

# **SIMULTANEOUS LOCALIZATION AND MAP BUILDING BY A MOBILE ROBOT USING SONAR SENSORS**

**Leonardo Shiguemi Dinnouti**

Department of Telematics - School of Electrical and Computer Engineering, UNICAMP, Campinas-SP Brazil  
dinnouti@dt.fee.unicamp.br

**Alessandro Corrêa Victorino**

Robotics and Computer Vision Laboratory/CenPRA PO Box 6162, 13081-970, Campinas-SP Brazil  
Alessandro.Victorino@cenpra.gov.br

**Geraldo F. Silveira**

Robotics and Computer Vision Laboratory/CenPRA PO Box 6162, 13081-970, Campinas-SP Brazil  
Geraldo.Silveira@cenpra.gov.br

***Abstract.** This paper presents a methodology to self localization of autonomous mobile robot using two distinct structures of metric maps. Ultrasonic sonar ranger sensors are used to build an occupancy grid, the first structure, and a map based on line segments extracted from the occupancy grid is built. The two maps are used to estimate an correct errors on odometric sensors.*

***Keywords.** Mobile robots, Map building, Automation.*

## **1 Introduction**

Safety navigation is an elementary condition absolutely necessary to operate autonomous vehicles in unknown environments. A mobile robot that safely displaces in an indoor environment must carry some essential capabilities such as the perception of the environment, a reliable localization and maintaining an accurate representation of its workspace (Victorino, 2002). These topics are the subject of this paper.

In many applications of mobile robotics, the localization can be carried out by integrating the odometry data provided by internal sensors like wheel odometers. However, due to the uncertainties on the robot model and the eventual wheels slippage (systematic and occasional errors (Borenstein and Feng, 1996)) the results of this localization procedure does not represent the real position of the robot and uncertainty increases in the integration process without bounds. An interesting idea for compensating these errors is to consider the perception from external sensors mounted on the robot that provides information about the environment where the robot moves. A metric representation (map) of the environment is constructing by the integration of range information delivered by sonar, laser or stereo vision system mounted on the robot and the robot positions are tracked in that representation. The map of the environment and the robot positions are incrementally estimated and updated by the integration of the new observations performed during the robot evolution.

A class of localization methods considers the mapping of the environment and the robot localization simultaneously in a same framework, this methodology is known as "Simultaneous Localization and Map building (SLAM)". The SLAM methods can be classified according to the nature of the metric representation of the environment used: geometric primitives or occupancy grid. In the first approach a model of the environment is construct by describing the geometry of its elements. Geometric primitives like points, line segments and planes are used (Crowley, 1989; Moutarlier and Chatila, 1989; Leonard, 1992; Castellanos et al., 1999). In an occupancy grid representation the environment is modeled by a discrete grid where it is evaluated the probability that a given cell in the grid belongs to an obstacle (Moravec and Elfes, 1985; Elfes, 1987; Konolige, 1997).

A probabilistic methodology is essential to consider the uncertainty either related to the model of the robot motion or in the sensor observation. An uncertainty model related to the observation of the environment performed by a mobile robot equipped with sonars was derived by (Crowley, 1989). In (Leonard, 1992) a localization and mapping algorithm is presented based on the fusion between the environment observation and the odometric data performed by an extended Kalman filter (EKF) where there is no correlation on the uncertainty models. The estimation algorithm is simple to be implemented, but the non-correlation hypothesis between the odometry errors and the environment observation errors introduces a bias on the estimation results. A consistent representation of the uncertainty propagation between different frames is developed in (Smith and Cheesman, 1986). A more robust version of the EKF is presented in (Moutarlier and Chatila, 1989) where the different correlation between the dynamic elements of the system are considered, with the notion of the uncertainty propagation between frames. The improvements added on the quality of the estimation results

are related to the degradation on the execution time.

In this context, this paper presents the theoretical formulation and results of the application of an simultaneous localization and map building algorithm for a mobile robot that carries a set of sonar sensors. The method is based on the fusion between the environment observation and the odometric data. Differently from the cited previous works, an hybrid metric representation of the environment is proposed. Two models, occupancy grid and line segment map, are constructed simultaneously during the robot evolution. A local occupancy grid, which is well adapted to integrate the sonar readings, is constructed at the localization points providing the local observations to the construction of the geometrical model. The geometrical model, which is incrementally built, is used to localize the robot. The positions of the robot during its navigation are estimated with the application of an EKF algorithm considering this hybrid model of the environment.

The paper is organized as follows. In the section 2, it is presented the modeling of the environment measurement used on the localization and mapping process. The measurement of the environment is provided by sonar sensors mounted on the robot. It is introduced the concept of the local occupancy grid that is an extension of the sonar model. A local geometric map, which is a set of line segments, is extracted from the local occupancy grid by performing a weighted Hough transform. The localization and mapping algorithm is presented in section 3. It consists of three steps: prediction of the states, the estimation phase and the update of the environment map. Results are presented and commented in the section 4. Some conclusions and comments are presented in the section 5 followed by the bibliography references.

## 2 Environment Modeling: A hybrid representation

The environment model in this paper is based on occupancy grid (Elfes, 1987; Moravec and Elfes, 1985) and a feature based map built from line segments. These models are described in this section.

### 2.1 Occupancy Grid

The occupancy grid is composed by a set of cells that contain a probability value in the  $[0..1]$  interval. The zero value represents a free cell, a one represents a occupied cell. Intermediate values represents different levels of occupancy, in particular, the value 0.5 represents a completely unknown level of occupancy. Each cell is a reference to a region on the Cartesian space around the robot.

#### The sensor model.

The sonar works emitting a signal to a direction and reading the resulting echo. The distance is computed based on the flight time of the acoustic sound wave. The Fig. 1 illustrate a sonar signal hitting a object at a distance  $r$  far from the sensor. The  $s$  distance represents a hypothetical distance to a cell being tested as the responsible to the echo sent back to the sonar sensor. The region before the recorded range  $r$  represents a free space: there is no objects before this range; the region with the same distance as the recorded range has an equal probability to contain the object responsible for the echo, forming an equidistant region from the sensor; the region farther than the recorded range  $r$  is a shadow region that has a unknown occupancy probability associated.

On each sonar reading, all cells on the sonar range are updated using a Bayes rule. In this work the implementation of such update process is subdivided in two phases: a Response Grid update and an Occupancy Grid update, as described in (Howard and Kitchen, 1996).

The Response Grid has the same structure of the occupancy grid, but contain in each cell a value of response probability from that cell. A cell is addressed by a  $(x, y)$  coordinate system and a sensor is represented by a integer  $\phi$ , numbering the reading sensor. The response of a cell given the reading of sensor  $\phi$  is represented by  $R(x, y, \phi) = [response, noresponse]$ . This state variable will be written as  $R_\phi$ , considering the update of just one cell. The response probability is then updated using the Eq. (1):

$$p(R_\phi|r) = \frac{p(r|R_\phi)p(R_\phi)}{p(r)}, \text{ and} \quad (1)$$

$$p(r|R_\phi) = \begin{cases} 0.05 & \text{se } s < r \\ 0.5 & \text{se } s > r \\ \alpha/r & \text{se } s = r \end{cases}, \quad (2)$$

where  $p(r|R_\phi)$ , in the Eq. 1, is the sensor model to a reading value  $r$  in the  $\phi$  direction to a cell in a distance  $s$  from the sensor. The Fig. (1) shows the variables  $r$  and  $s$ .

The Equation (2) has three cases:

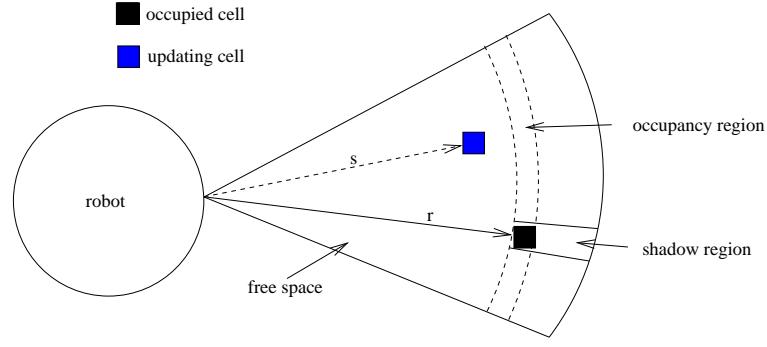


Figure 1: The sensor model.

- if the cell is closer to the sensor than the recorded range, illustrated as free region in Fig. 1, then cell probably does not generate responses in this direction and  $p(r|R_\phi) = 0.05$ ;
- if the cell is farther from the sensor than the recorded range, illustrated as shadow region in Fig. 1, then nothing is known about this cell and  $p(r|R_\phi) = 0.5$ ;
- if the cell is at same distance from the sensor as the recorded range, illustrated as occupancy region in Fig. 1, the cell may generate a response in this direction and  $p(r|R_\phi) = \alpha/r$ , where  $\alpha$  consists of a normalization factor to make the probability sum of the cell updated in one reading to be equal to 1. This value should be represented as  $\alpha = (2 \cdot \tan(\beta) \cdot r)$ , where  $\beta$  is the sonar aperture angle.

A single response probability to one cell, in the direction  $\theta$  is updated by:

$$p(R_\theta) \leftarrow \frac{p(r|R_\theta)p(R_\theta)}{p(r|R_\theta)p(R_\theta) + (1 - p(r|R_\theta))(1 - p(R_\theta))}. \quad (3)$$

Once the  $p(R_\theta)$  is obtained, the occupancy probability is updated in the Eq. (4):

$$p(O) = 1 - \prod_\phi (1 - p(R_\phi)). \quad (4)$$

The Equation (4) updates one cell in every direction  $\theta$  (number of sonar sensors available). The same computation is done for each cell in the sonar range.

The prior probabilities to  $p(R_\phi)$  and  $p(O)$  is given by Eq. (5):

$$\begin{aligned} p(R_\phi) &= 1 - (0.5)^{\frac{1}{n}}, \text{ and} \\ p(O) &= 0.5, \end{aligned} \quad (5)$$

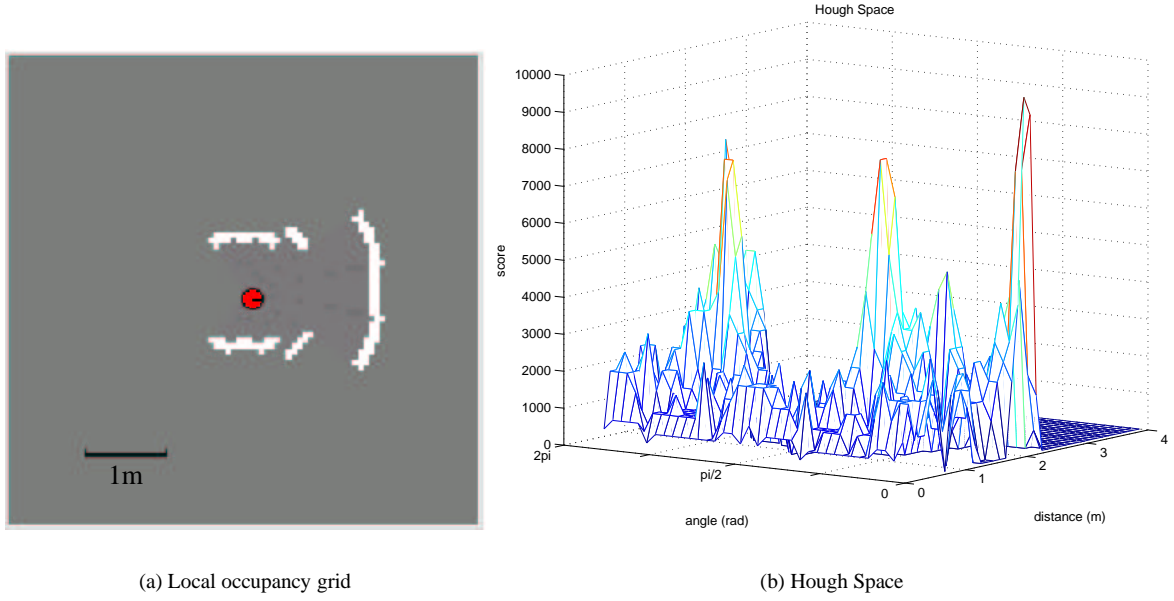
where  $n$  is the number of available sonar sensors. These prior probabilities ensure that  $p(O)$  to a given cell is 0.5 when nothing is known in the environment.

The occupancy grid obtained represents the probability of occupancy on the robot environment relative to its position. To use this construction to a global map is necessary to incorporate the odometric data or a better estimation based on fusion of odometric and geometric data from the robot observations.

## 2.2 Feature based map

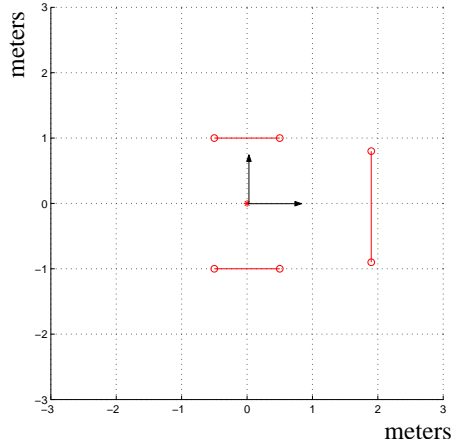
The model is a set of line segments that represent the geometry of objects in the robot free space. This geometry is used to describe the border of free space in a precise way on the localization of the robot. The construction of a geometric model from the sonar readings is explained in Fig. 2. At a given position of the robot, a local occupancy is constructed using the update techniques describe above (Fig. 2(a)). The lines are estimated from this local occupancy grid by applying a weighted Hough transform.

The basics of Hough transform is to create an Hough space where the coordinate system is polar and the space is discrete, displaced in a matrix where the rows address a distance value ( $\rho$ ) and columns an angle value ( $\theta$ ). Each cell in this matrix ( $\rho, \theta$ ) represents a line in Cartesian space. To process the occupancy grid cells in Cartesian space ( $x, y$ ), each



(a) Local occupancy grid

(b) Hough Space



(c) Local segments

Figure 2: Extracting line segments.

coordinate  $(\rho, \theta)$  in the Hough space accumulates the probability value of one cell that satisfies the relation in Eq. (6), that associates a  $(x, y)$  point to a  $(\rho, \theta)$  line.

$$f(x, y, \rho, \theta) = x \cos(\theta) + y \sin(\theta) - \rho = 0. \quad (6)$$

The Figure (2) shows an example obtained from a simulation and its corresponding Hough Space. The three higher peaks represents the best lines that fit the cells in the occupancy grid representing the walls.

There is an uncertainty associated to each line segment considering the distance of points from  $(\rho, \theta)$  lines given by the Hough transform. The covariance of  $(x, y)$  coordinates is computed over the distance from each point to the line in Eq. 7 and the covariance of  $\theta$  angle is computed in Eq. 8 using the inclination of a imaginary segment built over the projection of the farther point in the extremes of the segment. The figure (3) shows this estimated values.

$$\sigma_\rho^2 = \frac{1}{n} \left[ \sum_{i=1}^n (x_i - x_i \cos \theta)^2 - (y_i - y_i \sin \theta)^2 \right], \quad (7)$$

$$\sigma_\theta^2 = \left[ \arctan \left( \frac{\max((x_i - x_i \cos \theta)^2 - (y_i - y_i \sin \theta)^2)}{((x_1 - x_2)^2 - (y_1 - y_2)^2)/2} \right) \right]^2. \quad (8)$$

The feature map is finally represented by set of oct-tuples  $(x_1, y_1, x_2, y_2, \rho, \theta, \sigma_\rho^2, \sigma_\theta^2)$ , two points define a line seg-

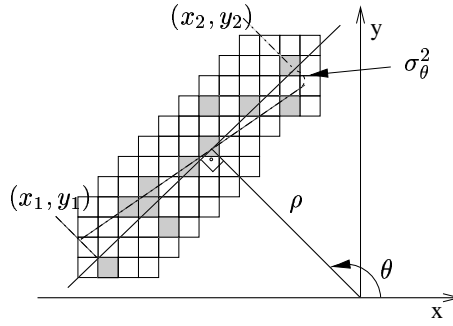


Figure 3: Feature map representation

ment,  $(\rho, \theta)$  a line in Cartesian space and  $(\sigma_\rho^2, \sigma_\theta^2)$  the covariances as illustrated in Fig. (3).

The Section 3 presents the methodology that uses these maps onto building of a global map, based on fusion of local maps.

### 3 Simultaneous Localization and Mapping Building - SLAM

The localization algorithm is based on the fusion between the environment observation delivered by the sonars and the odometry measurements using the extended version of the Kalman filter (Leonard, 1992; Moutarlier and Chatila, 1989). Suppose a relative displacement of the robot between two successive instants  $(k)$  and  $(k + 1)$ . The localization and mapping process, illustrated in figure 4, is composed of the following phases:

- A state prediction phase, where a prediction of the robot position  $(k + 1)$  is computed from the last estimation of the state and the relative displacement of the robot given by odometry.
- An observation prediction phase, where the map of the environment that was already built up-to- $(k)$  is projected into the local robot frame at the position  $(k + 1)$  by using the predicted robot position. A probabilistic correspondence test is then established between the projected global map and the local map constructed at the position  $(k + 1)$ . Two sets are constructed in this correspondence test: the couples of elements from global map and local map that were matched, and a set of elements from the local map which were not matched with the global.
- The estimation phase, where the results of the correspondence test, weighted by the Kalman filter gain, are used to correct the odometry information estimating the position at  $(k + 1)$ .
- The global map that is known at the position  $(k)$  is updated by the integration of the local map estimated at  $(k + 1)$ . The model of the elements in the global map that were reviewed are updated with the fusion on the set of matched couples. The new observation are added to the global map by including the set of unmatched elements.

#### 3.1 The state prediction

The robot displaces from a position  $(k)$ , where the optimal estimate is given by  $X_{k/k} = (x_{k/k} \ y_{k/k} \ \theta_{k/k})^T$  and its  $3 \times 3$  covariance matrix given by  $P_{k/k}$ , to a new position at the instant  $(k + 1)$  with a relative displacement given by odometry  $\Delta X = (\Delta x \ \Delta y \ \Delta \theta)^T$ . The prediction of this new position and its covariance matrix is given by:

$$\begin{aligned} X_{(k+1)/k} &= X_{k/k} + \Delta X, \text{ and} \\ P_{(k+1)/k} &= P_{k/k} + Q_{k+1}, \end{aligned} \quad (9)$$

where  $Q_{k+1}$  is the uncertainty related to the relative displacement  $\Delta X$  and calculated as

$$Q_{k+1} = \alpha \Delta X \begin{bmatrix} \epsilon_x & 0 & 0 \\ 0 & \epsilon_y & 0 \\ 0 & 0 & \epsilon_\theta \end{bmatrix},$$

where  $\alpha$  is a tuning factor and  $\epsilon_x$ ,  $\epsilon_y$  and  $\epsilon_\theta$  are constants values that bounds the uncertainty related to the model of the robot motion. In our case we set  $\epsilon_x = 0.001$ ,  $\epsilon_y = 0.001$  and  $\epsilon_\theta = 0.009$ . Note that the magnitude of the covariance matrix  $Q_{k+1}$  is proportional to the relative displacement  $\Delta X$ .

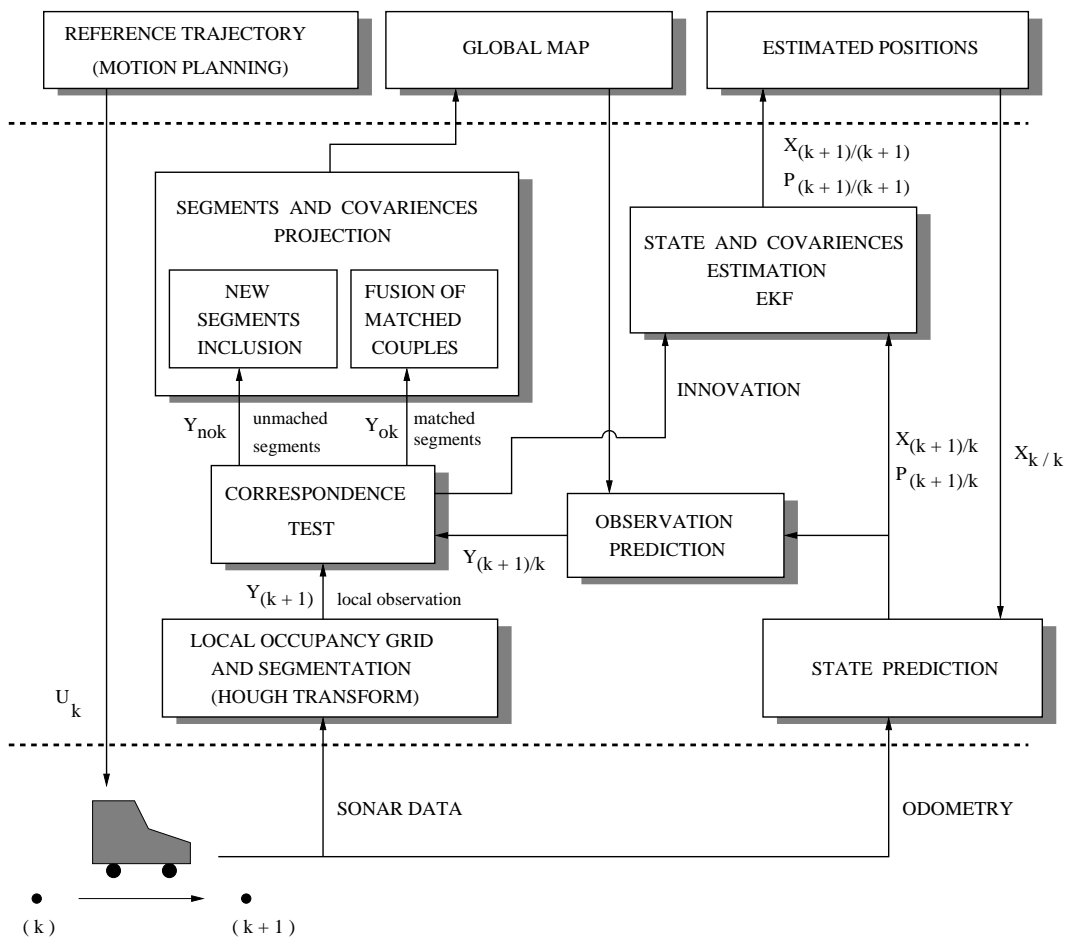


Figure 4: A description of the different parts of the simultaneous localization and map building algorithm.

### 3.2 The observation prediction and the state estimation

When the robot arrives to a new position at  $(k + 1)$ , a new local map is generated. This local map is a set of line segments called  $Y_{(k+1)}$  which consists of line parameters with its associated covariances. An element  $i$  of the observation vector  $Y_{(k+1)}$  is represented as  $l_{(k+1)}^i = (d_{(k+1)}^i, \phi_{(k+1)}^i)$ , where  $(d_{(k+1)}^i, \phi_{(k+1)}^i)$  are line parameters segmented from the sonar data following the techniques described in the section 2, and represented on the robot frame.

Let us consider now the set of objects,  $l_0^i = (d_0^i, \phi_0^i)$ , of the global map represented on the reference frame, which are potentially visible from the predicted position of the robot at  $(k + 1)$ ,  $X_{(k+1)/k}$ . Some objects of the observation vector  $Y_{(k+1)}$  can be matched to objects from the global map (others are viewed for the first time). This correspondence, which will be quantified at a probabilistic level, is used to correct the robot position. The objects already viewed, the global map, are projected to predicted robot frame by the transformation :

$$G(X_{(k+1)/k}, l_0) = \begin{bmatrix} d_0 - (k + 1)/k \cos(\phi_0) - y_{(k+1)/k} \sin(\phi_0) \\ \phi_0 - \theta_{(k+1)/k} \end{bmatrix}. \quad (10)$$

This set of objects is the prediction of the observation and are denoted by  $Y_{(k+1)/k} = G(X_{(k+1)/k}, l_0)$  and the observation equation is rewritten as :

$$Y_{(k+1)} = Y_{(k+1)/k} + E_{(k+1)}, \quad (11)$$

where  $E_{(k+1)}$  is the observation error.

The correspondence between  $Y_{(k+1)}$  and  $Y_{(k+1)/k}$  is given with the application of probabilistic methods (Moutarlier and Chatila, 1989) based on the Mahalanobis distance, there is a correspondence between  $Y_{(k+1)}$  and  $Y_{(k+1)/k}$  if

$$E_{(k+1)} S_{(k+1)/k}^{-1} E_{(k+1)}^T < \epsilon \quad (12)$$

where  $\epsilon$  is a threshold that defines a level of probability and  $S_{(k+1)/k}$  is the covariance associated to the prediction of the observation, given by:

$$S_{(k+1)/k} = \frac{\partial G}{\partial X_{(k+1)/k}} P_{(k+1)/k} \frac{\partial G}{\partial X_{(k+1)/k}}^T + R_{(k+1)}, \quad (13)$$

where  $R_{(k+1)}$  is the covariance matrix associated to  $Y_{(k+1)}$ .

The matched couples are considered in the update process of the global map, however only those with a given reliability, given by the frequency that they were viewed during the navigation, are considered to estimate the robot positions; the prediction error related to this subset is denoted by  $E'_{(k+1)}$ .

The application of the extended Kalman filter, taking the objects that were matched by the test described above as observation, provides the estimation of the new robot position at the instant  $(k + 1)$ ,

$$X_{(k+1)/(k+1)} = X_{(k+1)/k} + K_{(k+1)} E'_{(k+1)}$$

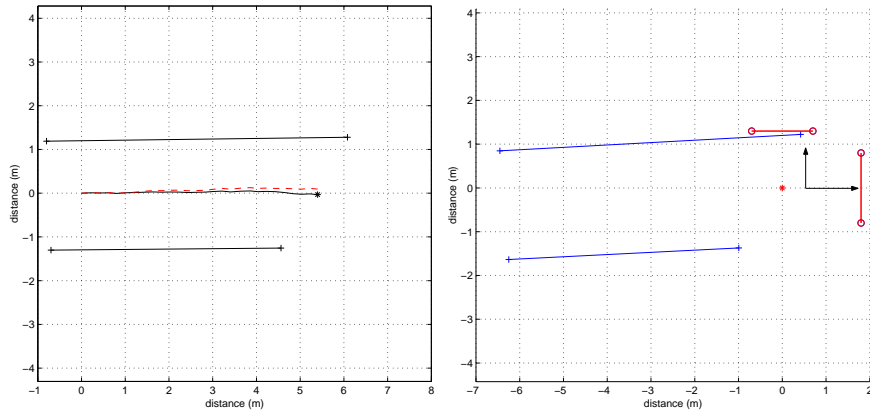
the gain  $K_{(k+1)}$  is given by

$$K_{(k+1)} = P_{(k+1)/k} \frac{\partial G}{\partial X_{(k+1)/k}}^T S_{(k+1)/k}^{-1}$$

and the new estimation covariance is given by,

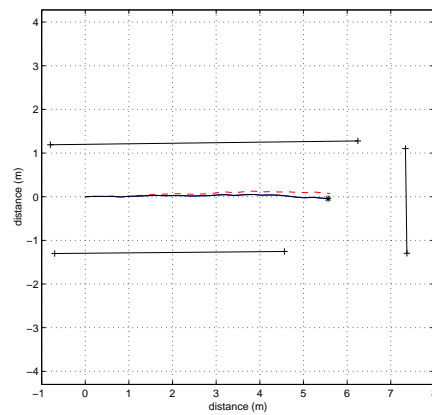
$$P_{(k+1)/(k+1)} = (I - K_{(k+1)} \frac{\partial G}{\partial X_{(k+1)/k}}) P_{(k+1)/k}.$$

The global map that represents the environment is updated with the fusion of the matched couples and/or by adding the objects that were observed by the first time, considered as new objects. The fusion of the segments already observed is done considering the end-points in a least-square estimation. The parameters of the resulted new line segment and its covariance matrix are added to the global map replacing the segment that was combined.



(a) Global map in  $k$  instant

(b) Local map in  $(k + 1)$  instant



(c) Global map in  $(k + 1)$  instant

Figure 5: Fusion of line segments.

## 4 Results

The main results reached with this methodology are:

- an occupancy grid based on odometric data corrected by an external observation;
- a geometric map as precise as possible considering the uncertainties around the sonar sensors lectures, hough transform and raw odometric data from the encoders.
- a robust and general procedure to build and maintain large metric maps, geometric or grid based.

The Figure 5(c) shows how the line fusion at a  $(k + 1)$  instant is obtained from the map at instant  $k$ , Fig. 5(a). Figure 5(b) shows projected segments from global map  $k$  to  $(k + 1)$  instant based only in the raw odometric data (blue line) and the actual sonar observation (red line). The odometric trajectory is drawn in dashed red lines and the estimated is drawn in solid black line.

The Figure 6 shows the global map in a simulated environment. The introduced errors are correct using the local observations by sonar sensors.

## 5 Conclusions

The work presented in this paper is applied to Simultaneous Localization and Map Building using a mobile robot equipped with a sonar sensors set.

The mapping methodology proposed is based on a hybrid structure using a occupancy grid and a geometric map, both



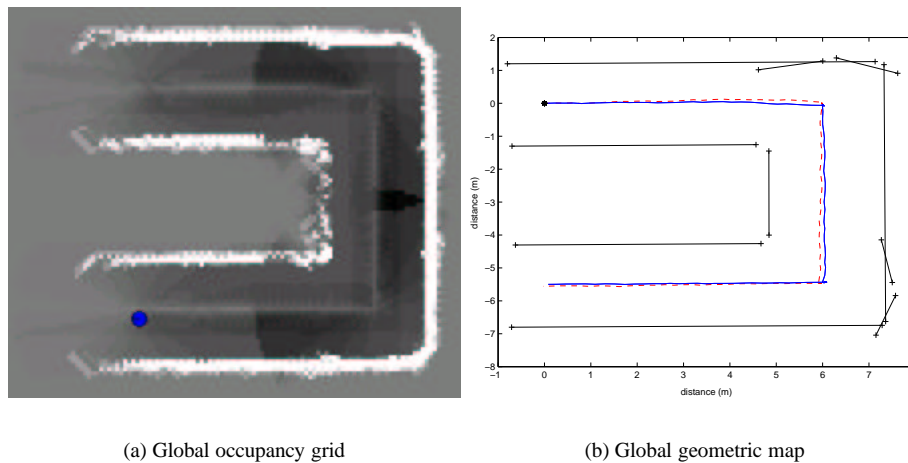


Figure 6: The resulting maps.

built simultaneously to the robot evolution. A new approach is presented when the local map is acquired as an occupancy grid and then converted to a geometric model, that is then applied to the odometric model. The geometric model is used to improve the robot localization process, combining the environment observation and odometric values with an extended Kalman filter.

The methodology must be improved to be used on a non structured, but indoor, environment, where hallways and walls could not be represented by line segments.

The experiments have been conducted in a Nomad 200 robot, available at Renato Archer Research Center, equipped with a belt of sixteen sonar sensors. A future work to do is apply this methodology to a real world application where precise navigation and map building are necessary.

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