of the three methods show similar trends, helping to validate the proposed method based on Fourier Transforms. An analysis in partial load condition indicates different sensitivities to that on design load operation.*

Keywords: *Sensitivity Analysis, Monte Carlo Analysis, Differential Analysis, Fourier Transform Analysis, Rankine Cycle, Thermal Systems*

1. Introduction

Thermal models have many sources of uncertainty and the evaluation of their impact over the results is an important task. Analysis of uncertainty and sensitivity is perform in order to find out witch parameters are relevant and therefore must be estimated with more caution or just taken in a simplified way. Besides this, sensitivity analysis can point out clues to improve system design.

In Brazil, the bureau charged to control electric energy distribution (ONS) of different power plant units over the country operates in a resolution of 0.1 MW. Thus, the uncertainty of the input data and operational parameters of this network should be consonant to the same order of magnitude. The study of this behavior is the main motivation of the present work. Three different methods of sensitivity analysis are applied to a numerical model of an electrical power plant: the Monte Carlo Method (MCM), the Differential Analysis Method (DAM) and finally a new approach based in a Fourier Transform, called here as Fourier Transform Method (FTM).

Hamby (1995) published a comparison of 14 methods of sensitivity analysis, where most of them generate very similar results, considering the same input dataset. MacDonald and Strachan (2001) constructed example cases using with MCM and DAM, showing the behavior of both methods. Herrador and González (2004), in a similar work, pointed out some limitations of DAM and some advantages of MCM. Results were pretty much the same when both methods were applied to weakly coupled thermal model, but in systems with strongest coupling the MCM leaded to better results. Lomas and Eppel (1992) signed out that MCM feats better to total sensitivity analysis and that DAM is more capable to handle individual sensibilities. Badar et al. (1993) elected MCM in order to evaluate output uncertainties in heat exchangers and its implementation is richly described.

2. Methods of sensitivity analysis

Both uncertainty and sensitivity analysis have common techniques, but their differences relay in the interpretation of results. Uncertainty analysis searches to evaluate the confidence associated to the results produced by a set of inputs that has its own range of uncertainty. Sensitivity analysis looks forward to identify how a given perturbation in any of the input data will be observed in the output dataset, and predict the main reactions of the system. As the uncertainty of the input dataset can be taken as a kind of perturbation the methodology of analysis of both methods can be taken as similar. The sensitivity methods employed in this work estimates the propagation of uncertainty by different ways, as well as the sensitivity index S_i, witch is the fractional contribution of each input data in respect of the variance of a output data.

2.1. Differential Analysis Method (DAM)

The Differential Analysis Method (ISO GUM 1995) uses the partial derivatives of first order of the outputs in relation to the independent inputs. To a given output Y, expressed as $Y = f(x_1,...,x_n)$, where $x_1,...,x_n$ are the independent input data. The uncertainty of Y is given by u_Y,

$$u_Y^2 = \sum_{i=1}^{n}\left(\frac{\partial Y}{\partial x_i}\right)^2 u_i^2 \tag{1}$$

where the uncertainty u_i is associated to x_i. If u_i is the standard deviation of a given quantity, then u_i^2 is the variance and the terms in the summation are the contribution of each individual input to the variance of Y, u_Y^2. The correspondent sensitivity index S_i is obtained by dividing Eq.1 by u_Y^2, as follows

$$S_i = \left(\frac{\partial Y}{\partial x_i}\right)^2 \frac{u_i^2}{u_Y^2} \tag{2}$$

This equation displays the fractional contribution of each input to the total uncertainty of Y. The important is this fraction, the biggest is the sensitivity of the system in respect to this parameter.

The error propagation equation (Eq. 2) is obtained by the expansion in Taylor series of the function Y, and the use of first order derivatives can lead to the linearization of the model, as showed by Herrador and González (2004).

2.2. Monte Carlo Method (MCM)

Based in multiple evaluations of a given model by a modified set of input data (Badar et al., 1993). To every individual run, a new input dataset is assembled by modifying each prescribed input by adding or subtracting random quantities that lie in its range of uncertainty. Results are analyzed by means of their medium values, standard deviation and associated probabilities. It can be seen as a method of combination of probability distributions. The system evaluation can be performed throughout a simultaneous variation of the complete input dataset, in order to analyze the total uncertainty of the system, or separately to each input data or subset of them, seeking for individual uncertainty.

The sensitivity index S_i is defined as

$$S_i = \frac{V_{Y|x_i}}{V_Y} \tag{3}$$

where V_Y is the variance of Y related to the total uncertainty. $V_{Y|x_i}$ is the variance of Y related to the uncertainty of x_i, that can be an individual data or a set of them with a common relation.

One clear disadvantage of this method relays on its computational effort, once it is necessary to run a great deal of simulations to build either total or individual uncertainties. In the other hand, it is an easy method to assemble, because there is no need of adding new functions to the original model.

2.3. Fourier Transform Method (FTM)

The Fourier transform is proposed in this work as an adaptation of the Monte Carlo Method. The main difference between them is that the input dataset is generated by a sinusoidal sequence in the place of a random one. To each input data an arbitrary frequency ω is proposed, like

$$x_{i,j} = \bar{x}_i + \sigma_i \sqrt{2}\,\mathrm{sen}(\omega_i t_j) \qquad \text{to } i=1,...,n \text{ and } j=1,...,k \tag{4}$$

where index i is associated to input data and index j to the evaluation of the model, $\bar{x}_i$, σ_i and ω_i are the medium value, the standard deviation and the characteristic frequency of the input data, and t_j is the time of a given evaluation of the system. The evaluation of the system is performed by varying simultaneously the entire input dataset. After a given number of evaluations of the system, a Fourier transform is applied to the output data, where some picks can be noticed related to the input data. Coefficients $A(\omega_i)$ and $B(\omega_i)$ of the Fourier transform, relative to each frequency ω_i, are used to calculate the picks amplitude and then the sensitivity index, as follows

$$S_i = \frac{A(\omega_i)^2 + B(\omega_i)^2}{\sum_{i=1}^{n}\left[A(\omega_i)^2 + B(\omega_i)^2\right]} \qquad S_i \tag{5}$$

The number of evaluations of the system is determinate by the Nyquist theorem.

3. Numerical model

A particular thermo electrical power plant, with reheating and regeneration, based in a real plant ("President Medice power plant - Fase B", Candiota, RS, Brazil) is the thermal system to be modeled. Its schematic diagram is presented in Fig. 1.

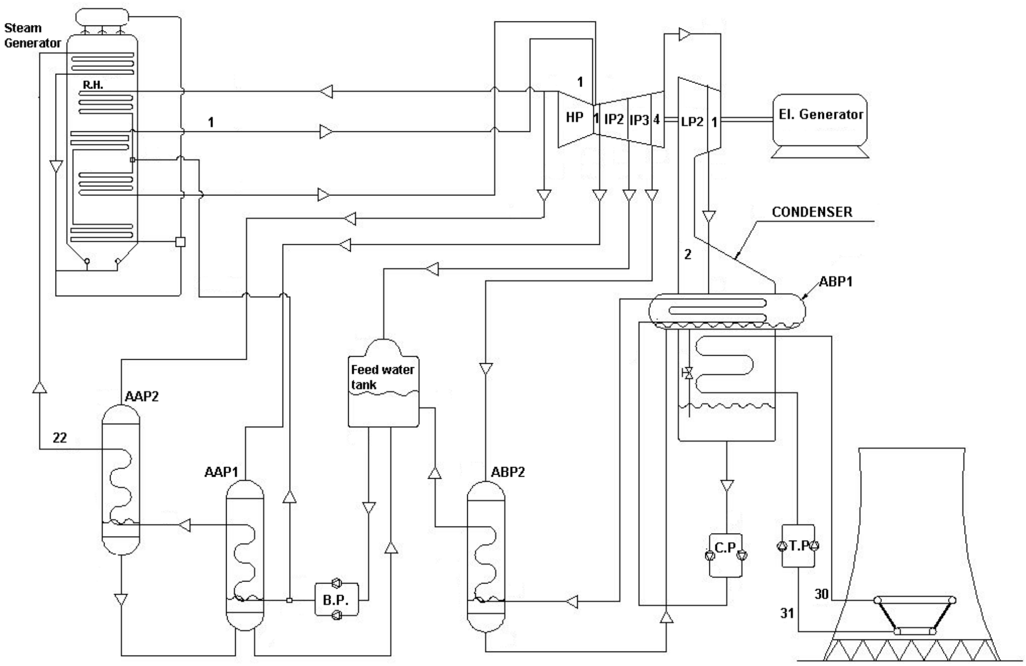

Figure 1- Schematic diagram of the modeled thermo electric power plant [UFRGS-CGTEE-ANEEL, 2004]..

3.1. Turbine

The total power delivered by the turbine W [kW] is the summation of its 7 stages contribution

$$W = \sum_{k=1}^{7} \dot{m}_k \left(h_{i,k} - h_{o,k}\right) \tag{6}$$

where $\dot{m}$ is the steam mass flow rate [kg/s] and h is its specific enthalpy [kJ/kg]. The under indices i and o are related to the input and output streams. The output specific enthalpy is calculated with the aid of the isentropic efficiency η of every turbine stage (Eq. 7).

$$\eta = \frac{h_i - h_o}{h_i - h_{s,o}} \tag{7}$$

where $h_{s,o}$ is the steam specific enthalpy through an isentropic expansion. The isentropic efficiency η varies according to the turbine load, and he curves of η x W for high (HP), intermediate (IP) and low pressure (LP) stages of a hypothetical machine are fitted by polynomials (Fig. 2).

In order to simulate off design conditions, an additional equation should be added to calculate some important relations of the turbine. Scheglиáiev (1978) developed expressions relating the mass flow $\dot{m}$ [kg/s], pressure p [kPa] and temperature T [K] of the steam flow, based on throttle analogy (Eq. 8)

$$\frac{\dot{m}}{\dot{m}_D} = \sqrt{\frac{\left(P_1^2 - P_2^2\right) - \sigma\left(P_1 - P_2\right)^2}{\left(P_{1D}^2 - P_{2D}^2\right) - \sigma\left(P_{1D} - P_{2D}\right)^2}} \sqrt{\frac{T_{1D}}{T_1}} \tag{8}$$

where the index D indicates design conditions and 1 and 2 are referred to steam input and output of a given stage. The σ factor is a dimensionless number related to the critical pressure ratio, as follows.

$$\sigma = \frac{\varepsilon_0}{1-\varepsilon_0} \quad \text{and} \quad \varepsilon_0 \text{ is defined as } \varepsilon_0 = \left(\frac{2}{k+1}\right)^{\frac{k}{k+1}} \tag{9}$$

where k is the isentropic coefficient of the process. Its value to water steam is 0.47822.

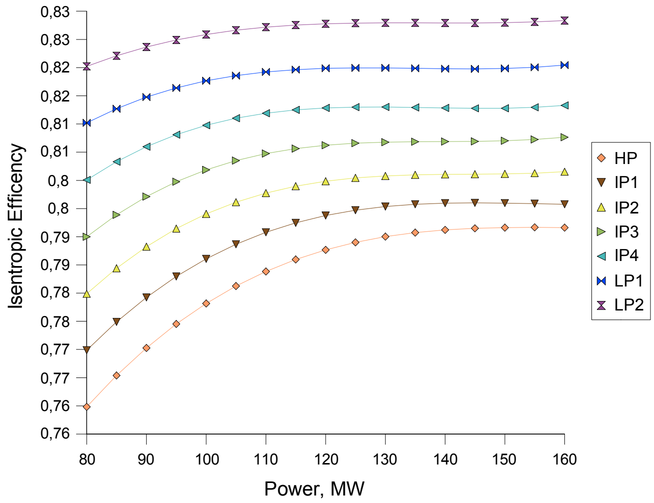

Figure 2- Isentropic efficiencies of turbine stages to different loads.

3.2. Condenser and preheater

Stoecker (1989) presents the following formulation to describe the heat exchange in a condenser

$$T_o = T_i + \left(T_c - T_i\right)\left(1 - e^{\frac{-UA}{\dot{m}c_p}}\right) \tag{11}$$

where T_o and T_i are the input and output temperatures of the cooling water [°C], T_c is the temperature of condensed water [°C], UA is the global heat transfer coefficient [kW/K] and c_p is the specific heat of water [kJ/kgK].

The heat exchanged by preheaters Q [kW] is expressed like

$$Q = U\,A\Delta T_{lm} \tag{12}$$

where ΔT_{lm} is the logarithmic mean temperature, given as

$$\Delta T_{lm} = \frac{\left(T_{h,i} - T_{c,o}\right) - \left(T_{h,o} - T_{c,i}\right)}{\ln\frac{\left(T_{h,i} - T_{c,o}\right)}{\left(T_{h,o} - T_{c,i}\right)}} \tag{13}$$

Indices h and c are related to the hot and cold sides of the fluid. Heat exchanged by the cooling water in the condenser or condensed in the preheaters is expressed as

$$Q_{PH,l} = \dot{m}_l \, c_p \left(T_o - T_i\right) \tag{14}$$

and for the steam side

$$Q_{PH,g} = \dot{m}_g \left(h_i - h_o\right) \tag{15}$$

The pressure drop ΔP [kPa] in the liquid side of the preheaters is given by (Stoecker, 1989)

$$\Delta P = kc \, \dot{m}^2 \tag{16}$$

where Kc is a flow coefficient [kPa kg^2/ s^2].

3.3. Pumps

Pump power W_P [kW] is defined as

$$W_P = \frac{\dot{m}\left(P_o - P_i\right)}{\rho \eta_E \eta_M} = \frac{\dot{m}\left(h_o - h_i\right)}{\eta_E \eta_M} \tag{17}$$

where P_o and P_i are the pressures in the outlet and the inlet of the machine [kPa], ρ is the water density [kg/m^3], η_E and η_M are the electric and mechanical efficiency of the pump, respectively. Change in temperature due to the pump work W is also expressed in the same equation, where h_o and h_i are the outlet and inlet specific enthalpies of water.

3.4. Steam generator, feed water tank, cooling tower and thermal efficiency

The steam generator is composed by boiling and superheating circuits, and also reheating. Heat exchanged by both equipments Q_{SG} and Q_{RH} [kW] of the whole set is given by

$$Q_{SG} = Q_{RH} = \frac{\dot{m}\left(h_o - h_i\right)}{\eta} \tag{19}$$

Feed water tank is modeled as a mixer, where

$$W_P = \frac{\dot{m}\left(h_o - h_i\right)}{\eta_E \eta_M} \tag{20}$$

The thermal efficiency η_{TH} of the cycle is calculated as

$$\eta_{TH} = \frac{W - \sum W_{PUMPS}}{Q_{SG} + Q_{RH}} \tag{21}$$

3.5. Input data uncertainty and implementation

Table 1, shows the most relevant parameters of this power plant, followed by their mean values and standard deviation. Mean values are close to operational data of the plant, and the probability distribution was assumed to be normal to all. Standard deviation was estimated 1% of the mean value divided by 2.58, giving a confidence interval of 99%. Data with common sense where got together in order to simplify the analysis.

In MCM, the estimated precision of the variance of any result is totally dependent on the sample size, witch can be approximately taken by the choice of an approximate confidence interval (Kreyszig, 1999). In the present work, samples generated to MCM have 12,000 points. In FTM, according to the Nyquist theorem, only 2,900 points are sufficient. All routines where built in Fortran 90 with IMSL libraries. Thermodynamic properties of water where computed following IAPWS-IF97 formulation (Wagner et al, 2000) with subroutines in Fortran 90 from Fonseca and Schneider (2004).

Table 1- Mean values and standard deviation of the input data (indices are referred to the plant in figure 1)

	Unit	***Mean value***	
Power	MW	80.0	160.0
T_1	°C	530.0	
P_2	kPa	0.02	
T_{31}	°C	20.0	
Turbine Eff.		80.0 MW	160.0 MW
η_{HP}		0.76	0.79
η_{IP1}		0.77	0.80
η_{IP2}		0.78	0.80
η_{IP3}		0.79	0.81
η_{IP4}		0.80	0.81
η_{LP1}		0.81	0.82
η_{LP2}		0.82	0.83
Pump Eff.			
η_E B.C		0.85	0.3295e-2
η_M B.C		0.60	0.2326e-2
η_E B.P		0.85	0.3295e-2
η_M B.P		0.64	0.2481e-2
η_E B.T		0.80	0.3101e-2
η_M B.T		0.64	0.2326e-2
SG Eff.			
η_{SG}		0.80	0.3101e-2
η_{RH}		0.80	0.3101e-2
UA			
UA_{Cond}	kW/K	25000	25000
UA_{ABP1}	kW/K	1108	1704
UA_{ABP2}	kW/K	2000	3071
UA_{AAP1}	kW/K	1322	1881
UA_{AAP2}	kW/K	1209	1713
Pressure drop			
Kc_{RH}	kPa s^2/kg^2	9.735e-6	
Kc_{ABP1}	kPa s^2/kg^2	8.02e-6	
Kc_{ABP2}	kPa s^2/kg^2	11.36e-6	
Kc_{AAP1}	kPa s^2/kg^2	4.42e-6	
Kc_{AAP2}	kPa s^2/kg^2	4.55e-6	

4. Results

Figure 3 displays the results to the power plant of Figure 1, obtained by means of the three methods of sensitivity analysis, always for design load. Bars indicate the fractional composition of the variance in percents of these results. The inspection of Figure 3 leads to the conclusion that the outputs from all the three methods are very close.

First bar from the left shows the relative variance of the thermal efficiency (Thermal Eff.), where the parameter steam generator efficiency (SG Eff.) has a sensitivity index of 0.81. This indicates its importance in the overall analysis. Even if the steam inlet temperature T1 determinates the cycle efficiency, to a given scenario, the most important sensitivity is the one from the parameter efficiency of the steam generator. Calculated steam inlet temperature T2 in the hot side and water outlet temperature T30 in the cold side of the condenser are strongly coupled to the parameter steam inlet pressure P2. Its important to notice that the calculated mass flow rate of water in the condenser m31 depends on the parameter outside air temperature T31. Whenever this temperature changes, following the expected climatic range of the site, its expected to observe an important variation of m31.

Concerning the pump power of the cooling water circuit (W_TP), the Differential Method leads to different results in respect to the other two methods. As the Differential Method takes into account only the first derivatives of any functional relation, it can be less accurate when applied to non linear equations. Results from the Fourier Transform Method proposed here are very close to those of the Monte Carlo Method. Its advantage in regard to the Monte Carlo Method is that the first one needs a smaller sample and both total and individual uncertainties of every data are determined in a single run.

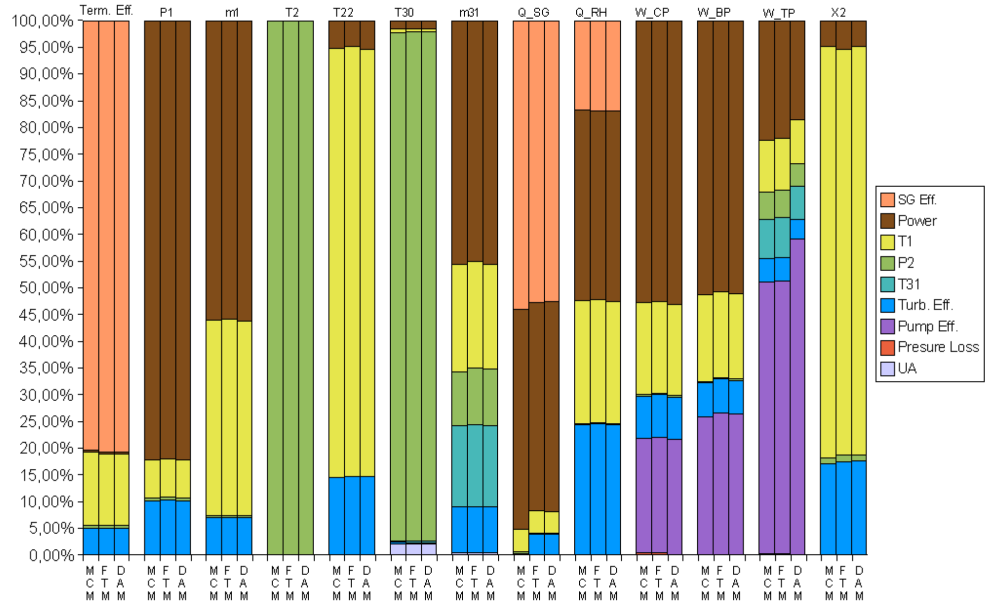

Figure 3- Fractional sensitivities for the three methods in regard to the main parameters of the system

Figure 4 displays the sensitivity analysis to design load conditions and 50% partial load, using Fourier Transform Method. It can be noticed that the sensitivity index of the parameter steam inlet temperature T1 in the turbine is higher to partial load. In this situation, steam outlet state goes from saturated to superheated, and the uncertainty associated to T1 becomes more important.

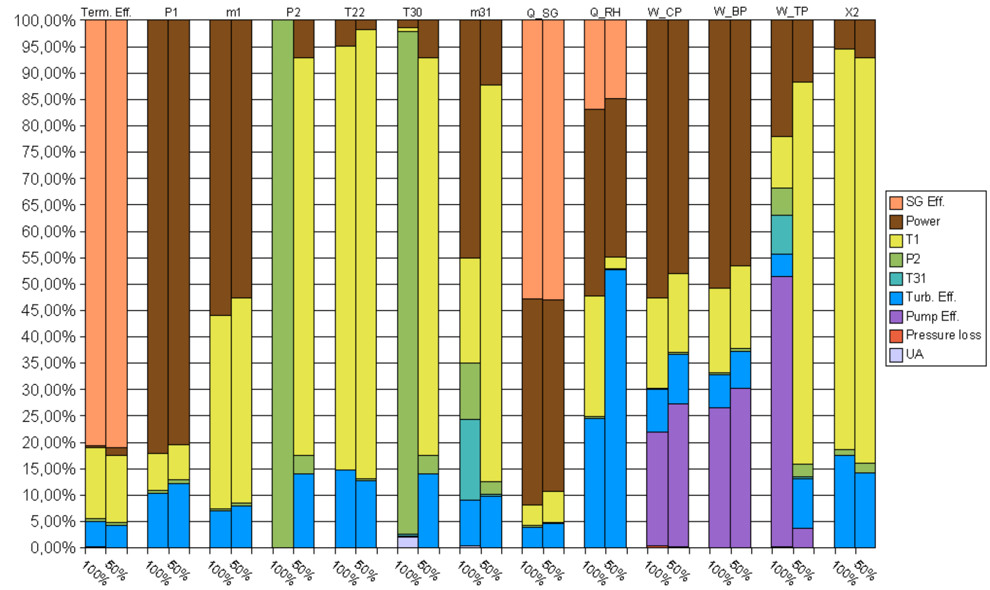

Figure 4- Fractional composition of variance obtained by FTM for design and partial load.

The same tendency is observed concerning the parameter turbine efficiency (Turb. Eff.) in partial load. It can be seen in Figure 2 that the derivative of this efficiency is higher towards smaller loads, affecting its variance (Eq. 2).

Cooling water temperature T31 is not anymore an important parameter to any output quantity in partial load, although it is an important parameter in design load operation.

To all scenarios, pressure drop in the preheaters has a non-important sensitivity index. This means that a more detailed modeling of this phenomenon can be avoided.

5. Conclusions

This paper shows the use of three different methods to estimate output data sensitivity, Differential Method, Monte Carlo Method and Fourier Transform Method, applied to a thermodynamic model of a thermo electrical power plant. The goal is to identify witch input data have more influence in the results of the model. All methods employ the sensitivity index, where the fractional composition of the variance of a given input data is observed in the output data.

First analysis where performed with all three methods at the same time, to a design condition plant operation. Most of the results present very close sensitivity indices. Biases were detected in respect to the Differential Method, once it only takes into account first derivatives for all relations. It can be stated that thermal efficiencies of many devices and turbine inlet steam temperature are the most important data to be collected.

Fourier Transform Method, as it was employed in this work, seems to be a valid method, and as accurate as some other well known ones.

Design and partial load conditions were compared using the Fourier Transform Method, and for the former condition the sensitivity index for turbine inlet steam temperature and its efficiency were more important than in design operation.

6. Acknowledgment

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7. References

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