**AN EXPERIMENTAL ANALYSIS OF SANDWICH COMPOSITES WITH FUNCTIONALLY GRADED CORE UNDER TRANSVERSE LOADING**

**Antonio F. Avila**  
Universidade Federal de Minas Gerais, Department of Mechanical Engineering, NanoComposites Laboratory, 6627 Antonio Carlos Avenue, 31270-901 Belo Horizonte, MG, Brazil  
aavila@netuno.lcc.ufmg.br

**Paulo César M. Rodrigues**  
Universidade Federal de Minas Gerais, Department of Mechanical Engineering, 6627 Antonio Carlos Avenue, 31270-901 Belo Horizonte, MG, Brazil  
paulocmr@demec.ufmg.br

**Ewerton A. S. Nogueira**  
Universidade Federal de Minas Gerais, Graduate Studies Program in Mechanical Engineering, 6627 Antonio Carlos Avenue, 31270-901 Belo Horizonte, MG, Brazil

**Abstract.** An experimental study on piece-wise functionally graded (PWFG) core of sandwich beams is performed considering the ASTM C 393-00 standard. A reference configuration is compared against two PWFG core groups, with densities varying from 150 to 300 kg/m³ in four steps, from upper to lower face-sheet and vice-versa. The upper to lower and lower to upper PWFG sandwich beams presented an average peak load 137% and 192% higher than the reference group, respectively. In both cases, it was noticed that the shear rigidity behavior is represented by an oscillating near asymptotic function associated to local densification/expansion phenomenon. The upper to lower PWFG core seems to be more adequate to bending loadings due to the compressive stress field present into upper face-sheet region. This configuration also leads to higher residual shear rigidity, around 5000 MPa.mm² for the present investigation, which leads to a rising slope for the force versus displacement diagram, and consequently, virtually eliminates the “plateau effect” present in conventional sandwich structures.

**Keywords:** Sandwich Composites, Beam, Functionally graded materials, Experimental analysis

1. Introduction

Functionally graded materials (FGM) are built in such manner that its microstructures vary gradually with location within the material. As a consequence of this microstructure non-uniform distribution the composite’s overall properties are not constant, in other words, its properties changes with from point to point inside the composite. This new concept of engineering material created in early 90’s allows designers to seek new applications not only in aerospace industry but also in automotive and biomedical fields. According to Chakraborty et al. (2003), due to their continuous variation of properties FGMs have gained large applicability as thermal-barriers structures and locations where corrosion is an issue rather than bonding dissimilar materials. To be able to better design mechanical components using FGMs an accurate mathematical modeling is required. Aboudi and Pindera (1995) are among those pioneers who had developed models to predict mechanical and thermal properties of FGM composites. The micromechanical model developed by Aboudi (1991) for laminated composites was extended by Aboudi and Pindera (1995) for FGM. In their case, the method of cells (Aboudi, 1991) was associated to a previously known fiber distribution through the material’s thickness. Moreover, by applying the classical plate theory (CPT), they were able to predict the mechanical properties with reasonable accuracy. By replacing the CPT by the higher order plate theory, Aboudi et al. (1999) not only improved their model but also they were able to incorporate to the mechanical analysis the thermal components.

An elasticity solution for simply supported FGM beams subjected to transverse loadings was proposed by Sankar (2001). In his solution, an exponential function was assumed to describe the beam’s stiffness variation through the thickness. Moreover, Sankar’s model was limited to long beams with slowly varying transverse loadings. A more comprehensive model was proposed by Reddy and Cheng (2001) where the asymptotic solution to uncoupled heat conduction problem was employed. According to them, the assumption of constant deflection of a rectangular FGM plate through the thickness is not valid for thermal loading, but it could be applied to mechanical loadings. Their model, however, can not be applied to FGMs where points of singularities are present, e.g. cracks. For this case, a finite element model (FEM) was developed by Kim and Paulino (2002), where a displacement correction technique and a modification of crack closure model was proposed for FGMs. The FEM implementations using those techniques lead to accurate stress intensity factors (SIF) with cracks either aligned with the principal directions or arbitrarily oriented. In any case, the concept of functionally graded materials brought an increase on composite’s performance.

The FGM concept was also applied to sandwich composites, where core is the functionally graded material. Apetre et al. (2003) extended the elasticity solution developed by Sankar (2001) to sandwich beams with functionally graded core. The main difference from Apetre’s work and the one developed by Sankar is the function that describes the stiffness variation through the core thickness, which in Apetre’s case was a polynomial expression. Apetre’s and Sankar’s conclusions were that by varying the core stiffness the core/face-sheet interface shear stress is reduced.
Anderson (2003) went further by developing an elasticity solution for FGM sandwich plates under low velocity impact loadings. He also noticed a decrease in shear stresses at core/facesheet interface generated during the impact loading when the core is functionally graded. Moreover, according to Chakraborty et al. (2003), cracks are more likely to initiate at interfaces and propagate into weaker material. Although the parametric study performed by Anderson (2003) showed the effects of graded core in stress and displacement fields, no comparison against experimental data was made. Kirugulige et al. (2005), however, performed not only an experimental investigation on sandwich structures under impact loading using high speed photography and interferometry, but also a numerical modeling was carried out in a parametric study. They showed a decrease of stress intensity factors was obtained by using a FGM core. Although the study accomplished by Kirugulige et al. (2005) lead important results, their conclusions were based on a sandwich structure with a bilinear variation of mechanical properties.

Although much work has been done by many researchers none of them performed an experimental investigation with sandwich structures with core where large variations of mechanical properties are observed. This paper is concerned with the mechanical behavior of sandwich beams with functionally graded core with step-wise density and modulus of elasticity variation.

2. Material Processing and Characterization

The sandwich beam dimensions followed the proportions suggested by the ASTM C393 standard (2000). The facesheets are made of a 6 layer woven fabric fiber glass/epoxy composites with 50% of fiber volume fraction. The epoxy formulation is based on two parts, part A (diglycidyl ether of bisphenol A) and part B - hardener aliphatic amine 
(triethylenetetramine). The weight mixing ratio suggested by the manufacturer is 100A:20B, and the average viscosity is around 900 centipoise (Hunstman, 2002), while fibers have a plain weave (Texiglass, 2001) configuration and gramature of 200 g/m². Moreover, when wet hand lay-up procedure is applied, an elastic modulus of around 11.0 GPa is obtained. The core is made of a polystyrene foam, Styrofoam™, from Dow Chemical (2000). However, its original density was changed by compression/densification. The FGM core is made of four polystyrene foam layers with distinct densities.

Three sandwich beams are prepared, i.e. conventional, FGM with density increasing from bottom to top, and another FGM sandwich beam with core density increasing from top to bottom. All beams are 410 mm long, 50.4 mm wide and with total thickness of 18 mm. The face-sheets have thickness of 1.5 mm each, while the core total thickness is 15 mm, as it is shown in Fig. 1. The conventional core is made of Styrofoam with density of 58.41 kg/m³, \( E \) equals to 1.49 MPa and \( G \) of 0.59 MPa. The elastic moduli for each FGM foam layer are based on equations developed by Gibson and Ashby (1997) for close cells and they are shown in Figs. 2A-2B.

![Figure 1. Sandwich beam main dimensions](image)

It is important to be aware of the compression/densification procedure generates a complete distinct core microstructure for each core layer, as it can be seen in Figures 3A-3B. Each core layer is bonded to the next one using a thin flexible adhesive, i.e. Araldite AW106/HV-953U from Huntsman Corporation (2003). The flexible adhesive was selected due to not only its capacity of load transfer but also its ability of accepting small deformations without damage the foams.

The three point bending tests are performed in an INSTRON 4482 universal testing machine and the strain rate applied follows the ASTM C393 standard (2000). The reference configuration will be called type #1, while the sandwich composite with large core density at the region close to the lower face-sheet will be named type #2, and type #3 will be referred to the core with large density close to the upper face-sheet. These three configurations are selected following the study performed by Tsotra and Friedrich (2003) whom also performed a three bending test. However, their experiments were done in carbon fibers functionally graded composites and not sandwich composites.
3. Results and Discussion

Following the ASTM C 393 standard, a set of at least five specimens for each configuration was prepared and tested. Figure 4 shows a typical force versus displacement curve for each of the three configurations studied. As it can be observed, the two FGM configurations lead to a better performance. Moreover, the type #3 configuration presented a
maximum loading 15.53% higher than the type #2 and 189.50% higher than the reference configuration (type 1). A much more interesting result is observed when the specimen’s failure is observed.

A failure at the upper face-sheet is noticed during the three bending tests of type #1 specimens, as it can be seen in Fig. 5A. This failure is clearly indicated by the peak force, \( \approx 240 \text{ N} \), and after this point it seems that a foam densification is observed. An evidence of this event is the load plateau from 20 to 60 mm of displacement. Another interesting fact is the spring back phenomenon detected, in other words, right after a sudden load drop a small increase on load is perceived. Figure 5B shows the spring back formation. According to Vinson (1999), this behavior is expected into conventional sandwich with foam core. However, when the two others configurations are observed an unexpected failure mechanism is observed.

The failure mechanism observed during the three points bending tests of type #2 specimens can be described as a combination of two simultaneous events. The first one is the upper face-sheet partial failure, and the second one is the foam non-linear densification. As the laminate failure progresses, the foams begin to experiment the densification process. However, this densification is directly proportional to the applied load and inversely to the distance. It is possible to assume the existence of a gradient on foam densification as the result of the applied load. This hypothesis can be demonstrated by Figures 6A through 6C and the plateau shown in Figure 4. Moreover, it is possible to note a densification around the applied load region, as the volume must remains constant, an expansion occurs at regions far from the loading, as it can demonstrated in Figure 6C. This is more evident at small density foam which for the type #2 specimens is the closest to the upper face-sheet.
When the type #3 specimens are analyzed some conclusions could be drawn. The higher loadings obtained could be due to a more favorable stiffness variation through the thickness. Experimental evidences of such better stiffness distribution can be confirmed by the fact that no laminate failure was noticed and the uniform densification region becomes larger than the one from type #2 specimens, as it is shown in Fig. 7A. However, as the sandwich cores are only continuous by parts, a displacement discontinuity is observed at the interfaces. As a consequence, of this non uniform stress distribution a series of cracks was created. These cracks shown in Fig. 7B could be the reason for functionally graded sandwich failure.

To understand the how the core density variation affects the overall beam behavior under bending, the issue of flexural and shear rigidities recalled by Cunningham and White (2001) must be addressed. According to them, the total center deflection in a three-point bending test can be described by two components, i.e. bending or primary partial deflection and the shear or secondary partial deflection. Moreover, they are expressed by the following equation:
\[ \Delta = \frac{P l^3}{48 \hat{E} I} + \frac{P l}{4S} \]  

(1)

where \( P \) is the applied load, \( l \) is defined as the distance between supports, \( \Delta \) represents the displacement at beam’s center, while \( \hat{S} \) and \( \hat{E}I \) are the shear and flexural rigidity, respectively.

Moreover, the total flexural rigidity is the sum of face and core rigidities. However, when the FGM core configuration is used the mass center is no longer coincident with the beam center line with respect to the thickness. In this case, the theorem of parallel axis (Zenkert, 1997) must be used. In this study, the two face-sheets have the same thickness and mechanical properties. Therefore, following expression can be written:

\[ \hat{E}I = E_f \left( \frac{b t_f^3}{12} + (b t) d_t^2 \right) + E_i \left( \frac{b t_i^3}{12} + (b t) d_i^2 \right) + \sum E_i \left( \frac{b t_i^3}{12} + (b t_i) d_i^2 \right) \]  

(2)

where the face-sheet and core elasticity moduli are represented by \( E_f \) and \( E_i \), respectively. The beam’s width is defined by \( b \), while the face-sheet and core thicknesses are given by \( t \) and \( t_i \), respectively. Furthermore, the distance between the beam’s center of mass and the center of mass of each face-sheet, bottom and top locations, and each core layer are represented in that order by \( d_b, d_t, \) and \( d_i \).

As said by Cunningham and White (2001), if the weak core and thin face-sheet condition are assumed, the dominant term in equation (2) is the one from face-sheet axial translation. In the present study, this term is two orders of magnitude higher than the two others. Therefore, the only significant contribution of the FGM core configuration has to be into shear rigidity component.

Figures 8-10 show the shear rigidity behavior as a function of the central displacement value. For the reference configuration (type #1 sandwich) the shear rigidity is negative, which leads to a decrease on peak force and overall bending resistance. However, in Fig 8, it is observed an abrupt change in \( \hat{S} \) due to the spring back effect. When the second configuration is studied some conclusion can be drawn. First, \( \hat{S} \) is two orders of magnitude higher than type #1 group. Additionally, a positive component is observed, which can be related to the overall increase on bending performance, i.e. higher collapse loadings. Figure 9A show \( \hat{S} \) as a function of displacement during the entire three-point bending test. As it can be observed, there is a sharp variation during the initial loadings. These variations can be attributed to the different behavior of each foam-layer, i.e. the compression of the small density foam near the upper face-sheet can lead to the shear rigidity. As the load is increasing the upper foam region collapse at same time that the other foam-layers become to experiment an increase on density around the load region and an expansion far from there. This dynamic phenomenon proceeds up to moment when a near asymptotic convergence around 40 mm displacement value is observed. The near asymptotic convergence is coincident to the plateau (foam densification phenomenon) noticed in Fig 4. In fact, there is an oscillation around a residual \( \hat{S} \) value of 1000 MPa.mm\(^2\), as shown in Fig 9B, that can be explained by the collapse and expansion of cells during the densification procedure.

![Figure 8. Shear rigidity variation for type #1 group](image-url)
The advantage of the FGM core configuration is noticed when the type #3 group is analyzed. The shear rigidity is around three orders of magnitude higher than type #1 configuration and close to 5 times higher than the type #2 group. As a consequence the average peak load for the type #3 sandwich structures is around 690 N while for the type #1 and #2 configurations this value is around 236 N and 563 N, respectively. The good performance can be explained not only by the shear rigidity itself but also by its distribution. In a classical three-point bending test the upper region is under compression, while the lower region is under tensile loadings. A core with larger density close to the upper region leads to a more resistant region to compression. Meanwhile, the tensile loadings at region close to the lower region is mostly carried by the face-sheet and only a small portion of foam close to adhesive interface is under the tensile loading influence. As it can be observed in Figs 10A and 10B, a sensible shear rigidity variation is experimented during the initial loadings. However, the $\bar{S}$ negative value can be attributed to the local densification process, which is more intense at the lower region (small density foams). The increase on shear rigidity is consequence of the continuous increase on loading, which results in step-wise foam densification, as it can be observed in Fig10B. As the result of this local densification an expansion into the regions far from the loading region is experimented. Again a near asymptotic behavior but with oscillation around 5000 MPa.mm² is observed. On the other hand, the force x displacement curve still has a rising slope, as it can be seen in Figure 4, which can be an indication of a “saturation” phenomenon. In other words, additionally to the near asymptotic oscillating (densification/expansion) an increase on density of the upper foam is observed due to the compressive fields into this region. When the foam saturation limit is reached cracks are generated and disseminated.
4. Closing Comments

An experimental investigation on sandwich beams with piece-wise functionally graded core was performed and compared against a conventional sandwich beam configuration. Both FGM core configurations, i.e. high density at the upper region and low density at upper region, performed better than conventional sandwich. The average peak load for low and high density configurations at upper region is 137% and 192% higher than the reference group. This better performance can be attributed not only to the increase on shear rigidity due to the core piece-wise functionally grade stiffness variation but also its distribution. The high density at upper region seems to be more efficient than the low density at upper region. This could be due to the compressive stress field present into the upper region when the three-point bending test is performed. By selecting higher densities at upper section, the core overall compressive strength is increased, which leads to a better performance. An oscillating near asymptotic shear rigidity variation as a function of central deflection was observed in both cases of FGM core pattern. This oscillating shear rigidity seems to be related to the densification/expansion process, which can lead to cracks and the overall failure. Finally, the piece-wise FGM core for sandwich structures seems to be a capable design.

5. Acknowledgments

The authors would like to acknowledge the financial support provided by the Brazilian Research Council (CNPq) and the Universidade Federal de Minas Gerais (UFMG).

6. References


7. Responsibility notice

The authors are responsible for the printed material included in this paper.