PARALLEL CFD APPLICATION: NATURAL CONVECTION IN A SQUARE CLOSE CAVITY

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Abstract. At the present work, natural convection of liquid metal inside a closed cavity is examined, based on the Boussinesq approximation. The upper and lower cavity walls are adiabatic, while the vertical walls are at uniform and different temperatures. The problem was numerically solved by the finite volume method, based on the power-law-interpolating scheme. The pressure-velocity coupling was solved with the algorithm SIMPLEC. The solution was validated comparing by with some experimental and numerical results available in the literature. The software was implemented using Fortran77 with message passing on twenty PIII processors Linux cluster. The results show that the introduction of parallelism does not affect the code precision and provides a significant reduction in the processing time.

Keywords*. Natural convection, parallel cfd.*

1. Introduction

Natural convection study in closed cavities is of a great concern due, in part, to the knowledge of the importance of this process in some industrial applications, as plate solar collectors, nuclear reactors design, purification of metals, crystal growth, metal solidification and melting, among others. Extensive review on natural convection in cavities had been made by Ostrach (1972) and Catton et. al (1983). Experiments using liquid metals had been led by Viskanta et. al (1986), Wolff et. al (1988) and Wang et. al (1999). Numerical results had been presented by Mohammad and Viskanta (1991), Rocha and Nieckele (1997), Arcidiacono et. al (2001), and many others.

There are many published works about natural convection in closed cavities, but, in their majority, the computational meshes are relatively coarse. The mesh refinement implies in the use of high performance computing in order to avoid a long time simulation. The necessary performance can be reached by three basic types of high performance computer architectures, Hwang (1998):

1. Vector architecture computer. The machines have parallel resources witch allow several operations, in general floating-point evaluations, to be executed simultaneously in just a machine instruction time.
2. SMP computers or Parallel Symmetric Multi-Processor. In this category of machines, several processors of the same computer are physically linked and they have access to shared memory. The processors use the memory to communicate.
3 Multi-computers. Several independent computers linked by an interconnection network compose these machines. These computers perform the processing nodes of the parallel machine, while the interconnection network allows the communications between the nodes. An important characteristic of this model is the fact that the nodes can work isolated like normal workstation.

This paper shows, with the help of a classic literature problem, that the introduction of parallelism greatly reduces the processing time without changing the result accuracy.

2. Physical and mathematical model

A Newtonian fluid flow in a two-dimensional closed cavity under natural convection effect is considered. The flow is laminar and steady state. The vertical walls are kept at uniform, constant, temperatures. The horizontal walls are perfectly insulated. A cavity diagram is shown in Fig. (1).

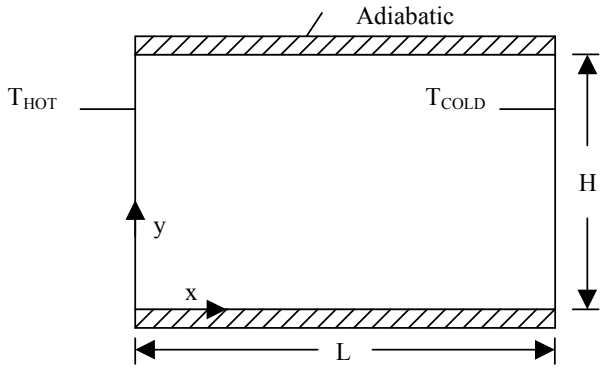

Figure 1. Coordinate axis, geometry and boundary conditions.

The fluid is assumed to have constant properties except insofar as the buoyancy is concerned, i.e. the Boussinesq approximation of linear temperature dependence of density is utilized, as defined in Eq. (1).

$$\rho(T) = \rho_c[1 - \beta(T - T_c)] \tag{1}$$

where ρ is the specific mass, β the coefficient of volumetric expansion and T the temperature. Subscript C is referring to the cold wall temperature, T_{COLD}.

To determine the velocity field and temperature, the equations of mass conservation, linear momentum and energy are solved with the pertinent boundary conditions.

Mass conservation.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2}$$

Component of linear momentum in x direction.

$$\frac{\partial(\rho u u)}{\partial x} + \frac{\partial(\rho v u)}{\partial y} = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial x}\left(\mu \frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right) \tag{3}$$

Component of linear momentum in y direction.

$$\frac{\partial(\rho u v)}{\partial x} + \frac{\partial(\rho v v)}{\partial y} = -\frac{\partial p}{\partial y} + \frac{\partial}{\partial x}\left(\mu \frac{\partial v}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mu \frac{\partial v}{\partial y}\right) - \rho_c \, g[1 - \beta(T - T_c)] \tag{4}$$

Energy conservation.

$$\frac{\partial(\rho u T)}{\partial x} + \frac{\partial(\rho v T)}{\partial y} = \frac{\partial}{\partial x}\left(\frac{k}{c_p}\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{k}{c_p}\frac{\partial T}{\partial y}\right) \tag{5}$$

The boundary conditions are given by:

$$u = v = 0; \qquad T = T_H; \qquad x = 0; \qquad 0 \le y \le H \tag{6}$$

$$u = v = 0; \qquad T = T_C; \qquad x = L; \qquad 0 \le y \le H \tag{7}$$

$$u = v = 0; \qquad \partial T/\partial y = 0; \qquad y = 0; \qquad 0 \le x \le L \tag{8}$$

$$u = v = 0; \qquad \partial T/\partial y = 0; \qquad y = H; \qquad 0 \le x \le L \tag{9}$$

The independent dimensionless parameters appearing in the problem are: aspect ratio, $AR = H/L$, Rayleigh number, $Ra = g\beta\Delta TH^3/\alpha\nu$ and Prandtl number, $Pr = \nu/\alpha$.

3. Numerical Method

Equations (2) to (9) have been discretized on a staggered grid by using a finite volume method (Patankar, 1980) and the interpolation method used is the Power Law (Patankar, 1980). The coupling pressure-velocity is solved by SIMPLEC algorithm (Van Doormaan and Raithby, 1984). The dependent variable in the momentum equation is the contra-variant velocity component (Pires and Nieckele, 1994), which is stored staggered of the others variables to prevent oscillations in the solution. The Newton-Raphson algorithm is used to solve the resultant equations system. All conservation equations are converged when the normalized residue are lesser or equal than $1x10^{-6}$. Uniform meshes are select in the present work.

A multi-computer is a parallel machine classified as MIMD (Flynn, 1966). These machines are also called multiprocessor because they have more then one processor. In this computer architecture each processor possesses a private memory. There is a communication network to support the multiprocessor communications. The programming model adopted is the message passing (Hwang 993), and implemented with PVM (Parallel Virtual Machine), Geist et. al (1994). This is an asynchronous parallel programming model communication that allows the data, process synchronism and interrupts signals to be exchanged between the processors, as show in Fig (2).

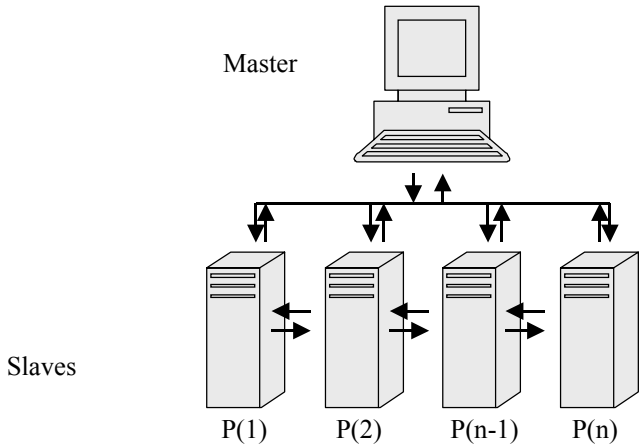

Figure 2. Virtual Parallel Machine.

The parallel algorithm is implemented as process farm Master-Slave (Hwang 1998) and it has a sequential phase and another parallel phase. The master sends the data to be processed to the slaves. After receiving the partial results from the each slave the final result is computed by the master.

During the sequential phase all the slaves executes the same task in the solution domain, Fig. (3a). In the parallel phase each slave in virtual machine executes its parcel of the processing. However, to avoid the discontinuity in the solution is necessary to create regions of interference between the slaves. This interference region is created by exchanging information of the limit columns between adjacent slaves, as show in Fig. (3b).

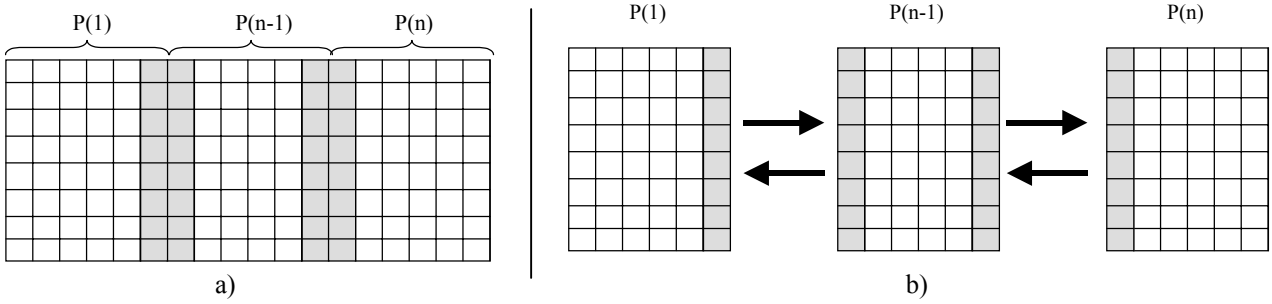

Figure 3 a) Computational domain. b) Domain of each slave showing the exchange of information in the interference region.

In this work the parallel machine is built by a set of 24 Pentium III 935 MHz computers with 128 Mb of RAM and 10 Gb of hard disk. The network is the Fast Ethernet 10/100 (IEEE 802.3u) switched. The operational system is the

LINUX, (Mandrake 8,1 kernel version 2.96-0.62), the library of message passing is the PVM 3.4.3 and the compiler is the GNU Fortran77 (0.5.2.6), all distributed freely.

4. Results

To validate the parallel algorithm the laminar natural convection in a closed cavity problem is solved. The dimensionless parameters are: $Pr = 2.08x10^{-2}$, $Ra = 1.06x10^{6}$ and $AR = 1.0$. Initially a test mesh is performed. Figure (4) presents the global Nusselt number estimations, defined as Eq. (10), with the meshes are refined.

$$\overline{Nu} = \int_0^1 -\frac{\partial \theta}{\partial X}\bigg|_{X=0} dY \tag{10}$$

The results show that the problem's solution remains practically unchanged from the mesh of 82x81 points, with a maximum deviation of 0.6 percent on the global Nusselt number value.

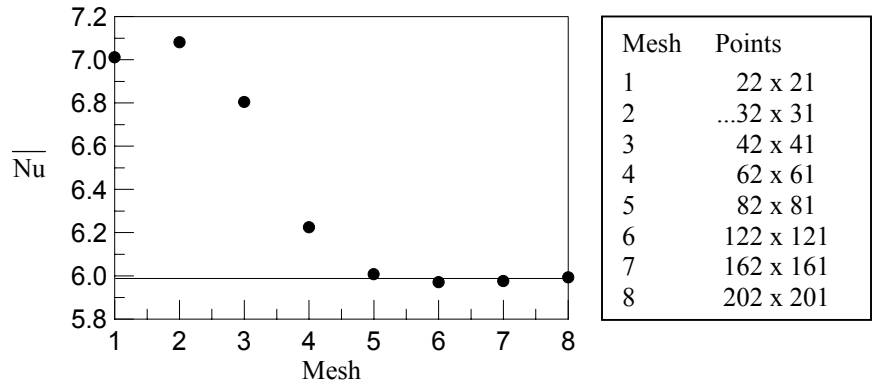

Figure 4. Mesh test - global Nusselt versus mesh refinement.

The solution is compared with the numerical results of Viskanta et. al (1986) through the global Nusselt number. In this comparison, the current paper uses a uniform 42x41 points mesh, and the global Nusselt number is $\overline{Nu} = 6.804$. Viskanta uses a 41x41 points mesh, and the global Nusselt number is $\overline{Nu} = 6.701$. The result presents 1.54 shunting line percent. Also, the numerical results are compared with the Wolff et al (1988) experimental data. The Fig. (5) shows the distribution of temperature along horizontal axe in three different positions. Good agreement between the results is observed where the greatest discrepancies are next to the center and to the top wall. However, in these regions, the numerical solution presents a qualitative correct behavior.

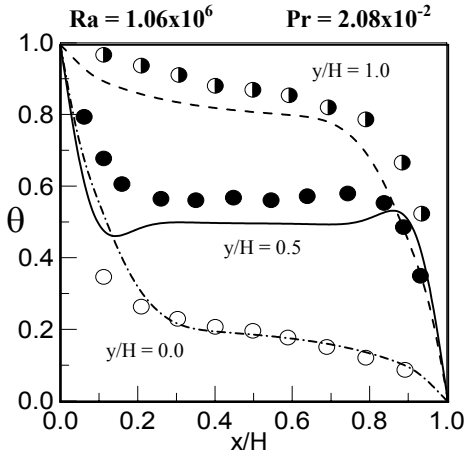

Figure 5. Comparison between numerical results and experimental data by Wolff et al (1988).

Figure (6) shows the isotherms and stream function lines. The temperature reduction in right top and left bottom is due small vortices in those places.

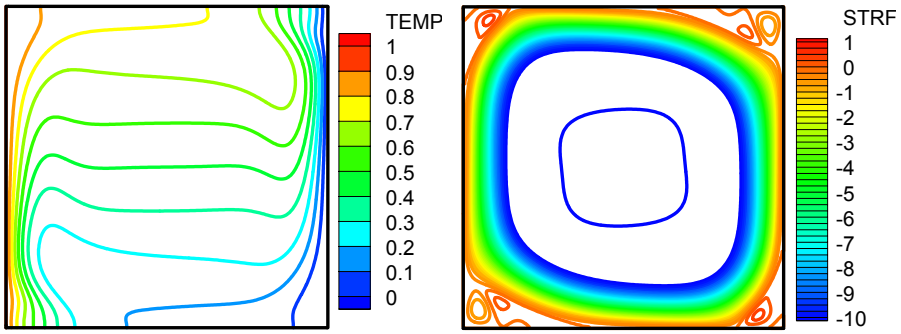

Figure 6. Isotherms and stream function.

In attainment of the numerical results, shown previously, the gradual mesh refinement results on a computational cost by raising the time. This rise of the computational time is shown in Fig. (7) and it discloses highly not linear. For coarse meshes, until 82x81 nodal points, the reply time goes approximately of some minutes until 2.5 hours. Refining further the mesh, the processing time grows and reaches 67 hours for 202x201 nodal points mesh.

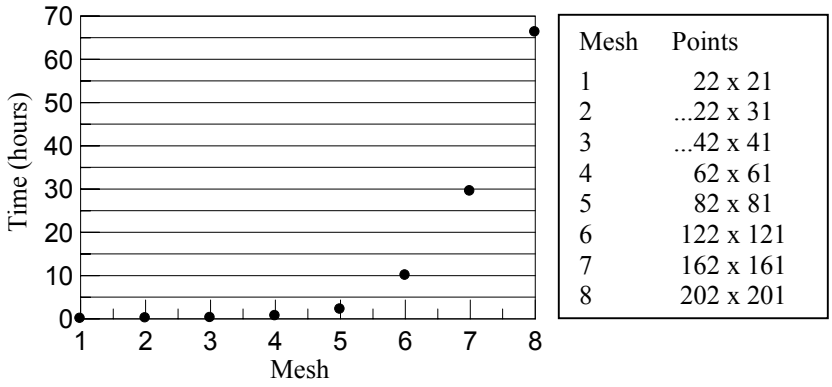

Figure 7. Execution time in function of mesh.

When parallel processing is applied successfully to a problem, the execution time decrease. The measure which relates this time reduction with the number of processors is called speedup and, is defined by the ratio between the execution time with one processor and the execution time with more than one processor. Speedup less then unit means that the processing time increases with the increase of the processors.

Figure (8) shows the speedup variation with the number of processors. For very small meshes, until 42x41, the increase of the number of processors makes the execution time to go up. It occurs because the communication time became bigger than the waste time in each unit processor. For the 62x61 mesh there is a small speedup that quickly converges to a same value resulted by just one processor. In this case, it can be affirmed that the use of parallelism is not justifiable. The 82x81 mesh already presents a reasonable improvement in the introduction of parallelism. Note that the execution time goes down approximately twice when 4 processors are used in the cluster. It can be notice that the gain achieved with more than 4 processors do not result in equivalent decreasing of processing time. The meshes 122x121, 162x161 and 202x201 presents good values of speedups until the inclusion of eight processors. The processing time decreased by a factor of 4. Beyond the 122x121 mesh the results presents stagnation in speedup, when the mesh reaches 162x161 a small improvement of 4.5 can be noted, at the end the mesh reaches 202x201 and the speedup hits a value little greater than 5. This is a maximum value achieved in this work.

In all curves reduction of the efficiency is observed. The efficiency is represented by curve inclination. The measure shows that the efficiency decreases when the number of processors increases. The explanation for this is that when increasing the number of processors occurs a work reduction that each unit processor must be executes. If the volume of work of each processor gradually goes down the processing time goes down too. When the processing time becomes too small the communication time becomes the most important whole part.

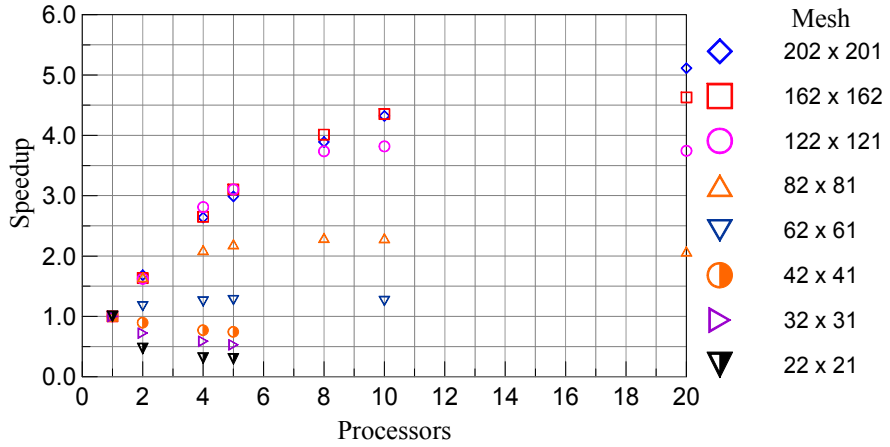

Figure 8. Speedup versus number of processors.

Table (1) presents the processing time hours. In this experiment the maximum value of speedup is not so high how as those found by Tanenbaum (1999). However, this value becomes relevant when the time of processing is very high with one processor in the cluster. Also, this table helps in the virtual machine management.

Table 1. Processing time variation (in hours) with mesh and number of processors.

	22x21	32x31	42x41	62x61	82x81	122x121	162x161	202x201
1	0,03	0,09	0,19	0,67	2,18	10,01	29,48	66,26
5	0,12	0,18	0,25	0,52	0,99	3,23	9,50	22,18
10	---	---	---	0,53	0,95	2,62	6,77	15,33
20	---	---	---	---	1,05	2,67	6,37	12,96

Finally is analyzed the solution accuracy when the parallelism is used. Table (2) presents the global Nusselt number relation, Eq. (10), for the several meshes and choices in the number of processors in cluster. The result shows that independent of the number of processors the problem‘s result is always the same, conferring to the code the necessary precision.

Table 2. Global Nusselt number variation with mesh and processors number.

	22x21	32x31	42x41	62x61	82x81	122x121	162x161	202x201
1	7,0111	7,0804	6,8038	6,2247	6,0078	5,9703	5,9760	5,9923
2	7,0111	7,0804	6,8038	6,2247	6,0078	5,9703	5,9760	5,9923
5	7,0111	7,0804	6,8038	6,2247	6,0078	5,9703	5,9760	5,9923
10	7,0111	7,0804	6,8038	6,2247	6,0078	5,9703	5,9760	5,9923
20	7,0111	7,0804	6,8038	6,2247	6,0078	5,9703	5,9760	5,9923

5. Conclusions

The result shows that the use of the cluster with the message-passing interface, PVM, was successfully applied to the closed cavity’s problem. The numerical results were compare to experimental data and with another numerical result getting good agreement. The parallel processing made possible to reduce the computing time from 66 to about 13 hours. The results show that the introduction of parallelism does not affect the precision of the code and provides a significant reduction in the processing time. Comparing the time processing achieved with different number of processors and the different mesh sizes is possible to scale the parallel machine in order to improve the best efficiency. In Tab (1) the best efficiency for 202x201 mesh occurs to 10 processors. When 20 processors are applied, the computation time is reduced to 2.3 hours. The decision in use 10 or 20 processors depends on the availability or not of the computational resources.

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